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It follows from common sense that a site suitable for the installation of wind turbines should be 'windy'. However, the windiness of a site needs to be specified in quantitative terms. In order to do so, one needs to first obtain data on wind speeds and directions by installing measuring instruments at potential sites. Since wind speed and direction vary continuously, estimation of the power generation potential requires some statistical analysis. Finally, one has to obtain a proper match between the characteristics of the wind turbine and those of the site. We will take up these issues in this chapter.

# 2.1 Wind Speed Measurements

The device used for wind speed measurement is called an *anemometer*. There are three different techniques for wind speed measurement. In general, any measurable phenomenon that has strong dependence on wind velocity can be used for wind speed measurement. Experience has shown that thrust, pressure, and the cooling effect, are the three most convenient parameters using which wind speed can be directly measured.

### 2.1.1 Robinson Cup Anemometer

The Robinson cup anemometer consists of a vertical shaft carrying three or four horizontal arms, at the ends of which there are hemispherical cups of thin sheet metal. The circular rims of the cups are in vertical planes passing through the common axis of rotation. The thrust of wind is greater on the concave sides than on the convex ones, thereby leading to the rotation of the vertical shaft (Fig. 2.1).



**Fig. 2.1** The Robinson cup anemometer

As this is a vertical-axis device, there is no problem of orientation along the wind direction. The wind velocity has a linear relationship with the speed of rotation, which is measured by a photocell-operated digital counter. The display can be precalibrated to give the wind speed directly. Modern devices have facilities for continuous data logging and storage, from which data can be retrieved later for analysis.

At very low wind speeds, the readings of the cup anemometer can be erroneous due to the friction of the bearings. During fast variations of wind speed, the inertia effect may be significant; e.g., when the wind speed drops quickly, the anemometer tends to rotate faster and takes time to slow down. In spite of these minor drawbacks, the Robinson cup anemometer is the most extensively used instrument for wind speed measurement.

# 2.1.2 Pressure Tube Anemometer

The pressure tube anemometer a simple mechanical device suitable for stand-alone application in remote windy locations.

Structure-wise, it has two distinct parts. The head, which is usually mounted on a mast at the desired height, consists of a horizontal tube, bent at one end and supported by two concentric vertical tubes. The horizontal end is connected to the inner tube. At the other side of the outer tube, there are a few holes a little below the horizontal tube. The entire head is free to rotate, which is turned to face the wind by a vane.



Fig. 2.2 The pressure tube anemometer

The wind blowing into the horizontal tube creates pressure, which is communicated through a flexible tube to a recorder. Again, the wind blowing over the small holes in the concentric tube creates a suction effect, which is also communicated to the recorder through a second flexible tube.

In the recording apparatus, a copper vessel, closed at one end, floats inverted in a cylindrical metal container partly filled with water and sealed from the outside air. The wind pressure from the horizontal tube of the head is transmitted to the space inside the float, causing it to rise as wind blows. This is assisted by the suction that is applied to the space above the float.

Thus, as the wind speed rises and falls, the float also rises and falls, and its motion is transferred to a pen tracing a record on

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a sheet of paper by means of a rod passing through an airtight passage at the top of the cylinder. If the paper movement is spring operated, the device does not need any electrical supply.

The float can be so shaped, in accordance with the law relating pressure to wind velocity, that the velocity scale on the chart is linear. Most pressure tube anemometers also have wind direction recorders taking signals from the tail-vane, so that both the speed and direction of wind are recorded.

# 2.1.3 Hot Wire Anemometer

A hot wire anemometer uses the cooling effect of wind on an electrically heated platinum or tungsten wire to measure wind velocities. The wire is heated by a constant-current source. With the variation of wind speed, the wire temperature varies, which varies the resistance of the wire. Naturally, in order to find the wind speed, it suffices to measure the resistance of the wire using any standard method. The calibration has to take into account the resistance–temperature characteristics of the wire and the ambient temperature of air.

In a hot wire anemometer, the temperature difference between the wire and the ambient air is inversely proportional to the square root of the wind velocity:

$$T_w - T_a \propto \frac{1}{\sqrt{v}} \tag{2.1}$$

Because of this relation, this anemometer is useful especially for measuring small wind velocities.

# 2.2 Wind Speed Statistics

Since the power contained in wind varies with the cube of the wind speed, the average wind speed available at a particular site is the first criterion to be considered in site selection. During the site identification process, the measuring instruments described in the previous section are installed at the site. The annual average wind speed is calculated according to the equation

$$\overline{v} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} v \, dt \tag{2.2}$$

where  $\overline{v}$  is the annual average wind speed (m/s), v is the instantaneous wind speed (m/s), and  $t_2 - t_1$  is the duration of one year (8760 hrs).

In the case of a digital data logger recording wind speed data at regular intervals, the average wind speed can be calculated as

$$\overline{v} = \frac{1}{n} \sum_{i=1}^{n} v_i$$

where  $v_i$  is the wind speed at the *i*th observation and *n* is the number of observations.

At any given site, the wind speed varies with the height from the ground level. It is generally not possible to install measuring instruments at all heights, but an empirical formula can be used to find the mean wind speed at a certain height using the observed mean wind speed at 10 m:

$$\overline{v}_H = \overline{v}_{10} \left(\frac{H}{10}\right)^x \tag{2.3}$$

where  $\overline{v}_H$  is the annual average wind speed at height H (m/s),  $\overline{v}_{10}$  is the annual average wind speed at 10 m (m/s), and x is an exponent that depends on the roughness of the ground. The values of x are given in Table 2.1.

The measuring instruments record the wind speed continuously against time. If the data are collected throughout a year, the

**Table 2.1**Values of the boundary layer exponent x for varying ground<br/>roughness

Description of land	Exponent $x$
Smooth land with very few obstacles (e.g., sea, coast, desert, snow)	0.10-0.13
Moderately rough, (e.g., agricultural fields with very few trees, grasslands, rural areas)	0.13-0.20
Land with uniformly distributed obstacles 10–15 m high (e.g., forests, small towns, agricultural fields with tree plantations)	0.20-0.28
Land with big and non-uniform obstacles (e.g., big cities, plateaus)	0.28 - 0.40

resulting chart would look like a long wavy line, as shown in Fig. 2.3.



Fig. 2.3 Typical anemometer recording of wind speed versus time

It is clear that this plot is too wavy and irregular for us to obtain any useful information from it. The next step is to obtain, from Fig. 2.3, the plot of wind speed v versus the total time during a year for which the wind speed is v (Fig. 2.4), called the *wind speed distribution curve*. Of course, the period of time for which the wind speed assumes an exact value is infinitely small. So, the vertical axis actually gives the annual duration for which the wind speed falls within certain limits, for instance 0.5 m/s below and above v. The y-axis of the wind speed distribution curve should be given in hours per annum per metres per second. Thus the integral of the function (or the area under the curve) will always be 8760 hrs, corresponding to the number of hours in a year.



**Fig. 2.4** Plot with wind speed on the *x*-axis and the duration in a year for which wind assumes that speed on the *y*-axis

It is easy to plot the energy distribution curve for a site—the energy available at a particular wind speed is the power contained in the wind (proportional to  $v^3$ , the value of the proportionality constant is unimportant for our purpose) multiplied by the duration for which wind blows at that speed. This curve, shown in Fig. 2.5, gives the value of the wind speed at which the maximum energy is available—it is the wind speed at which the wind turbine should normally be rated. Note that the wind speed for maximum energy is different from and higher than the most frequent wind speed.



Fig. 2.5 The energy distribution curve

#### 2.2.1 Statistical Wind Speed Distributions

In certain cases, the total data of wind speeds against time over a year may not be available, but the yearly average wind speed may be known. In such cases, the wind speed distribution curves can be obtained approximately from the magnitude of the average wind speed, by using a standard statistical distribution function, such as the Rayleigh distribution function. It is observed that the wind speed distributions of different sites have certain similarities and can be approximated by the Rayleigh distribution function.

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The distribution function is given by

$$t = 8760 \frac{\pi}{2} \frac{v}{\overline{v}^2} \exp\left(\frac{-\pi v^2}{4\overline{v}^2}\right)$$
(2.4)

where t is the time (hours per year), v is the wind speed (m/s), and  $\overline{v}$  is the annual average wind speed (m/s). Equation (2.4) predicts the total number of hours per year for which wind will blow at speed v at a site with mean wind speed  $\overline{v}$ . It can be shown analytically that for a Rayleigh distribution, the most frequent wind speed occurs at  $v_{\rm mf} = 0.8\overline{v}$  and the maximum energy is available at 1.6  $\overline{v}$ . These relations give a very quick method of finding the wind speed at which the maximum energy is available, that is, the speed at which a wind turbine should be rated. It should be noted, however, that the Rayleigh distribution becomes inappropriate at wind speeds below 10 mph, and therefore should not be used for sites where the mean annual wind speed is below 10 mph.

A more general distribution function is required to obtain a better approximation for wind speed distribution on a daily or a still shorter time scale. In such cases, one may apply the Weibull distribution, given by

$$f(v) = \left(\frac{k}{c}\right) \left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right]$$
(2.5)

where c is a scale factor often taken equal to the mean wind speed calculated at hub height, k is the shape factor, ranging between 1.8 and 2.3, and

$$f(v) = \frac{\text{fraction of time the wind speed is between } v \text{ and } (v + \Delta v)}{\Delta v}$$

The curves calculated using Eqn (2.5) with k = 1.8 and k = 2.3 are shown in Fig. 2.6(a). The value of k is chosen to fit the actual curve in the best way. It may be noted that for k = 2, the Weibull distribution reduces to the Rayleigh distribution when converted into the proper units.

The dependence of the distribution function on the choice of the scale factor c is shown in Fig. 2.6(b). For greater values of c, the curve shifts to the right, to higher wind speeds, which implies that high wind speeds are experienced for a greater number of

days. As stated earlier, a good choice of c for a particular site is the annual average wind speed  $\overline{v}$ .



**Fig. 2.6** The Weibull distribution for (a) k = 1.8 and 2.3, with c set at mean wind speed  $\overline{v} = 10$  kmph, and (b) c = 10 and 20 kmph, with k set at 2

We can get an idea about the energy potential of a site from the root mean cube (rmc) speed, which is given by

$$V_{\rm rmc} = \left(\frac{1}{8760} \int_0^\infty f(v) v^3 dv\right)^{1/3}$$
(2.6)

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In terms of discrete observations of anemometer readings, the rmc wind speed can be calculated using the formula

$$\overline{v} = \left(\frac{\sum_{j=1}^{N} V_j^3}{N}\right)^{1/3} \tag{2.7}$$

where  $V_j$  is the wind speed at the *j*th observation and N is the number of wind speed observations. The value of  $V_{\rm rmc}$  is very useful in estimating the annual average power of a site, given by

$$P_{\rm rmc} \approx rac{1}{4} 
ho V_{\rm rmc}^3 ~({
m W/m^2})$$

We thus obtain, either from direct measurement or by using this statistical distribution formula, the wind speed distribution curve shown in Fig. 2.4.

**Example 2.1** The annual average wind velocity at a height of 10 m over a flat terrain is 6 m/s. The boundary layer exponent is x = 0.13. Find the annual average power density (W/m<sup>2</sup>) in the wind at a height of 50 m. Assume the Rayleigh distribution as an approximation to the wind velocity–duration distribution over the terrain and 1.225 kg/m<sup>3</sup> as the density of air.

Solution The Rayleigh distribution is a special case of the Weibull function when the shape parameter k in Eqn (2.5) is equal to 2. Hence the Rayleigh distribution function is given by

$$f(v) = \frac{2v}{c^2} e^{-(v/c)^2}$$
(2.8)

The average wind speed is

$$\overline{v} = \int_0^\infty v f(v) dv \tag{2.9}$$

Substituting the expression for f(v) in Eqn (2.9) and carrying out the integration give the average wind speed

$$\overline{v} = c\Gamma(1.5) \tag{2.10}$$

where  $\Gamma(1.5)$  is the gamma function. Using tables for the gamma function used in Eqn (2.10) gives

$$c = 1.12\overline{v} \tag{2.11}$$

The annual average wind velocity at a height of 50 m is

$$\overline{v}_{50} = \overline{v}_{10} \left(\frac{50}{10}\right)^{0.13} = 7.52 \text{ m/s}$$
 (2.12)

At this wind velocity, the scale factor from Eqn (2.11) is

$$c = 1.12 \times 7.52 = 8.42 \text{ m/s} \tag{2.13}$$

The power in the wind is

$$P_w = \frac{1}{2}\rho A v_{50}^3 \,\,(\mathrm{W}) \tag{2.14}$$

The average power in the wind is given by

$$P_{\rm avg} = \frac{1}{2} \rho A \int_0^\infty v^3 f(v) dv \ (W)$$
 (2.15)

The use of Eqn (2.8) in Eqn (2.15) yields

$$P_{\rm avg} = \frac{1}{2} \rho A \int_0^\infty \frac{2v^4}{c^2} e^{-(v/c)^2} dv$$
 (2.16)

Introducing the change in variable  $x = (v/c)^2$  in Eqn (2.16) yields

$$P_{\rm avg} = \frac{1}{2}\rho Ac^3 \Gamma \left(1 + \frac{3}{2}\right) \tag{2.17}$$

Dividing Eqn (2.17) by A, consulting the table for gamma functions, and substituting the required values yield the following average power density:

$$P_{\text{avg}} = \frac{1}{2} \times 1.225 \times 8.42^3 \times 1.3$$
  
= 475.3 W/m<sup>2</sup>

# 2.3 Site and Turbine Selection

Site selection involves not only the choice of the geographical location for a wind turbine or a wind farm, but also the model of the turbine that is best suited to a particular site.

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For the final selection process, that is, while choosing the wind turbine that is best suited for a particular site, a modification of the curve shown in Fig. 2.4 is necessary. At this stage, we plot the speed-duration curve—the graph of v versus the total duration for which the wind speed exceeds or equals v (Fig. 2.7). Naturally, the largest coordinate on the y-axis is the number of hours in a year (8760), when the wind speed exceeds zero. If the wind speed is measured using a digital recorder with data logging facility, the wind speed distribution and duration curves can be obtained directly or generated by a computer later using the stored data.



**Fig. 2.7** The wind speed-duration curve: plot with wind speed along the *x*-axis and the duration for which the wind speed equals or exceeds that speed along the *y*-axis

The productivity of any wind generator at a particular site depends on the characteristics of the site (given by Fig. 2.7) and those of the wind machine. The latter are given as the power versus wind speed characteristics (such as that shown in Fig. 2.8), which are generally available for all commercially produced wind machines.

Every wind turbine model has a specific cut-in speed, a rated speed, a furling speed, and power versus wind speed characteristics within the wind speed range between the cut-in speed and the furling speed. At the cut-in speed the wind generator starts generating power. As the wind speed increases, the power output increases in proportion with the power contained in the wind. After the rated speed is reached, the speed-regulating mechanism



Fig. 2.8 Typical power versus wind speed characteristics of constant-speed wind machines

comes into action, and there is a region of constant speed. Beyond a certain wind speed, the maximum power handling capacity of the generator is reached, and thereafter the system works in the constant-power output mode. In some machines the constantspeed region is small (or negligible) and the speed-regulating mechanism works only in constant-power mode. In such cases the characteristics can be approximately expressed as

$$P(v) \approx \begin{cases} 0.5\eta_v C_p \rho A v^3 \text{ for } V_c \le v < V_r \\ 0.5\eta_v C_p \rho A V_r^3 \text{ for } V_r \le v < V_f \end{cases}$$
(2.18)

where  $V_c$  is the cut-in speed,  $V_r$  is the rated speed,  $V_f$  is the furling speed,  $\eta_v$  is the efficiency of generator and mechanical transmission,  $C_p$  is the wind turbine coefficient of performance,  $\rho$  is the density of air, A is the blade swept area, and v is the wind speed. At the furling wind speed, the plant is shut down to avoid damage.

From Figs 2.8 and 2.7, the wind generator's characteristics, as weighted by the site's wind speed–duration curve, yield the power–duration characteristics. For each value of wind speed shown in Fig. 2.7, the corresponding value of the output power is obtained from Fig. 2.8. The typical output power–duration curve for a wind turbine is shown in Fig. 2.9. To illustrate, we have also shown the wind power–duration curve (obtained by the relation  $0.5\eta_v C_p \rho A v^3$ ), so that the energy loss due to cut-in and furling becomes clear.

The area under the output power–duration curve measures the energy output of a particular machine at a given site. By plotting





Fig. 2.9 The output power-duration characteristics of a wind generator at a given site

similar curves for different machines at a particular site, one can choose the appropriate machine. One generally chooses the model that gives the maximum output for a specific rated power at a particular site.

# 2.4 Capacity Factor

Wind power plants differ in a variety of ways from power plants that burn fuel. In spite of the downtime in a year, a coal plant can be run day and night at almost its rated capacity during any season of the year. In contrast, the wind speed varies with the time of the day and with the season. At times the wind speed may even be insufficient to drive the turbine. Consequently, a wind turbine cannot operate 24 hrs a day, 365 days a year at full power. A wind farm generally runs 65-80% of the time in a year with variation in output power. Because wind farms get paid for the total energy production, the annual energy output is a more relevant measure

for evaluating a wind turbine than its rated power at a certain speed.

The term capacity factor refers to the capability of a wind turbine to produce energy in a year. It is defined as the ratio of the actual energy output to the energy that would be produced if it operated at rated power throughout the year.

 $\label{eq:Capacity factor} \text{Capacity factor} = \frac{\text{annual energy output}}{\text{rated power} \times \text{time in a year}}$ 

Thus, the capacity factor is the ratio of the average output power, computed over a year, to the rated power.

The capacity factor is influenced by the same factors that affect the production of electricity by a wind turbine. These factors include the mean wind speeds at different hours of the day, the type and characteristics of the turbine (such as the cut-in, rated, and furling speeds), and the nature of the variation of output power between the cut-in speed and the rated speed.

The average power output from a wind turbine is obtained from the product of the power produced at each wind speed and the fraction of time for which this speed prevails, integrated over all possible wind speeds. In terms of the probability distribution f(v)of the wind speed–duration curve, the average power is

$$P_{\rm avg} = \int_0^\infty P(v)f(v)\,dv \tag{2.19}$$

When the total data of wind speeds over a year are not available but the yearly average wind speed at a particular site is known, the Weibull distribution function given by Eqn (2.5) may be used instead of f(v). The equation describing the variation in output power between the cut-in and furling speeds is given by Eqn (2.18). Substituting P(v) from Eqn (2.18) into Eqn (2.19) and dividing the result by the rated power, we get the capacity factor (CF):

$$CF = \frac{1}{V_r^3} \int_{V_c}^{V_r} v^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv$$

If a closed form expression cannot be obtained after integration, numerical integration will be required. Capacity factors of successful wind farms usually lie in the range 0.20–0.35, which is quite low compared to the capacity factors of 0.5–0.75 for fossil fuel plants.

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This chapter introduces the various techniques of wind speed measurement. Of these techniques, the Robinson cup anemometer is most popular, because the output is obtained in the form of an electrical signal. Data loggers can then be used to keep a record of the wind speeds occurring at short time intervals, which can later be loaded into a computer for further analysis.

The chapter also introduces the techniques of analysing the mass of data that is obtained from direct measurement at any given site. Statistical techniques to obtain the approximate wind speed distribution curve at any site are presented.

A major problem faced by wind system planners is to choose a wind turbine that will be suit the specific characteristics of a particular site. The technique to achieve this matching has also been introduced.

# **Problems**

Refer to Fig. 2.10 for solving the following problems.

- 1. Replot the wind speed distribution curve for Chandipur with the x-axis expressed in m/s and the y-axis expressed in hours per annum per m/s.
- 2. Plot the energy distribution curve for the four sites and obtain the wind speeds at which the maximum amount of energy is available.
- 3. For the wind speed distribution curve for Chandipur, obtain the annual average wind speed. Draw the energy distribution curve using the Rayleigh distribution. Compare it with the actual distribution for the site.
- 4. Estimate the parameters k and c of the Weibull distribution function for Puri. Plot the curve and compare it with the actual wind speed distribution for the site.
- 5. Assuming a boundary layer exponent of 0.14, obtain the wind speed distribution at a hub height of 18 m for the site of Gopalpur.

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Fig. 2.10 The measured wind speed distribution curves for four sites in the state of Orissa, at a height of 10 m (Mani 1990)

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- 6. Draw the wind speed duration curves for the four sites.
- 7. Table 2.2 gives the wind data for a site in terms of the percentage of time over a year for different speed groups. Calculate the annual average power in the wind passing normally through the swept area of a turbine of diameter 30 m. Take  $1.225 \text{ kg/m}^3$  as the density of air.

Table 2.2

Speed group (m/s	$\begin{array}{c} 0 < v \leq 3 \\ \mathbf{s} \end{array}$	$3 \! < \! v \! \le \! 6$	$6 \! < \! v \! \le \! 9$	$9\!<\!v\!\le\!12$	$12\!<\!v\!\le\!16$	$16\!<\!v\!\le\!20$
Percentage of time	12.36	28.25	29.37	18.96	9.31	1.67

- 8. The wind speed distribution over a year at a site is described by the two-parameter Weibull density function in which the shape parameter k is 2.6 while the scale parameter is 1.1 times the mean speed  $\overline{v}$ . If the rated wind speed to cut-in wind speed ratio is 2.5, find the value of the rated wind speed in terms of the mean wind speed  $\overline{v}$  for which the energy extracted per year will be maximum.
- 9. The specifications of a few commercially available wind turbines are given in Table 2.3. Using the power versus wind speed characteristics shown in Fig. 2.8, obtain the capacity factor and find out which windmill is most suitable for each of the four sites.
- 10. A wind turbine rated at 100 kW has a rated wind speed of 10.5 m/s, a cut-in speed of 4.5 m/s, and a furling speed of 22 m/s. The wind speed frequency distribution over a year is given by a Weibull distribution having the shape parameter k = 5 and the scale parameter c = 7 m/s. Determine the capacity factor and the yearly energy production.
- 11. For the wind frequency distribution of Problem 10, find the optimum value of the rated wind speed for which the energy production per year will be maximum.
- 12. The coefficient of performance  $C_p$  versus tip speed ratio  $\lambda$  for a fixed-speed (42 rpm) turbine of diameter 30 m is given by

$$C_p = C_{p,\max} \frac{\lambda}{\lambda_{opt}} \left(2 - \frac{\lambda}{\lambda_{opt}}\right)$$

Windmill	Rated	Cut-in wind	Rated wind	Furling wind	$\mathbf{Hub}$
	power	speed	speed	speed	$\mathbf{height}$
	(kW)	(m/s)	(m/s)	(m/s)	(m)
1	75	4.5	8.9	22.3	21.3
2	3000	6.3	11.8	24	100
3	50	5.4	9.8	19.7	18.3
4	2500	6.3	12.2	26.8	60.9
5	2000	6	13	21	80
6	100	5	13	20	18.3
7	15	4.5	9.5	25	15
8	15	4	10.3	27	18.3
9	4	3.6	12	17.8	24.4
10	0.75	6	7	14	10
11	5.5	4	14	25.3	18.7
12	3.5	3	11	20	18.7
13	2.2	4	11	12	10
14	10	5	12	12.1	14

# Table 2.3 Specifications of a few commercially available wind turbines

 $C_p$  reaches the maximum value  $C_{p,\text{max}} = 0.38$  at  $\lambda = \lambda_{\text{opt}}$  (= 6). Plot the shaft power output of the turbine at standard conditions (0° and 101.3 kPa), which yield a density of 1.293 kg/m<sup>3</sup> for dry air.