Engineering Mechanics

Contents

CHAPTER

1

Fundamentals of Mechanics

CHAPTER OBJECTIVES

After reading this chapter, the readers will be able to understand

- *the fundamentals of mechanics of rigid bodies*
- *how dimensional analysis can be useful in understanding equations of an engineering problem*
- *different laws and principles to study the conditions of equilibrium, motion, and inertial effects on a body*
- *how vector mathematics can be used to solve the problems of mechanics*

1.1 INTRODUCTION

Mechanics is the science that describes and predicts the conditions of inertia and motion of bodies due to the action of forces.

1.1.1 Mechanics and Its Classification

Depending upon the nature of the body, the transmission of forces may cause the body to deform internally or may not produce any deformation but the body may tend to move due to it. Accordingly, the subject of mechanics can be broadly classified into

(a) Mechanics of rigid bodies
(b) Mechanics of deformable bodies
(c) Mechanics of fluids

The broad classifications of mechanics are shown in Fig. 1.1. General rigid body mechanics is divided into two groups, namely *statics* and *dynamics*. In this part of study of mechanics, the body is assumed to be perfectly rigid. In mechanics of machines, we study *kinematics* and *dynamics*, which deal with rigid bodies to obtain desired motion by transmission of forces (for example, mechanism that converts reciprocatory motion into rotary motion). In practice, structures and machines are never absolutely rigid and undergo small deformations under the action of external loads. Since the deformations are very small, they do not appreciably affect the condition of equilibrium, which may be under static loading or motion of the structure under dynamic loading.

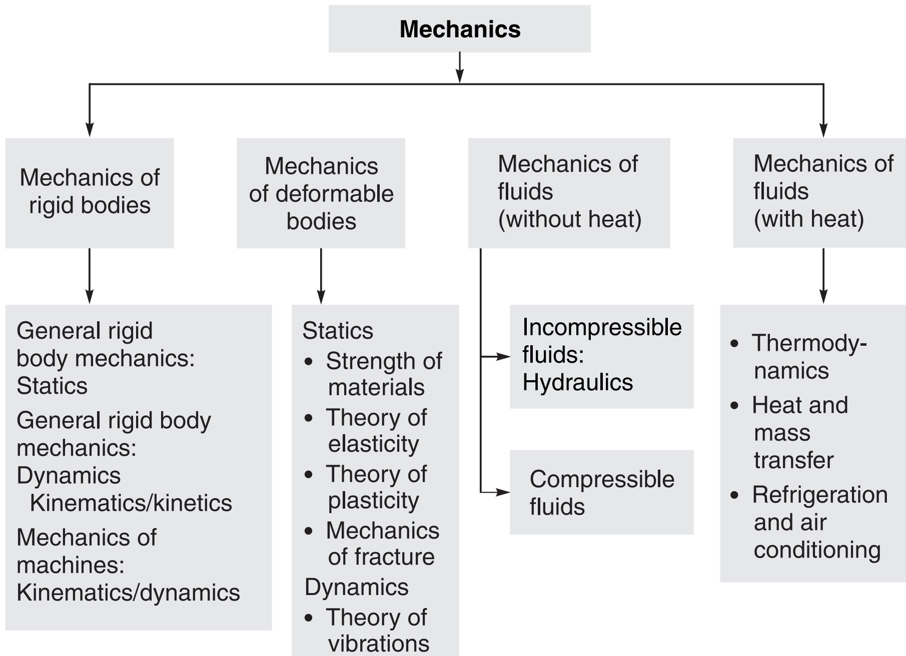

Fig. 1.1 Mechanics and its classification

As far as the resistance of a structure to failure is concerned, it is important to relate the deformation to external loading and geometry of the structure and these are being studied in a separate subject called mechanics of deformable bodies. Strength of materials and theory of elasticity deal with deformable bodies under static loading and recoverable shapes after unloading.

Theory of plasticity deals with deformable bodies under static loading and irrecoverable shapes after unloading. Mechanics of fracture deals with deformable physical bodies containing a definite size of crack and subjected to static or dynamic loading and the recoverable or irrecoverable shapes after unloading. Theory of vibrations deals with deformable bodies under dynamic loading and recoverable shapes after unloading.

Another part of mechanics is mechanics of fluids, which deals with the fluids having no heat transfer into or out of the system consisting of static fluid or fluid flow. Again, this particular subject can be subdivided into the study of incompressible fluids (hydraulics, which deals with problems involving liquids) and compressible fluids. Thermodynamics, heat and mass transfer, and refrigeration and air conditioning, etc. deal with fluids considering the effect of heat transfer into or out of the system.

1.1.2 Historical Development of Mechanics

Many researchers have contributed various concepts of rigid body mechanics to establish this particular course, which is being studied as rigid body mechanics or engineering mechanics at undergraduate level.

Among various researchers, Archimedes (287–212 BC) developed the concept of buoyancy forces. Kepler (1571–1630) established the fundamentals of astronomy for planetary motion and the principles were named after him as Kepler's laws.

Newton (1642–1727) was the first person in history to establish the laws of mechanics applicable to solids and his principles were named after him as Newton's three laws and the mechanics developed by him was called Newtonian mechanics.

Bernoulli (1667–1748) developed the principle of virtual work, which is applied in fluid mechanics as Bernoulli's equation for the total energy at any point in a fluid flow.

D'Alembert (1717–1783) established a very important principle for a dynamic system to be brought to equilibrium. His principle was named after him as D'Alembert's principle applied to a dynamic system.

1.1.3 Fundamental Concepts: Space, Time, Mass, and Force

In Newtonian mechanics, space, time, and mass are absolute quantities and independent of each other.

Space is associated with the conception of the position of a point, say *P*. Three coordinates, with reference to a particular point or the origin in three mutually perpendicular directions, may define the position of *P* and are called coordinates of *P*.

The **time** of an event in case of the dynamic condition of a point is to be defined.

Mass is used to quantify the amount of resistance that is exerted by a body while changing its state of rest or motion.

Force is defined as the ability to translate a body into action or as the action of one body on another (for example, gravitational forces, magnetic forces, and so on).

The force can be characterized with magnitude, point of application, and direction. It is a vector quantity. A force on the *xy*-plane can be represented as

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} \tag{1.1}$$

The magnitude of the resultant force F is $\sqrt{F_x^2 + F_y^2}$ and its direction with respect to the x-axis is $\theta = \tan^{-1} F_y/F_x$ as shown in Fig. 1.2.

Similarly, a force in the space can be represented with three components in the *x*-, *y*-, and *z*-directions in vector form as

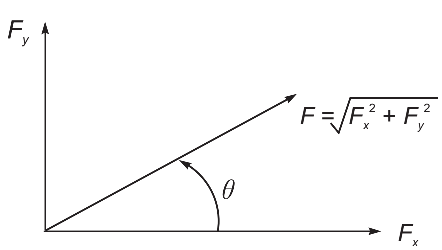

Fig. 1.2 Graphical representation of a force on the *xy*-plane

$$\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_x\boldsymbol{k} \tag{1.2}$$

The magnitude of the resultant force F is $\sqrt{F_x^2 + F_y^2 + F_z^2}$ and its directions with respect the to the *x*-, *y*-, and *z*-axes respectively are $\theta_x = \cos^{-1} F_x/F$, $\theta_y = \cos^{-1} F_y/F$, and $\theta_z = \cos^{-1} F_z/F$, as shown in Fig. 1.3, where $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are called *directional cosines* and are also represented as *l*, *m*, and *n* respectively. Mathematically, it can be shown that

$$l^2 + m^2 + n^2 = 1 \tag{1.3}$$

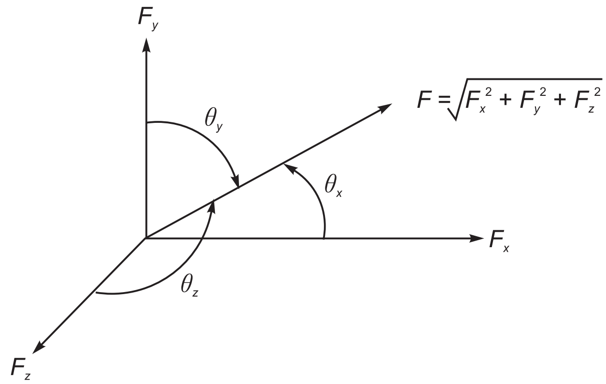

Fig. 1.3 Graphical representation of a force in space

1.1.4 Conceptualization of Rigid Body Mechanics

Rigid bodies are made up of atoms and molecules and these can be physically defined by their shape. At micro-level, the behaviour of these atoms and molecules is too complex to study and hence mass can be assumed to be continuously distributed within the body. The body's behaviour can be measured with its dimension or position with respect to certain coordinate system and time. This method of description of a body at its macro-level is called *continuum*. It can be rigid or deformable, holding its shape or continuously deforming and changing its shape depending upon the matter under study or consideration.

Rigid body mechanics

A body that does not undergo any deformation due to the action of the forces applied on it is considered to be rigid. Or, in other words, if the deformation of a body due to the external forces applied on it is negligible as compared with its dimension/shape, then the body can be considered as rigid. Hence, the system of external forces and moments due to the forces applied over the body and its support reaction keep the body in equilibrium under static condition.

A system of external forces and moments due to the forces applied over the body will be in equilibrium with its inertia forces and inertia moments respectively under dynamic condition of the body.

Generally, a system of non-concurrent forces, which may be of coplanar or non-coplanar forces (discussed later in Chapter 2, Section 2.1.1), is discussed under rigid body mechanics.

Particle mechanics

A body is idealized as a particle when the whole mass of the body is concentrated at its centroid and lines of action of a system of forces, including the support reactions applied on the body, pass through the centroid. These forces have only the force effect on the body and no moment effect. Generally, a system of concurrent forces, which may be of coplanar or non-coplanar forces, is discussed under particle mechanics.

Mechanics of statics

If in a body, the external forces, support reactions, moments due to forces, and reaction moments at supports, which are acting on the body, keep the body in equilibrium, then the body is said to be in static condition. A body moving with constant velocity is also treated as an equivalent static problem. The equilibrium conditions for various cases are discussed below.

(i) A plane body idealized as a particle and applied with a system of concurrent, coplanar forces, as shown in Fig. 1.4, should satisfy the following equilibrium conditions:

$$\sum F_x = 0 \text{ and } \sum F_y = 0 \tag{1.4}$$

(ii) A plane body idealized as a particle and applied with a system of concurrent, non-coplanar forces, as shown in Fig. 1.5, should satisfy the following equilibrium conditions:

$$\sum F_x = 0,\ \sum F_y = 0, \text{ and } \sum F_z = 0 \tag{1.5}$$

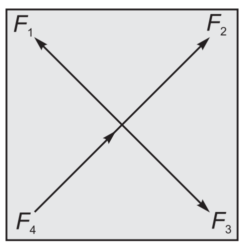

Fig. 1.4 A system of concurrent, coplanar forces

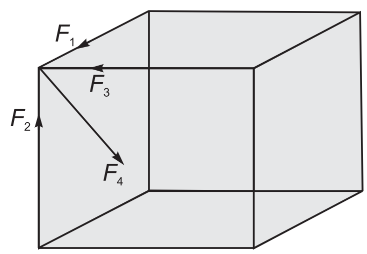

Fig. 1.5 A system of concurrent, non-coplanar forces

(iii) A rigid body idealized as a plane rigid body and applied with a system of non-concurrent, coplanar forces, as shown in Fig. 1.6, should satisfy the following equilibrium conditions:

$$\sum F_x = 0,\ \sum F_y = 0, \text{ and } \sum M_{\text{about any point}} = 0 \tag{1.6}$$

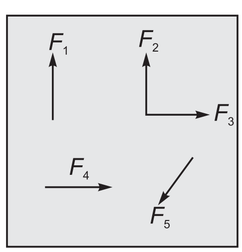

Fig. 1.6 A system of non-concurrent, coplanar forces

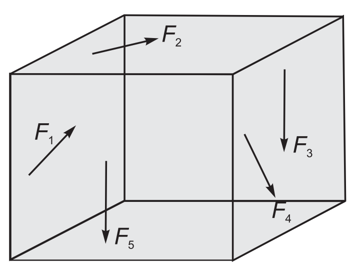

Fig. 1.7 A system of non-concurrent, non-coplanar forces

(iv) A rigid body applied with a system of non-concurrent, non-coplanar forces, as shown in Fig. 1.7, should satisfy the following equilibrium conditions:

$$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0, \Sigma M_x = 0, \Sigma M_y, \text{ and } \Sigma M_z = 0 \quad (1.7)$$

Mechanics of dynamics

In a body applied with external forces and support reactions, the moments due to forces and reaction moments at supports will cause the body to accelerate or decelerate in the direction along which it can move. However, a body getting translated or rotated or getting both translated and rotated (general plane motion) will be under equilibrium with its inertia forces and inertia moments.

Dynamics is divided into two groups— kinematics and kinetics.

Under *kinematics*, only the geometry of the motion relating to the various motion parameters, such as position, velocity, and acceleration of the body, with respect to time is studied irrespective of the cause of the motion (which is either forces or the moments due to forces).

Under *kinetics*, the geometry of the motion such as acceleration relating to the cause of the motion, which is either force or moment due to force, is studied.

A body under constant acceleration with constraints has the following cases, which can be discussed for dynamic equilibrium over the period of time.

(a) A plane body idealized as a particle under the action of concurrent, coplanar forces should satisfy the following dynamic and static equilibrium conditions for different cases.

(i) For the case when the body is free to move in the x-direction and constrained to move in the y-direction as shown in Fig. 1.8, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = ma_x \text{ and } \Sigma F_y = 0 \quad (1.8)$$

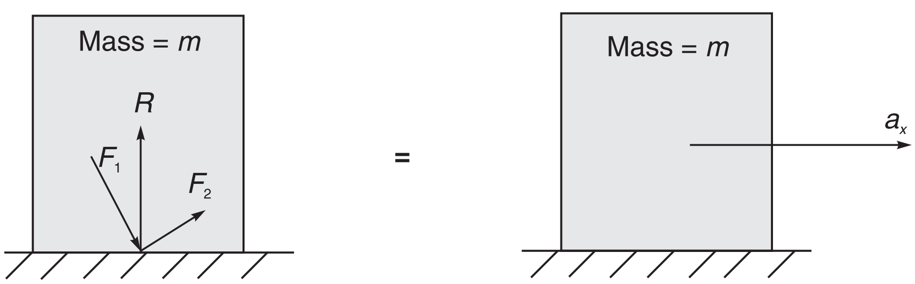

Fig. 1.8 A system of concurrent, coplanar forces causing the body to accelerate in the x-direction

(ii) For the case when the body is free to move in the y-direction and constrained to move in the x-direction as shown in Fig. 1.9, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = ma_y \quad (1.9)$$

(iii) If the body is free to move in an inclined direction and constrained to move in the direction perpendicular to the inclined plane as shown in Fig. 1.10, the equilibrium conditions can be written as

$$\Sigma F_{\text{along the motion}} = ma_{\text{along the motion}} \text{ and } \Sigma F_{\perp \text{ to the motion}} = 0 \quad (1.10)$$

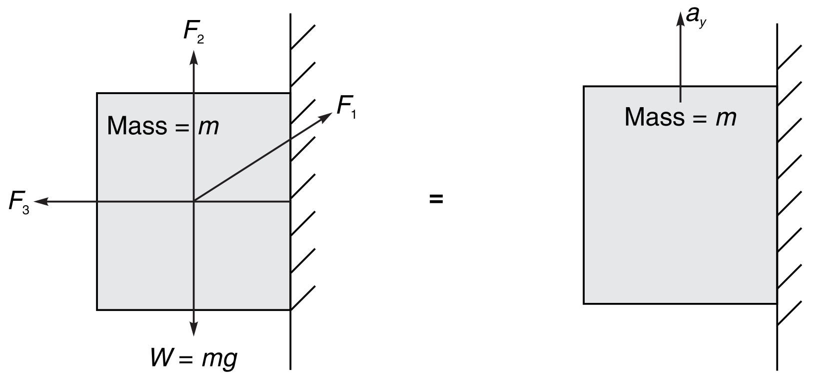

Fig. 1.9 A system of concurrent, coplanar forces causing the body to accelerate in the y-direction

(b) A body idealized as a particle under the action of concurrent, non-coplanar system of forces should satisfy the following dynamic and static equilibrium conditions for the different cases.

(i) For the case when the body is free to move in the x-direction and constrained to move in the y- and z-directions, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = ma_x,\ \sum F_y = 0,\ \text{and}\ \sum F_z = 0 \tag{1.11}$$

(ii) For the case when the body is free to move in the y-direction and constrained to move in the x- and z-directions, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = 0,\ \sum F_y = ma_y,\ \text{and}\ \sum F_z = 0 \tag{1.12}$$

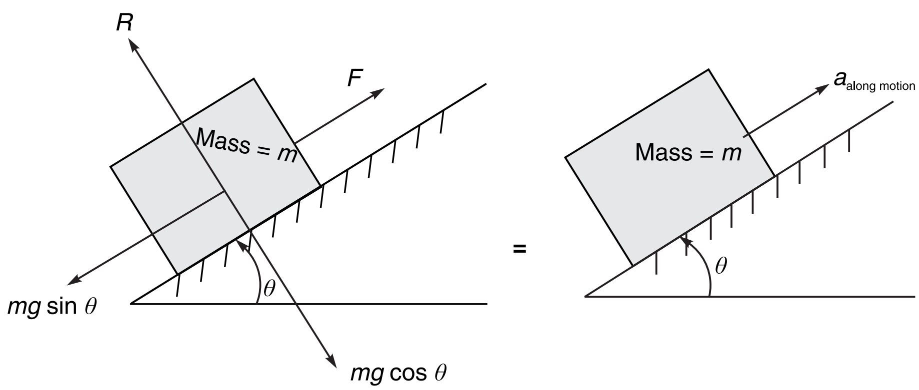

Fig. 1.10 A system of concurrent, coplanar forces causing the body to accelerate along the plane

(iii) For the case when the body is free to move in the z-direction and constrained to move in other two perpendicular directions, the dynamic and static equilibrium conditions can be written as

$$\sum F_x = 0,\ \sum F_y = 0,\ \text{and}\ \sum F_z = ma_z \tag{1.13}$$

(iv) For the case when the body is free to move in an inclined direction and constrained to move in other two perpendicular directions to the inclined plane, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_{\text{along motion}} = ma_{\text{along motion}},\ \Sigma F_{\perp\text{-I to the motion}} = 0,$$
$$\text{and } \Sigma F_{\perp\text{-II to the motion}} = 0 \tag{1.14}$$

(c) A plane body under the action of non-concurrent, coplanar system of forces should satisfy the following dynamic and static equilibrium conditions for different cases.

(i) For the case when the body is free to translate in the x-direction and constrained to move in the y-direction and constrained to rotate about the z-axis, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = ma_x,\ \Sigma F_y = 0, \text{ and } \Sigma M_{\text{about } z\text{-axis taken at any point}} = 0 \tag{1.15}$$

(ii) For the case when the body is free to translate in the x-direction and constrained to move in the y-direction and free to rotate about the z-axis, the dynamic and static equilibrium conditions can be written as

$$\Sigma F_x = ma_x,\ \Sigma F_y = 0, \text{ and } \Sigma M_{\text{about } z\text{-axis and along rotation}} = \mathrm{I}\alpha_{\text{along motion}} \tag{1.16}$$

where I is the mass moment of inertia of an axis about which the body is rotating.

Similarly, the equilibrium equations can be written for the case when the body is free to translate in the y-direction and constrained to move in the x-direction and either free to rotate or constrained to rotate about the z-axis.

(d) A solid body under the action of non-concurrent coplanar or non-coplanar system of forces should satisfy the equilibrium conditions according to the constraints. The static and dynamic equilibrium equations for the same can be written accordingly as discussed above.

1.2 UNITS AND DIMENSIONS

1.2.1 System of Units (SI Units)

In Section 1.1.3, the four fundamental concepts, namely space, time, mass, and force, were introduced. The space (which is referred in terms of length), time, and mass are called base units, whereas the unit of force is a derived one. Force is equal to the multiplication of mass and acceleration. A force of 1 N gives an acceleration of 1 m/s^2 for a body of mass of 1 kg (Fig. 1.11). Similarly, when a body of mass 1 kg falls freely, it accelerates with 9.81 m/s^2 and exerts the force of 9.81 N, which is called the weight of the body, as shown in Fig. 1.12.

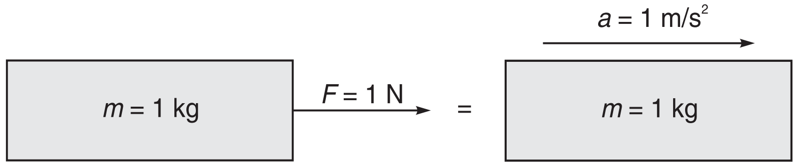

Fig. 1.11 One newton force accelerates the mass of 1 kg with 1 m/s^2

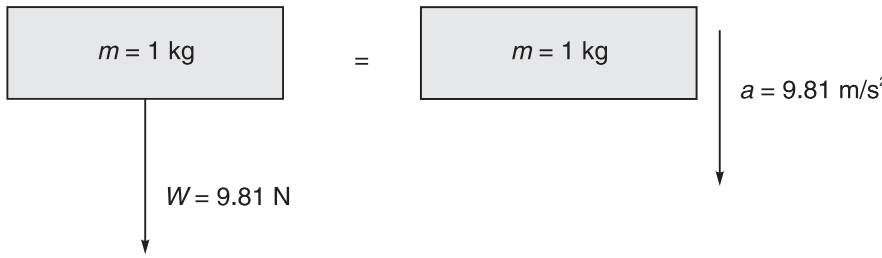

Fig. 1.12 A body of 1 kg mass accelerates with 9.81 m/s^2 and exerts a force of 9.81 N

The following four systems of units were generally followed:

(a) SI units: International system of units

(b) MKS units: Metre, kilogram, second system

(c) CGS units: Centimetre, gram, second system

(d) FPS units: Foot, pound, second system

Presently, the SI system of units is followed in many countries and hence in this textbook the same SI units are used. The important SI units used for various quantities that are used in mechanics are given in Table 1.1. In addition to the SI units for various quantities, some prefixes, which are given in Table 1.2, are also used under the SI system of units to represent very large or very small numbers. For example, Young's modulus of steel may be given as 200 GPa and it is equal to 200×10^9 N/m^2. Yield strength of steel may be given as 250 MPa, which is equal to 250×10^6 N/m^2. Speed may be given, for example, as 100 kmph, which is equal to $100 \times 1000 / 3600$ m/s.

Table 1.1 Various quantities used in mechanics and their SI units

Sl. No.	Quantity	Dimensions in terms of MLT	Unit	Representation of the unit
	Base units			
1	Length	L	Metre	m
2	Mass	M	Kilogram	kg
3	Time	T	Second	s
	Derived units			
4	Area	L^2	Square metre	m^2
5	Acceleration	LT^{-2}	Metre per square second	m/s^2
6	Angle	–	Radian	rad
7	Angular velocity	T^{-1}	Radian per second	rad/s
8	Angular acceleration	T^{-2}	Radian per square second	rad/s^2
9	Area moment of inertia	L^4	(metre)^4	m^4
10	Absolute viscosity	$ML^{-1}T^{-1}$	Newton-second per square metre or Pascal-second	N-s/m^2 or Pa-s
11	Density	ML^{-3}	Kilogram per cublic metre	kg/m^3
12	Energy/work	ML^2T^{-2}	Joule	J

Table 1.1 contd.

Table 1.1 contd.

Sl. No.	Quantity	Dimensions in terms of MLT	Unit	Representation of the unit
13	Flow rate or discharge	L^3T^{-1}	Cubic metre per second	m^3/s
14	Frequency	T^{-1}	Hertz (cycles per second)	1/s
15	Force/weight	MLT^{-2}	Newton	N
16	Impulse/momentum	MLT^{-1}	Newton-second	Ns or kg-m/s
17	Kinematic viscosity	L^2T^{-1}	Square metre per second	m^2/s
18	Mass moment of inertia	ML^2	Kilogram-Square metre	kg-m^2
19	Modulus of elasticity/ modulus of rigidity	$ML^{-1}T^{-2}$	Pascal	N/m^2 or Pa
20	Moment of force	ML^2T^{-2}	Newton-metre	N-m
21	Pressure/Stress	$ML^{-1}T^{-2}$	Pascal	N/m^2 or Pa
22	Power	ML^2T^{-3}	Watt	W or J/s
23	Specific weight	$ML^{-2}T^{-2}$	Newton per cubic metre	N/m^3
24	Surface tension	MT^{-3}	Newton per metre	N/m
25	Volume of a solid	L^3	cubic metre	m^3
26	Volume of a liquid	L^3	Litre	10^{-3} m^3 (1000 cubic centimetre)
27	Velocity	LT^{-1}	Metre per second	m/s

Table 1.2 Various prefixes used in the SI system

Sl. No.	Prefix	Value	Symbol used
1	Nano	10^{-9}	n
2	Micro	10^{-6}	μ
3	Milli	10^{-3}	m
4	Kilo	10^{3}	k
5	Mega	10^{6}	M
6	Giga	10^{9}	G

1.2.2 Conversion of One System of Units to Another

Let us take an example. The SI unit of pressure is Pa or N/m^2.

This unit can be converted into a different unit in terms of N/mm^2 as follows:

$$1\ N/m^2 = 1\ N/(1000\ mm)^2 = 1\times10^{-6}\ N/mm^2$$

or $$1\ N/mm^2 = 1\times10^{6}\ N/m^2 = 1\ MPa$$

Thus, a unit in one system can be converted into its equivalent in other system. The US customary units and their SI equivalents are given in Table 1.3.

Table 1.3 US customary FPS units and their SI equivalents

Sl. No.	Quantity	US customary unit	SI equivalent
1	Acceleration	ft/s^2	0.3048 m/s^2
		in/s^2	0.0254 m/s^2
2	Area	ft^2	0.0929 m^2
		in^2	645.2 mm^2
3	Energy	lb-ft	1.356 J

Table 1.3 contd.

Table 1.3 contd.

Sl. No.	Quantity	US customary unit	SI equivalent
4	Force	kip	4.448 kN
		lb	4.448 N
		oz	0.2780 N
5	Impulse	lb-s	4.448 N-s
6	Length	ft	0.3048 m

1.2.3 Dimensional Analysis

Dimensions of various quantities are given in Table 1.1 in terms of absolute MLT (mass, length, and time) system. There is another system called gravitational FLT (force, length, and time) system to represent various quantities. The absolute MLT system is generally followed. The variables governing the phenomena expressed in a mathematical equation give the relationship between the variables. The variables of the equation may be dimensional or non-dimensional. The qualitative description of the a variable/quantity is known as *dimension* and the quantitative description is called *unit*. For example, the weight of a body, which is equal to the product of its mass and the acceleration due to gravity, has the dimension MLT^{-2}. If the mass of the body is 1 kg, then its weight equals 1 kg × 9.81 m/s^2 = 9.81 N, which is the quantitative value in the SI system of unit.

To be correct, the governing equation should satisfy the dimensions on both sides of the equation. This condition is called *dimensional homogeneity*.

Using dimensional analysis, the dimensions of unknown variables can be determined for physical phenomenon that is expressed in mathematical equation. The homogeneous equation of a physical phenomenon can be converted into a non-dimensional form. For example, the deflection of a helical spring is expressed as

$$\delta = \frac{8FD^3n}{Gd^4} \tag{1.17}$$

where δ is the deflection, F is the force, D is the mean diameter of the coil, n is the total number of turns of the coil, d is the diameter of the wire of the helical spring, and G is the shear modulus of the spring material.

Substituting the dimensions in Eq. (1.17),

$$\mathrm{L} = \frac{\left(\mathrm{MLT^{-2}}\right)\mathrm{L^3}}{\frac{\mathrm{MLT^{-2}}}{\mathrm{L^2}}\mathrm{L^4}}$$

or
$$\mathrm{L} = \mathrm{L} \tag{1.18}$$

Example 1.1 Check the dimensional homogeneity for the equation $v^2 - u^2 = 2aS$, where S is the displacement travelled by the particle when the velocity of the particle changes from u to v with constant acceleration of a.

Solution Substituting the dimensions in the given equation,

$$(\mathrm{LT^{-1}})^2 - (\mathrm{LT^{-1}})^2 = (\mathrm{LT^{-2}})\mathrm{L}$$

or
$$\mathrm{L^2T^{-2}} - \mathrm{L^2T^{-2}} = \mathrm{L^2T^{-2}}$$

or
$$\mathrm{L^2T^{-2}} = \mathrm{L^2T^{-2}} \tag{1.19}$$

1.3 LAWS OF MECHANICS

Following are the fundamental laws/principles of mechanics:

(a) Newton's first law
(b) Newton's second law
(c) Newton's third law
(d) Lami's theorem
(e) Parallelogram law for addition of forces
(f) Triangular law of forces
(g) Principle of transmissibility
(h) Newton's law of gravitation

1.3.1 Newton's Laws

Newton's first, second, and third laws are discussed below.

Newton's first law of motion If a body has the state of rest or uniform motion, then it will continue to have the same state of condition until and unless an external force influences it.

Newton's second law of motion When a body is under acceleration or deceleration, then the rate of change of momentum of the body in the direction of the motion is equal to the algebraic sum of the forces acting along the same direction of the motion. For the case of a rigid body,

$$\Sigma F_{\text{along motion}} = \frac{d}{dt}(mV) = ma_{\text{along motion}} \tag{1.20}$$

Newton's third law of motion The action of forces and the reaction developed have the same magnitude and are opposite to each other, and they lie along the same line of action. In simple terms, for every action, there is an equal and opposite reaction. For example, when a gun is fired, the spring force on the bullet that is in contact with the surface of the barrel develops the reactive force, which kicks the shoulder.

1.3.2 Lami's Theorem

Lami's theorem states that 'if three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle between the other two forces'.

Explanation

Three concurrent, coplanar forces, which are acting at a point with their directions represented away from the point, keep the point in static equilibrium. According to Lami's theorem, for the forces as shown in Fig. 1.13, each force is proportional to the sine of the included angle between the other two forces. This is basically the trigonometric sine rule applicable to a triangle.

Mathematically, Lami's theorem can be written as

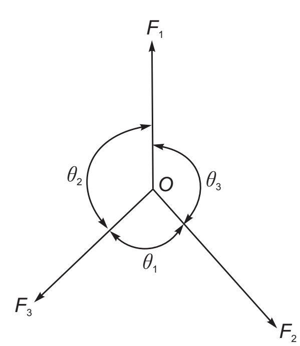

Fig. 1.13 Equilibrium of three concurrent, coplanar forces: Lami's theorem

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3} \tag{1.21}$$

1.3.3 Parallelogram Law for Addition of Forces

According to this law, if an equivalent single force, which is called resultant, can replace the two forces acting on a particle, then the resultant can be found by drawing the diagonal of the parallelogram, which has sides equal to the given forces. This is illustrated in Fig. 1.14.

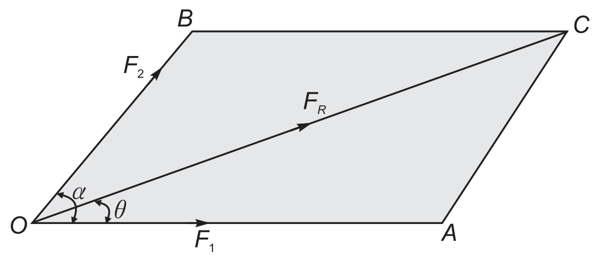

Fig. 1.14 Two forces and their resultant represented in a parallelogram

1.3.4 Triangular Law of Forces

This law states that if F_1 and F_2 are two forces acting on a particle that can be represented by the two sides of a triangle in the magnitude and direction taken one after the other, then the side that closes the triangle represents the resultant in opposite direction. This is illustrated in Fig. 1.15.

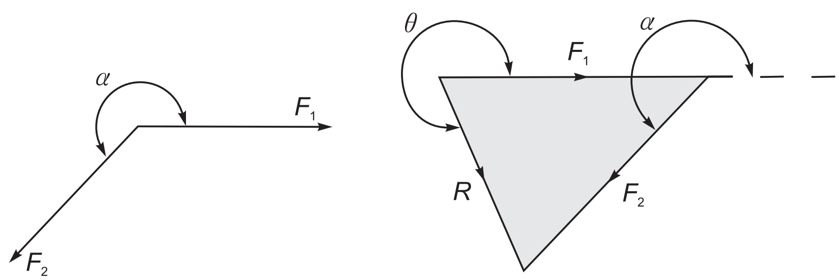

Fig. 1.15 Finding resultant of two forces by the triangular law of forces

1.3.5 Principle of Transmissibility

According to this principle, 'the condition of equilibrium or motion of a rigid body will remain unchanged if the force acting at a point on the rigid body is transmitted to another point on the same line of action in the same direction'. For example, pushing a vehicle from behind has the same effect as if it is pulled from front with the force of same magnitude and along the same line of action as shown in Fig. 1.16.

1.3.6 Newton's Law of Gravitation

It states that the gravitational force of attraction between two bodies is proportional to the product of the masses of the two bodies and inversely proportional to the square of the distance between them. This law is illustrated in Fig. 1.17.

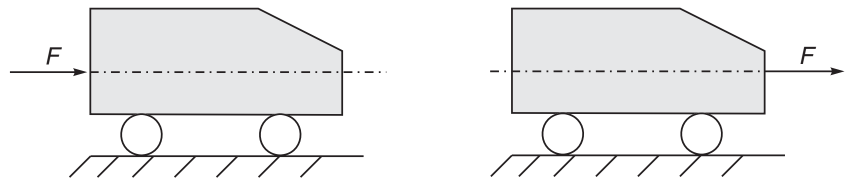

Fig. 1.16 Principle of transmissibility

Mathematically, Newton's law of gravitation can be written as

$$F \propto \frac{m_1 m_2}{r^2} \Rightarrow F = G\frac{m_1 m_2}{r^2} \quad (1.22)$$

where G is the universal constant or constant of gravitation. Its value is equal to 66.73×10^{-12} m^3/kg-s^2.

If the particle lies on the earth, then Eq. (1.22) can be rewritten for the force of attraction by the earth on the particle as

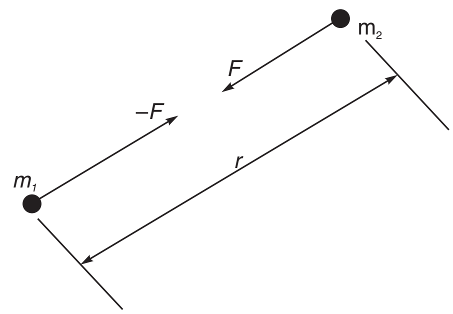

Fig. 1.17 Newton's law of gravitation

$$F = G\frac{M\,m}{R^2} \quad (1.23)$$

where M is the mass of the earth in kg, which is equal to 5.98×10^{24} kg, m is the mass of the particle, and R is the distance between the centre of the earth and the centre of the particle and is equal to the radius of the earth (6.378×10^6 m). Equation (1.23) can be further rewritten to calculate the weight of a body W of mass m, which is on the earth, or the force of attraction F of the earth on the particle as follows:

$$F = m\left(\frac{GM}{R^2}\right) \Rightarrow W = mg \quad (1.24)$$

where g is called the acceleration due to gravity and is equal to

$$\left(\frac{GM_{\text{Earth}}}{R^2_{\text{Earth}}}\right) = 9.81 \text{ m/s}^2 \quad (1.25)$$

All the principles discussed in this section will be introduced in subsequent chapters as and when they are needed.

1.4 VECTOR OPERATIONS

In this section, we will deal with vector operations and vector representation of quantities.

1.4.1 Addition and Subtraction

First we will discuss how vectors are added.

Addition

The parallelogram law or the traingular law can be used for vector addition. Two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ as shown in Fig. 1.18(a) can be added using the parallelogram law. The vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ are joined at their tails and parallel lines are drawn (shown with dotted lines) from the arrowhead of each vector as shown in Fig. 1.18(b) to form a parallelogram. The diagonal of the parallelogarm represents the resultant force vector $\boldsymbol{R}$ of the two vectors, which is given by

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$$

Using the triangular law, the head of vector $\boldsymbol{P}$ is connected to the tail of $\boldsymbol{Q}$ as shown in Fig. 1.18(c).

The resultant $\boldsymbol{R}$ can be obtained by connecting the head of vector $\boldsymbol{Q}$ to the tail of $\boldsymbol{P}$. In other way, $\boldsymbol{R}$ can be obtained from Fig. 1.18(d).

Hence, vector addition is commutative, i.e.,

$$\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{Q} + \boldsymbol{P} \tag{1.26}$$

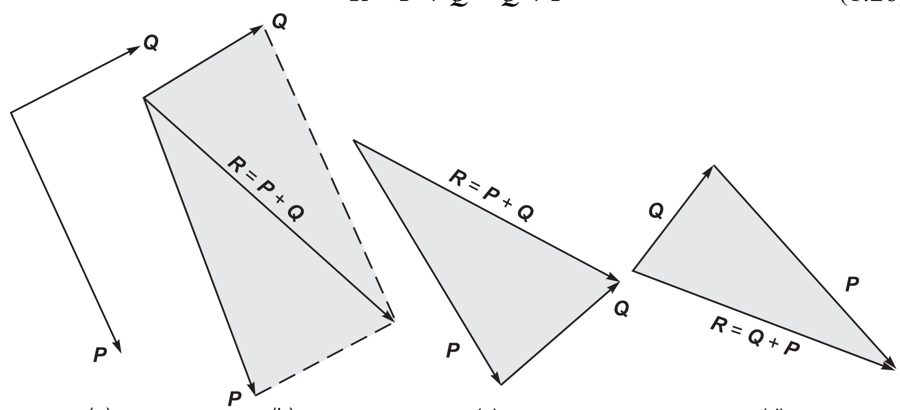

Fig 1.18 Representation of forces using the parallelogram and triangular laws: vector addition

Subtraction

The parallelogram law or the triangular law can be used for vector subtraction.

From Figures 1.19(a), (b), and (c),

$$\begin{aligned}\boldsymbol{R} &= -\boldsymbol{P} + \boldsymbol{Q} = (-\boldsymbol{P}) + \boldsymbol{Q}\\ &= \boldsymbol{Q} + (-\boldsymbol{P})\end{aligned}$$

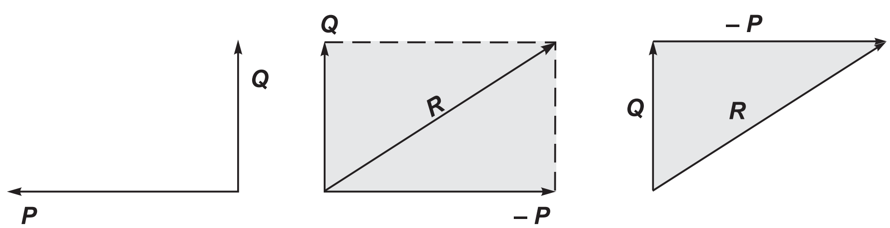

Fig. 1.19 Representation of forces using the parallelogram and triangular vector subtraction

The vector subtraction is also commutative, similar to the vector addition.

Representation of a vector quantity (1D, 2D, or 3D) using unit vectors Unit vectors, denoted by $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$, are used to represent the direction along the x-, y-, and z-axes respectively and their magnitudes are equal to 1. A one-dimensional vector is represented with a unit vector of $\boldsymbol{i}$. A two-dimensional vector is represented with unit vectors of $\boldsymbol{i}$ and $\boldsymbol{j}$. Similarly, a three-dimensional vector is represented with unit vectors of $\boldsymbol{i}, \boldsymbol{j}$, and $\boldsymbol{k}$. For example,

Velocity, $\boldsymbol{V} = 5\boldsymbol{i} + 2\boldsymbol{j}$ m/s (two-dimensional)

Force, $\boldsymbol{F} = 3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ N (three-dimensional)

Example 1.2 If $\boldsymbol{P} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$, $\boldsymbol{Q} = 5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}$, and $\boldsymbol{R} = 3\boldsymbol{i} - 5\boldsymbol{j} + 2\boldsymbol{k}$, find the vector $3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R}$ and its magnitude.

Solution We have

$$\boldsymbol{P} = 3\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k},\ \boldsymbol{Q} = 5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}, \text{ and } \boldsymbol{R} = 3\boldsymbol{i} - 5\boldsymbol{j} + 2\boldsymbol{k}$$

So, $3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R} = (9\boldsymbol{i} + 6\boldsymbol{j} - 15\boldsymbol{k}) + (5\boldsymbol{i} + 3\boldsymbol{j} - 3\boldsymbol{k}) - (9\boldsymbol{i} - 15\boldsymbol{j} + 6\boldsymbol{k})$

$$= 5\boldsymbol{i} + 24\boldsymbol{j} - 24\boldsymbol{k} \quad (Ans)$$

Magnitude of $3\boldsymbol{P} + \boldsymbol{Q} - 3\boldsymbol{R}$ is $\sqrt{5^2 + (24)^2 + (-24)^2} = \mathbf{34.31}$ (Ans)

Example 1.3 Determine the unit vector parallel to the resultant of vectors $\boldsymbol{P} = 3\boldsymbol{i} + 6\boldsymbol{j} - 9\boldsymbol{k}$ and $\boldsymbol{Q} = 3\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$.

Solution Resultant vector $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q} = 6\boldsymbol{i} + 9\boldsymbol{j} - 4\boldsymbol{k}$

Magnitude of the resultant, $\boldsymbol{R} = \sqrt{6^2 + 9^2 + (-4)^2} = 11.53$

The unit vector parallel to $\boldsymbol{R}$ is $\dfrac{\boldsymbol{R}}{|R|} = \dfrac{6\boldsymbol{i} + 9\boldsymbol{j} - 4\boldsymbol{k}}{11.53}$

$$= 0.52\boldsymbol{i} + 0.78\boldsymbol{j} - 0.34\boldsymbol{k} \quad (Ans)$$

To check the answer, the magnitude of the unit vector should be 1, i.e.,

$$\sqrt{(0.52)^2 + (0.78)^2 + (-0.34)^2} = 1$$

Example 1.4 Determine the unit vector parallel to the line that starts at the point A whose position is defined as $(x_A, y_A, z_A) = (3, 2, -3)$ and passes through the point B whose position is defined as $(x_B, y_B, z_B) = (2, 1, 6)$.

Solution The unit vector $\boldsymbol{\lambda}_{AB}$ parallel to the line $\boldsymbol{AB}$ is equal to $\dfrac{\boldsymbol{AB}}{|AB|}$.

$$\boldsymbol{AB} = (x_B - x_A)\boldsymbol{i} + (y_B - y_A)\boldsymbol{j} + (z_B - z_A)\boldsymbol{k}$$
$$= (2 - 3)\boldsymbol{i} + (1 - 2)\boldsymbol{j} + [6 - (-3)]\boldsymbol{k} = -\boldsymbol{i} - \boldsymbol{j} + 9\boldsymbol{k}$$

Magnitude of $\boldsymbol{AB} = |AB| = \sqrt{(-1)^2 + (-1)^2 + 9^2} = 9.11$

So, the unit vector $\boldsymbol{\lambda}_{AB} = \dfrac{-\boldsymbol{i} - \boldsymbol{j} + 9\boldsymbol{k}}{9.11}$

or $\boldsymbol{\lambda}_{AB} = -0.1097\boldsymbol{i} - 0.1097\boldsymbol{j} + 0.988\boldsymbol{k}$ (Ans)

1.4.2 Vector Representation of a Force

First we will discuss vector representation of a force on a plane.

A force on a plane (two-dimensional force)

Figure 1.20 illustrates the two-dimensional vector representation of a force.

In Fig. 1.20(a), the force vector $\boldsymbol{F}$, which is in the first quadrant, has two components along the x- and y-directions.

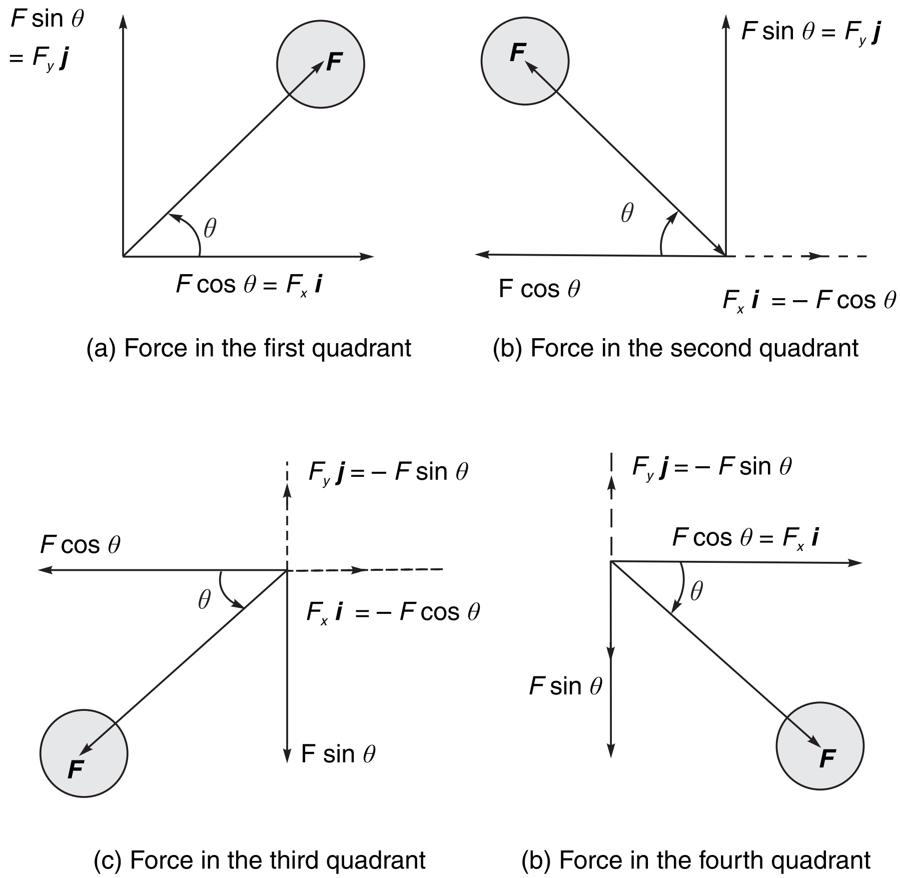

Fig. 1.20 Vector representation of a force on a plane (two-dimensional force)

This can be represented in vector form as

$$\begin{aligned} \boldsymbol{F} &= F_x\boldsymbol{i} + F_y\boldsymbol{j} \\ &= (F\cos\theta)\boldsymbol{i} + (F\sin\theta)\boldsymbol{j} \end{aligned} \tag{1.27}$$

where $F_x = F\cos\theta$ and $F_y = F\sin\theta$

Similarly, from Fig. 1.20(b),

$$\boldsymbol{F} = (-F\cos\theta)\boldsymbol{i} + (F\sin\theta)\boldsymbol{j}$$

where $F_x = -F\cos\theta$ and $F_y = F\sin\theta$ (1.28)

From Fig. 1.20(c),

$$\boldsymbol{F} = (-\,F\cos\theta)\boldsymbol{i} + (-\,F\sin\theta)\boldsymbol{j} \tag{1.29}$$

where $F_x = -F\cos\theta$ and $F_y = -F\sin\theta$

From Fig. 1.20(d),

$$\boldsymbol{F} = (F\cos\theta)\boldsymbol{i} + (-F\sin\theta)\boldsymbol{j} \tag{1.30}$$

where $F_x = F\cos\theta$ and $F_y = -F\sin\theta$

Vector representation of a force in space (three-dimensional force)

Figure 1.21 illustrates the vector representation of a force in space.

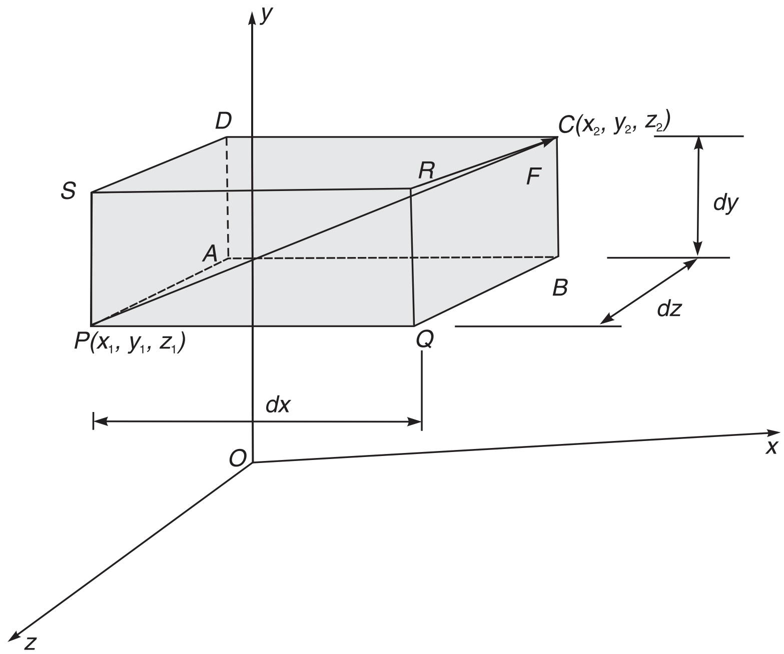

Fig. 1.21 Representation of a force in space (three-dimensional force)

In Fig. 1.21, the force vector $\boldsymbol{F}$ is acting from the point P to the point C. Here, $\boldsymbol{\lambda}_{PC}$ is a unit vector along the line PC.

Vector representation of the force F can be written as

$$\boldsymbol{F} = (\text{magnitude of } \boldsymbol{F})\, \boldsymbol{\lambda}_{PC} \tag{1.31}$$

where $$\boldsymbol{\lambda}_{PC} = \frac{\text{Position vector of } \boldsymbol{PC}}{\text{Magnitude of } \boldsymbol{PC}} \tag{1.32}$$

$$\text{Position vector } \boldsymbol{PC} = dx\,\boldsymbol{i} + dy\,\boldsymbol{j} + dz\,\boldsymbol{k} \tag{1.33}$$

$$\text{Magnitude of } \boldsymbol{PC} = \sqrt{(dx)^2 + (dy)^2 + (dz)^2} \tag{1.34}$$

Example 1.5 Express the vector form for a force of 200 N acting from the point $P(x_1, y_1, z_1) = (0,0,0)$ to the point $C(x_2, y_2, z_2) = (2,3,4)$ cm.

Solution From Fig. 1.16 and Eq. (1.34),

$$\text{Magnitude of } \boldsymbol{PC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{2^2 + 3^2 + 4^2} = 5.38 \text{ cm}$$

$$\text{Unit vector } \boldsymbol{\lambda}_{PC} = \frac{dx\boldsymbol{i} + dy\boldsymbol{j} + dz\boldsymbol{k}}{\text{Magnitude of } \boldsymbol{PC}}$$

$$= \frac{2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}}{5.38}$$

or $\boldsymbol{\lambda}_{PC} = 0.37\boldsymbol{i} + 0.56\boldsymbol{j} + 0.74\boldsymbol{k}$

From Eq. (1.31),

$$\boldsymbol{F} = 200(0.37\boldsymbol{i} + 0.56\boldsymbol{j} + 0.74\boldsymbol{k})$$

or $\boldsymbol{F} = 74\,\boldsymbol{i} + 112\boldsymbol{j} + 148.5\,\boldsymbol{k}$ (*Ans*)

Example 1.6 Position vectors of the points A and B shown in Fig. 1.22 are given by $\boldsymbol{r}_1 = 4\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{r}_2 = 5\boldsymbol{i} + 4\boldsymbol{j} - 3.5\boldsymbol{k}$. Determine $\boldsymbol{AB}$ in terms of $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ and also find the distance between the points A and B.

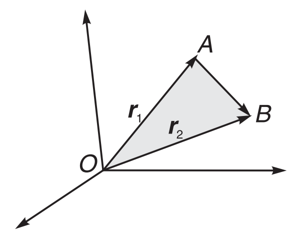

Fig. 1.22

Solution We have

$$\boldsymbol{r}_2 = \boldsymbol{r}_1 + \boldsymbol{AB}$$

or
$$\boldsymbol{AB} = \boldsymbol{r}_2 - \boldsymbol{r}_1 = (5\boldsymbol{i} + 4\boldsymbol{j} - 3.5\boldsymbol{k}) - (4\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k})$$

or $\boldsymbol{AB} = \boldsymbol{i} + \boldsymbol{j} - 1.5\boldsymbol{k}$ (*Ans*)

The distance $AB = \sqrt{1^2 + 1^2 - (-1.5)^2} = 2.06$ units (*Ans*)

Using the vector mathematics, the problems in subsequent chapters are worked out.

1.4.3 Dot Product and Cross Product

We will first deal with dot product.

Dot product (scalar)

Let us consider two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ as shown in Fig. 1.23. The dot product of these vectors is given by

$$\boldsymbol{P} \cdot \boldsymbol{Q} = PQ \cos\theta \quad (1.35)$$

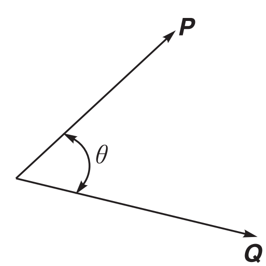

Fig. 1.23

Thus, the dot product of $\boldsymbol{P}$ and $\boldsymbol{Q}$ is equal to the magnitude of $\boldsymbol{P}$ multiplied by the component $\boldsymbol{Q}\cos\theta$ of vector $\boldsymbol{Q}$ in the direction of $\boldsymbol{P}$ or the magnitude of $\boldsymbol{Q}$ multiplied by the component $\boldsymbol{P}\cos\theta$ of $\boldsymbol{P}$ in the direction of $\boldsymbol{Q}$. It satisfies the commutative law, i.e.,

$$\boldsymbol{P} \cdot \boldsymbol{Q} = \boldsymbol{Q} \cdot \boldsymbol{P} \quad (1.36)$$

For unit vectors,

$$\boldsymbol{i} \cdot \boldsymbol{i} = \boldsymbol{j} \cdot \boldsymbol{j} = \boldsymbol{k} \cdot \boldsymbol{k} = 1 \quad (1.37)$$

(since cos 0° = 1)

$$\boldsymbol{i} \cdot \boldsymbol{j} = \boldsymbol{j} \cdot \boldsymbol{i} = \boldsymbol{j} \cdot \boldsymbol{k} = \boldsymbol{k} \cdot \boldsymbol{j} = \boldsymbol{k} \cdot \boldsymbol{i} = \boldsymbol{i} \cdot \boldsymbol{k} = 0 \quad (1.38)$$

(since cos 90° = 0)

The vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be expressed as

$$\boldsymbol{P} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}$$

and
$$\boldsymbol{Q} = Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}$$

So,
$$\boldsymbol{P} \cdot \boldsymbol{Q} = P_x Q_x + P_y Q_y + P_z Q_z \quad (1.39)$$

Also
$$\boldsymbol{P} \cdot \boldsymbol{P} = P_x^2 + P_y^2 + P_z^2 \quad (1.40)$$

Vectors also satisfy the distributive law, i.e.,

$$\boldsymbol{P} \cdot (\boldsymbol{Q} + \boldsymbol{R}) = \boldsymbol{P} \cdot \boldsymbol{Q} + \boldsymbol{P} \cdot \boldsymbol{R} \tag{1.41}$$

Cross product (vector)

The cross product $\boldsymbol{P} \times \boldsymbol{Q}$ of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is another vector and its magnitude is given as

$$|\boldsymbol{P} \times \boldsymbol{Q}| = \boldsymbol{PQ} \sin\theta \tag{1.42}$$

$$\boldsymbol{P} \times \boldsymbol{Q} = -\boldsymbol{Q} \times \boldsymbol{P} \tag{1.43}$$

The cross product satisfies the distributive law, i.e.,

$$\boldsymbol{P} \times (\boldsymbol{Q} + \boldsymbol{R}) = \boldsymbol{P} \times \boldsymbol{Q} + \boldsymbol{P} \times \boldsymbol{R} \tag{1.44}$$

The vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ can be expressed as

$$\boldsymbol{P} = P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}$$

and

$$\boldsymbol{Q} = Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}$$

So, $\boldsymbol{P} \times \boldsymbol{Q} = (P_y Q_z - P_z Q_y)\,\boldsymbol{i} + (P_z Q_x - P_x Q_z)\,\boldsymbol{j} + (P_x Q_y - P_y Q_x)\boldsymbol{k}$ (1.45)

The cross product $\boldsymbol{P} \times \boldsymbol{Q}$ can be expressed in the determinant of matrix as

$$\boldsymbol{P} \times \boldsymbol{Q} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \tag{1.46}$$

Applying Eq. (1.42) for unit vectors,

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}, \quad \boldsymbol{j} \times \boldsymbol{i} = -\boldsymbol{k} \tag{1.47}$$

$$\boldsymbol{j} \times \boldsymbol{k} = \boldsymbol{i}, \quad \boldsymbol{k} \times \boldsymbol{j} = -\boldsymbol{i} \tag{1.48}$$

$$\boldsymbol{k} \times \boldsymbol{i} = \boldsymbol{j}, \quad \boldsymbol{i} \times \boldsymbol{k} = -\boldsymbol{j} \tag{1.49}$$

1.4.4 Vector Representation of Moment

A force can translate and rotate a body about an axis. This rotational tendency of the force about an axis is called *moment* and is denoted by M. The moment is the cross product of the position vector and the force vector:

$$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F} \tag{1.50}$$

Figure 1.24 shows a force vector $\boldsymbol{F}$ on a plane (two-dimensional), which is acting at a point defined by a position vector $\boldsymbol{r}_{OA}$. The moment due to the force about the z-axis, which is perpendicular to the plane taken at point $\boldsymbol{O}$, can be expressed as

$$M_{z\text{-axis taken at point } O} = \boldsymbol{r}_{OA} \times \boldsymbol{F} \tag{1.51}$$

where $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$ and

$\boldsymbol{r}_{OA} = dx\,\boldsymbol{i} + dy\,\boldsymbol{j} = x_1\boldsymbol{i} + y_1\boldsymbol{j}$

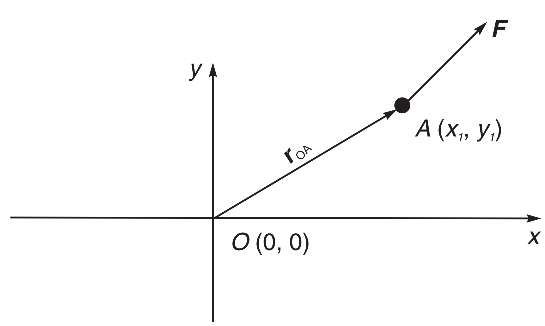

Fig. 1.24 Force $\boldsymbol{F}$ on a plane acting at a point $\boldsymbol{A}$ defined by a position vector $\boldsymbol{r}_{OA}$

$$M_{z\text{-axis taken at point } O} = \begin{vmatrix} x_1 & y_1 \\ F_x & F_y \end{vmatrix} \tag{1.52}$$

Similarly, Fig. 1.25 shows force vector $\boldsymbol{F}$ in space (three-dimensional) acting at a point defined by a position vector $\boldsymbol{r}_{PC}$ and the moment due to the force about point P can be expressed as

$$\boldsymbol{M}_P = \boldsymbol{r}_{PC} \times \boldsymbol{F} \tag{1.53}$$

where $\boldsymbol{F} = F_x \boldsymbol{i} + F_y \boldsymbol{j} + F_z \boldsymbol{k}$ and $\boldsymbol{r}_{PC} = dx\, \boldsymbol{i} + dy\, \boldsymbol{j} + dz \boldsymbol{k} = x_1 \boldsymbol{i} + y_1 \boldsymbol{j} + z_1 \boldsymbol{k}$

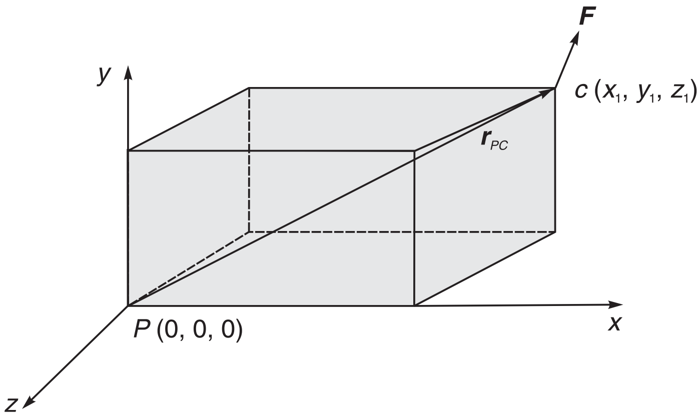

Fig. 1.25 Force $\boldsymbol{F}$ in space acting at a point defined by a position vector $\boldsymbol{r}_{PC}$

From Eq. (1.46), the moment can be expressed in the determinant of matrix form as

$$\boldsymbol{M}_P = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_1 & y_1 & z_1 \\ F_x & F_y & F_z \end{vmatrix} \tag{1.54}$$

Problems are worked out in Chapter 4 using Eq. (1.54) to find out the moment of force about the origin.

1.5 HOW TO SOLVE AN ENGINEERING MECHANICS PROBLEM

Problems in engineering mechanics are to be approached as an actual engineerng situation and with individual experience and common sense, it is easy to understand and formulate the problem. The first step involved in formulating the problem is stating the problem, which contains the given data and the details regarding what is to be determined.

The physical quantities are to be represented in a neat line diagram. The independent line diagrams for all the bodies representing the magnitudes and the directions of the forces acting on the body (which are known as free-body diagrams) are to be drawn.

The next step is the solution part, which is based on the fundamental principles/laws of mechanics stated in Section 1.3. They are used further in later chapters to write a set of equations for a given numerical data of the problem. Thus, by solving the equations, the required unknown values could be found out. Use of suitable principles and correct computations in practical engineering problems are highly important since they influence the design safety of the structure and its behaviour as well as the manufacturing cost of the entire structure. Most static problems of rigid body mechanics are related to the mass, geometry, and type of the constraints/supports of the body. For these problems, the reaction forces and reaction moments that are developed can be determined by using the static equilibrium equations. After finding the support reactions, supports can be designed for the structure.

The dynamic problems of rigid body mechanics can be solved by two methods: force (or moment due to force) method and energy method.

The solution to the dynamic problems is also related with the mass and geometry of the body, geometry of the motion (rectilinear/curvilinear/projectile), type of the motion (uniform motion/uniformly accelerated motion/uniformly decelerated motion/combination of acceleration and deceleration, uniform motion, etc.), external forces, and reactions, and moments due to the forces.

Kinematics of a body can be expressed in terms of mathematical equations relating to its position, distance travelled, velocity, and acceleration with respect to time. Kinetics of a body can be expressed in terms of mathematical equations relating to the geometry of the motion (acceleration/deceleration) and mass of the body, to determine the inertia forces/inertia moments of the body under motion satisfying the condition of the dynamic equilibrium. Further, the body may be in static equilibrium in the directions perpendicular to the motion. From all these relations, a set of equations is formulated, which will be solved to determine the unknown quantities in a specific problem.

In a similar way, an alternate energy method can be utilized to formulate a set of equations to a specific problem from which the unknown quantities can be found.

Recapitulation

- Mechanics of bodies is classified as mechanics of rigid bodies, mechanics of deformable bodies, and mechanics of fluids.
- In engineering mechanics, depending on the nature of the external forces applied on the body, bodies are idealized as particle and rigid body. Hence, this subject is studied under two groups—particle mechanics and rigid body mechanics.
- The fundamental laws/principles of mechanics are
  - (i) Newton’s three laws of motion
  - (ii) Lami’s theorem
  - (iii) Parallelogram law for addition of forces
  - (iv) Triangular law of forces
  - (v) Principle of transmissibility
  - (vi) Newton's law of gravitation
- Various systems of units are SI, MKS, CGS, and FPS.
- A body is idealized as a ‘particle’ in statics when its shape and size does not affect the solution to the given problem and the mass of the body is assumed to be concentrated to a specific point. When a system of concurrent forces is applied on a body, then the body can be idealized as a particle.
- The idealizing situation of a rigid body is that the shape and size of the body will not change at any condition of loading. A rigid body undergoes deformation and changes its shape under a system of external forces acting on it. The deformation and the change in shape are small and have negligible effect to

develop the reactions required to maintain the equilibrium conditions of the body. And hence in a structure of engineering applications, the members of the structure are assumed to be rigid and the reactions at contact points are determined. In rigid body mechanics, the effects of a system of non-concurrent, coplanar or non-concurrent, non-coplanar forces acting on a body are studied.

- Statics of rigid body deals with a system of external forces applied on a body in equilibrium with support reactions along the constrained direction. Further, considering the unconstrained directions to be the directions perpendicular to the constrained directions along which the condition of equilibrium is to be justified.
- Dynamics of rigid body is studied under two groups, namely kinematics and kinetics. Kinematics deals with the geometry of the motion such as position/distance travelled, and velocity and acceleration of the body with respect to time irrespective of the cause of the motion (force or moment due to the force). In kinetics, the geometry of the motion, such as acceleration/deceleration, is related with the cause of the motion (force or moment due to the force) by which it satisfies the condition of the dynamic equilibrium.
- Vector addition and subtraction can be expressed as $\boldsymbol{R} = \boldsymbol{P} + \boldsymbol{Q}$ and $\boldsymbol{R} = \boldsymbol{P} - \boldsymbol{Q}$.
- Vector representation of a force in one dimension, two dimensions (on a plane), and three dimensions (in space) can be expressed respectively as $\boldsymbol{F} = F_x\boldsymbol{i}$, $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j}$, and $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$, where F_x, F_y, and F_z are the forces along the x-, y-, and z-directions, respectively.
- The unit vector $\boldsymbol{\lambda}_{AB}$ of line AB whose coordinates are (x_A, y_A, z_A) and (x_B, y_B, z_B) is equal to $\boldsymbol{AB}/|\boldsymbol{AB}|$, where

  $\boldsymbol{AB} = (x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (z_B - z_A)\,\boldsymbol{k}$

  and $|\boldsymbol{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$

  A force of magnitude F acting along line AB can be expressed in vector form as $\boldsymbol{F} = F(\boldsymbol{\lambda}_{AB}) = F\,[(x_B - x_A)\,\boldsymbol{i} + (y_B - y_A)\,\boldsymbol{j} + (x_B - z_A)\boldsymbol{k}]$.
- Moment can be expressed in vector form as $\boldsymbol{M} = M_x\boldsymbol{i} + M_y\boldsymbol{j} + M_z\boldsymbol{k}$, where M_x, M_y, and M_z are moments due to force about the x-, y-, and z-axes respectively.
- Moment about the origin O due to the force $\boldsymbol{F} = F_x\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\boldsymbol{k}$, which is acting at a point, say A, whose coordinates are (x_A, y_A, z_A), can be expressed as determinent of the matrix as

$$\boldsymbol{M}_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_A & y_A & z_A \\ F_x & F_y & F_z \end{vmatrix}$$

- The problem-solving technique in engineering mechanics involves first drawing free-body diagram(s) showing the forces and the moments due to the forces with their directions. In case of static problems, a set of equations can be formulated by satisfying the equilibrium conditions and hence the unknown quantities of the problem can be found. In case of dynamic problems, a set of equations can be formulated in terms of kinematics and kinetics of the body satisfying the condition of the dynamic equilibrium along the direction of the motion and of the static equilibrium in the direction perpendicular to the motion. From these equations, the unknown quantities of the problem can be found.

Review Questions

1. Differentiate between rigid body, deformable body, and fluid.
2. Differentiate between statics and dynamics of rigid body.
3. Define force and its units.
4. How is a force represented in vector form?
5. Differentiate between particle and rigid body.
6. Write the dimensions in MLT system for the following quantities:
(i) Acceleration (ii) Force (iii) Moment
7. What is Lami's theorem?
8. Define unit vector.
9. How can the force shown in Fig. 1.26 be represented in vector form?

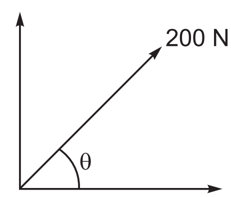

Fig. 1.26

Ans: $200\cos\theta \boldsymbol{i} + 200\sin\theta \boldsymbol{j}$

10. 'Most of the mechanics problems are governed by equilibrium conditions'. Do you agree? Justify your answer.