Structural Analysis

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Structural Systems

1.1 INTRODUCTION

Structures are built to facilitate the performance of various activities connected with residence, office, education, healthcare, sports and recreation, transportation, storage, power generation, irrigation, etc. We see a variety of structures in our midst. Some are monumental, some residential, some commercial, some recreational, some mobile, etc. All of them have certain common features; they form systems consisting of a load-resisting component, which is called *super structure* and a load-distributing component to the ground which is known as *substructure*.

All the structures should sustain the loads coming on them during their service life by possessing adequate *strength* and also limit the deformation by possessing enough *stiffness*. Strength of a structure depends on the characteristics of the material with which it is constructed. Stiffness depends on the cross section and the geometrical configuration of the structure. A structure is not a single entity; it consists of many parts that are assembled together as a system. The parts are called *elements* or *members*. The loads coming on a structure degenerate into forces in these elements because of the deformation they undergo. The members should be designed to resist these forces induced in them as per the relevant codes of practices prevalent in a country. Besides, the structure should be stable against overturning moments caused by some kind of horizontal loads like that caused by earthquake or wind. Moreover, all the loads applied on the structure should be safely transmitted to the ground through its foundation. Therefore, safety is of prime importance in the existence of structures. Because human beings occupy the structure eventually one should not compromise on the safety aspect of the structure. Otherwise distress in the structure will endanger lives of occupants. Transmission of loads coming on the *global* system through its *local* members to the subsystem consisting of the foundation for eventual distribution on the ground is called *load path*. Any interruption in the load path will lead to collapse of the structure. So, the safety of a structure can be assured with the right choice of appropriate load path.

Structural analysis, therefore, deals with the mechanism of degeneration of loads applied on the system into local element forces, using various theories and theorems enunciated by eminent engineers and investigators. It also deals with the computation of deformations these members suffer under the action of the induced forces. We will discuss all these aspects in details in the following chapters in this book along with a number of illustrative examples.

First, we discuss about various types of loads applied on the structural systems.

1.1.1 Loads

A structural system experiences many kinds of loads during its service life. The system should be designed for the worst combination of loads that is going to act on it throughout its service life. Many types of loads that are applied on the structure are as follows:

1. Dead loads
2. Live loads
3. Construction loads
4. Snow and ice loads
5. Earth pressures or soil pressures
6. Water pressures or hydrostatic pressures
7. Loads due to subsidence
8. Loads caused by thermal changes and misfit
9. Wind loads
10. Earthquake loads or seismic loads
11. Impact loads
12. Blast loads
13. Dynamic loads induced by machines
14. Fatigue loads induced by their repeated application, e.g., due to traffic, waves, etc.
15. Loads due to transportation of fluids in pipelines
16. Loads caused by floods, landslides, etc.
17. Loads caused by fire
18. Centrifugal forces in bridges
19. Longitudinal forces by braking of vehicles on bridges

The dead load is *fixed* to the structure and is mostly invariant with time. For example, self-weight of the structure; furniture; stored materials like books, stationery items, etc., constitute the dead loads. They reside permanently in the structure and are the basic parameter for design in all structural systems. The basic information about these loads is prescribed in relevant codes and standards as well as in reference books.

All the other loads are somewhat variable in nature, both in time and magnitude. These loads are applied over and above the dead or *fixed loads*. Hence, they are called by a generic name, *imposed loads*. These loads are specified in various codes and standards by countries across the globe.

Some of the loads due to wind, earthquake, floods, landslides, etc., are caused by natural phenomena; therefore, they are categorized as *geophysical loads*. These loads are mostly non-deterministic because of the uncertainties associated with their occurrence, time and magnitude, and terrestrial conditions.

Some other loads like impact loads, blast loads, dynamic loads, etc., are caused by human activities and hence are called *man-made loads*.

Loads such as dead loads, live loads, snow and ice loads, act vertically downwards and hence are called *gravity loads*. Wind loads, earthquake loads, and soil and hydrostatic pressures act horizontally and hence are called *lateral loads*. Loads caused by thermal changes and misfit as well as by blasting are *directionless*.

Loads that do not vary with time and always maintain the same sense are called *static loads* or *monotonic loads*, e.g., dead loads, construction loads, snow loads, earth pressure, etc. However, those that vary with time and in sense are called *cyclic loads* as well as *dynamic loads*, e.g., earthquake loads, impact loads, blast loads, machine-induced loads, etc. *Fatigue load* is a kind of cyclic load which is applied repeatedly on the structure for a long duration and it is a special case of dynamic load. For example, traffic- and wave-induced loads fall under the category of fatigue loads. Dynamic load sets the structure to vibrate.

1.1.2 Material

The materials form the backbone of the carrying capacity of structures. We have a variety of materials that can be used in construction. Popular materials that are being used over the decades are

1. Wood
2. Stone
3. Brick
4. Concrete
5. Steel
6. Aluminium
7. Brass
8. Cement products
9. Plastic
10. Smart and intelligent materials

The oldest material of construction is timber. At the beginning of habitation, as plenty of timber was available, the construction was mostly based on this material. Therefore, most of the buildings in Europe and America, even today, are built with timber.

With the advent of binders, like lime and gypsum, stone was also used in construction. With the invention of cement, brick became a popular construction material. Stone and brick construction is called *masonry*. Stone and brick are quite strong in resisting compression.

In the early nineteenth century, *concrete* emerged as a popular material for construction. Because of its inherent weakness in carrying tensile force, it is normally strengthened with steel bars. This combination of concrete and steel is called *reinforced concrete*. In this form of construction, initially steel remains passive or dormant. Only upon loading, the steel becomes active. An advanced form of reinforced concrete is the *prestressed concrete* in which the steel is made active by imparting strength in it before carrying any load.

Steel has dominated the civil engineering construction over the past several decades. Our earth is currently dotted with many prominent steel structures such as The Eiffel Tower, The St. Louis Arch, etc. Of late, plastic is extensively used particularly in the form of *fibre reinforced plastics* (FRP).

All these materials discussed previously are called *dumb materials* because they are not adaptive to their environment and do not forewarn any impending distress in the structure. Like plants and animals which adapt to their environment, some materials called *smart materials* are being evolved. These materials when incorporated in construction can adjust itself to the environment and indicate any distress occurring inside. These are called *intelligent materials* or smart materials. Shape memory alloy (SMA) is a popular example and it falls under the category of smart materials.

Adoption of a particular material for construction depends on the importance of the building, economy, type of use, etc.

1.2 FORMS OF STRUCTURES

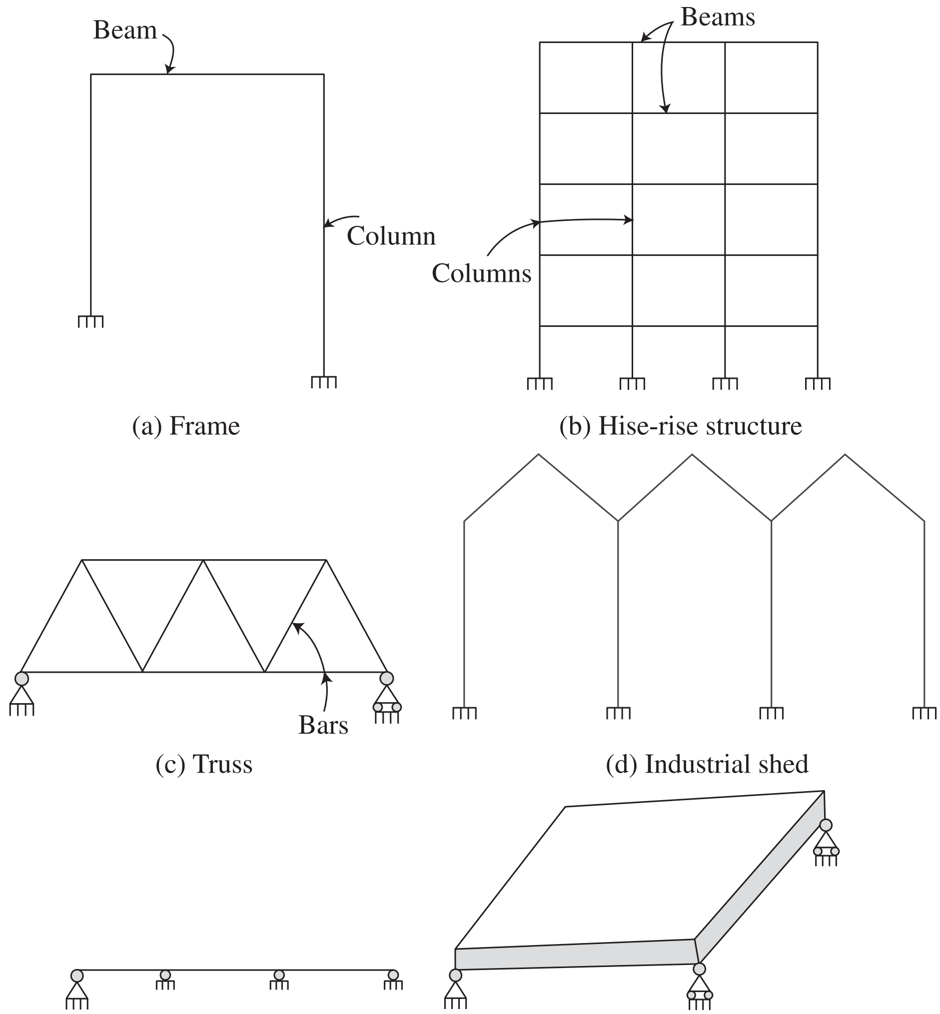
We have constructed structures of many forms and shapes. All structural forms used for load transfer from one point to another are three-dimensional (3D) in nature. Generally, they can be categorized as *linear forms* (Fig. 1.1) and *curvilinear*

Fig. 1.1 Chicago downtown buildings (linear form).

forms (Fig. 1.2). The type of functions and aesthetics dictate the forms of structures. For instance, linear forms are preferred for residential, official, and educational purposes. The linear form is called *skeletal structures*. They are articulated structures assembled with parts consisting of linear elements, such as bars and beams, the connection between them being bolted or riveted or welded.

Fig. 1.2 Balloon structure (curvilinear form).

Assemblage of members forming a frame to support the forces acting is called the *framed structure*. A framework is the skeleton of the complete structure and it supports all intended loads safely and economically. Some structural examples are frames [Fig. 1.3(a)], high-rise structures [Fig. 1.3(b)], trusses [Fig. 1.3(c)], industrial shed [Fig. 1.3(d)], bridge deck [Fig. 1.3(e)], plates [Fig. 1.3(f)], etc. Generally, these structures are two-dimensional (2D) lying in one plane along two coordinate axes. However, the parts by which they are assembled are one-dimensional (1D) lying in a single plane along one coordinate axis.



(a) Frame

(b) Hise-rise structure

(c) Truss

(d) Industrial shed

(e) Bridge deck

(f) Plate

Fig. 1.3 Skeletal structures.

Curvilinear forms as single entities mostly occupy a space. For structural analysis purposes these structures are idealized as continuous system. Continuous system structures transfer loads through the in-plane or membrane action to the boundaries. Assemblages of continuous members like shells, domes, etc., are called *continuous system*. They are 3D structures. The examples for continuous system are domes, shells, arches, cables, cylindrical members, cooling towers, space crafts, aircrafts, etc. These are shown in Fig. 1.4. Structures in curvilinear form are called *surface structures*.

The most suitable structural form is the one which provides satisfactory solutions to functional, economic, sociological, aesthetic, and other requirements to

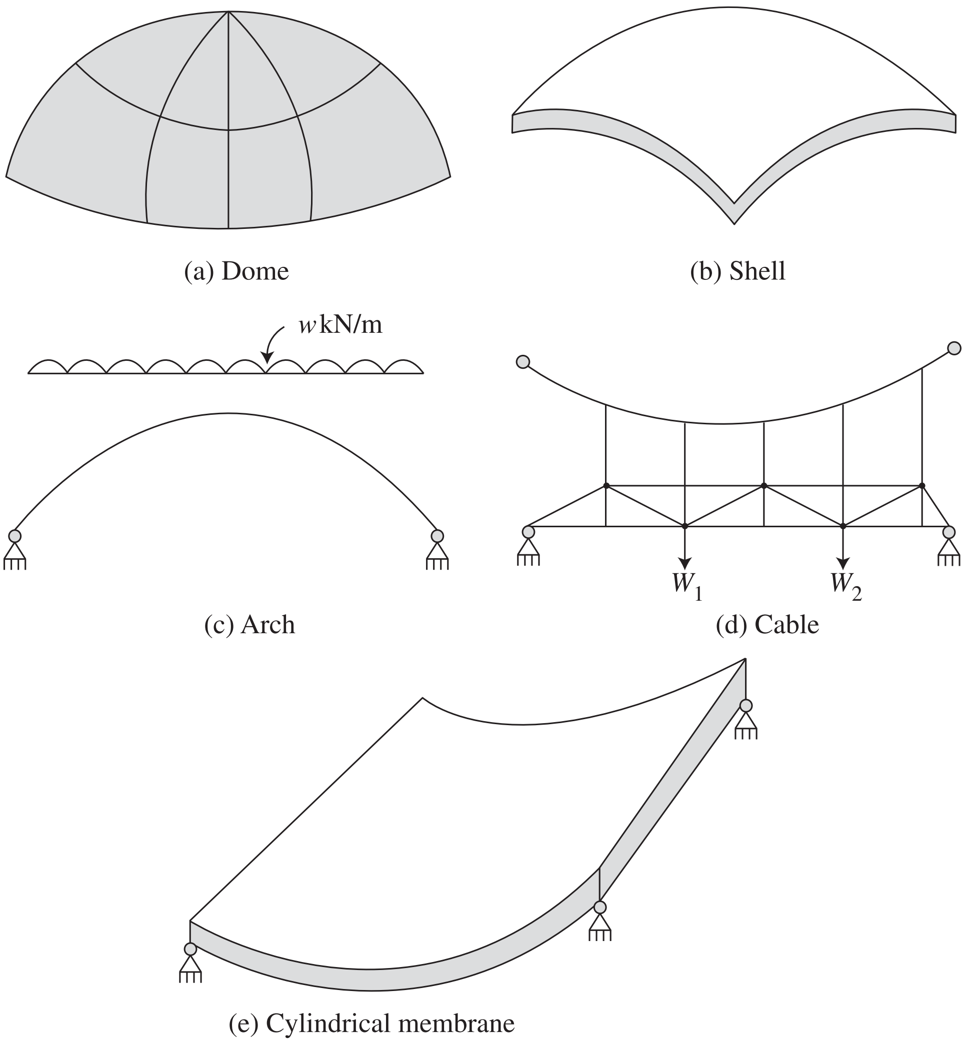


Fig. 1.4 Surface structures.

the highest degree and that can be economically and reliably built, using the most appropriate structural materials and construction methods that are available.

On the basis of the dominant stress conditions developed under their most significant design loads and conditions, structural forms may be classified as *uniform stress forms* and *varying stress forms*. When the stress across a section is uniform over the depth of a member or over the thickness of a panel, e.g., cables, arches, truss members, membranes, and shells, such a form is called a uniform stress form. When the stress varies over the depth or thickness, from a maximum compressive stress on one surface to a maximum tensile stress on the other, e.g., in the case of beams, rigid frames, slabs, plates, etc., such a form is called a varying stress form.

1.3 DIFFERENT STRUCTURAL SYSTEMS

The term *structural system* or *structural frame* in structural engineering refers to load-resisting subsystem of a structure. Structural system transfers loads through interconnected structural components or members. In general, structural systems are designed as a combination of different elements. The basic ones are the members that are organized and connected in a way to define units of different kinds like frames, trusses, floors. A series of units, connected and stabilized by auxiliary members, determines the system. All the components are arranged in such a way as to allow the ensemble to assume a configuration capable to withstand the actions like loads, settlements of the construction, dimensional variations caused by hygrothermal fluctuations, etc., more or less matching with the stiffness, strength, and relative motion between adjacent parts. All structural systems resist forces in three basic ways: by flexure or bending, shear, and axial tension and compression. Broadly speaking, we can categorize the structural system based on applied loads as (i) gravity load-resisting system and (ii) lateral load-resisting system.

1.3.1 Gravity Load-resisting System

In this system, normally load is applied on the horizontal elements of the system like floors which is then transferred to vertical supporting elements like walls.

1.3.1.1 Structural System Types for Simple Structures

Simple structures are mostly *gravity load-resisting* type. Structural systems normally employed in relatively simple structures are wood frame, masonry bearing wall and concrete plank, steel, precast concrete, cast-in situ concrete, etc. The selection of the appropriate system or combination of system is usually the result of an evaluation of the following 12 factors:

1. Soil conditions
2. The programme and concept
3. Relevant codes
4. Potential changes in codes
5. Flexibility
6. Impact on finished ceiling and building height
7. Delivery of material and construction time
8. Construction capabilities locally available and preferences
9. Ease of construction and schedule
10. Cost of selected system
11. Cost impact on other systems
12. Elegance and aesthetic issues

On the basis of the factors listed previously some issues normally crop up. Nine of the most common structural systems in which application of the above-mentioned factors may give rise to problems are briefed as follows.

Wood Frame This system is typically inexpensive; it can be implemented by a wide variety of contractors; it is fast and it can be flexible. The limitations regarding span can be circumvented with the use of trusses, laminated beam, heavy timber, or mixing with steel or other systems in larger space. The most common disadvantage in this system is that timber is combustible. Another potential disadvantage is the vibration of the floor in high traffic areas.

Bearing Wall and Concrete Plank Here, we use a combination of masonry bearing walls and precast concrete slab. It is a simple method, familiar to many contractors, relatively low cost, and relatively fast in construction. The span limitations can be overcome by combining with other systems for large spaces. The major drawbacks in this system are its relative lack of flexibility; performance on unstable soil; height limitations; impact on the distribution of mechanical, electrical, plumbing, and fire protection systems; and the occasional shortages of masons. Non-availability of precast companies is another hurdle in the implementation of this system.

Steel and Concrete Plank This system is quite flexible; it performs adequately on unstable soil, and hardly suffers from a lack of skilled manpower on the other hand, steel requires fireproofing and its delivery time is more.

Steel and Poured-Concrete Deck It is more expensive and the depth of the system is more than that for steel and concrete plank due to the inclusion of the intermediate beams.

Precast Concrete Precast concrete can be used for more than exterior wall system. It is also used in some locations of the columns, beams, and bearing walls. It is a common structural choice for garages, site bridges, and other simple long-span, heavy-load structures. The major drawback in this system is the non-availability of precast companies in the vicinity.

'Beam-and-slab' Cast-in-place Concrete This is a non-combustible system. Particularly, it is best suited for situations where the lateral loads are significant. It is relatively easy to build with locally available material. It can produce flexible buildings. Cast-in-situ tends to be relatively expensive and very often it has a greater thickness than that of steel-and-concrete plank.

'Flat-slab' Cast-in situ Concrete For taller residential buildings, a two-way, flat-slab concrete structure is a popular choice because it minimizes floor-to-floor height, is fast to build, creates a finished ceiling with the underside of the slab, permits flexibility in placing columns, and is relatively easy to brace or stiffen for lateral loads. However, this system is more expensive than some of the other options.

Prestressed and Post-tensioned Concrete In this system, the steel reinforcement is tensioned even before it is loaded and hence it is an active system. In prestressed concrete, steel is tensioned before pouring concrete. In post-tensioned concrete, the member is cast first. After hardening of concrete, the reinforcing steel is tensioned against the hardened concrete. The system can be thinner than flat-plate and is more expensive. The system is less flexible because concrete cannot be cut for future modifications. The construction industry in many parts of the country does not have experience to implement these systems.

Combination of Systems We can employ two or more systems in a single project. On the whole, the selection and design of a structural system, or combination of systems, is an issue with significant cost, aesthetic, and functional implications.

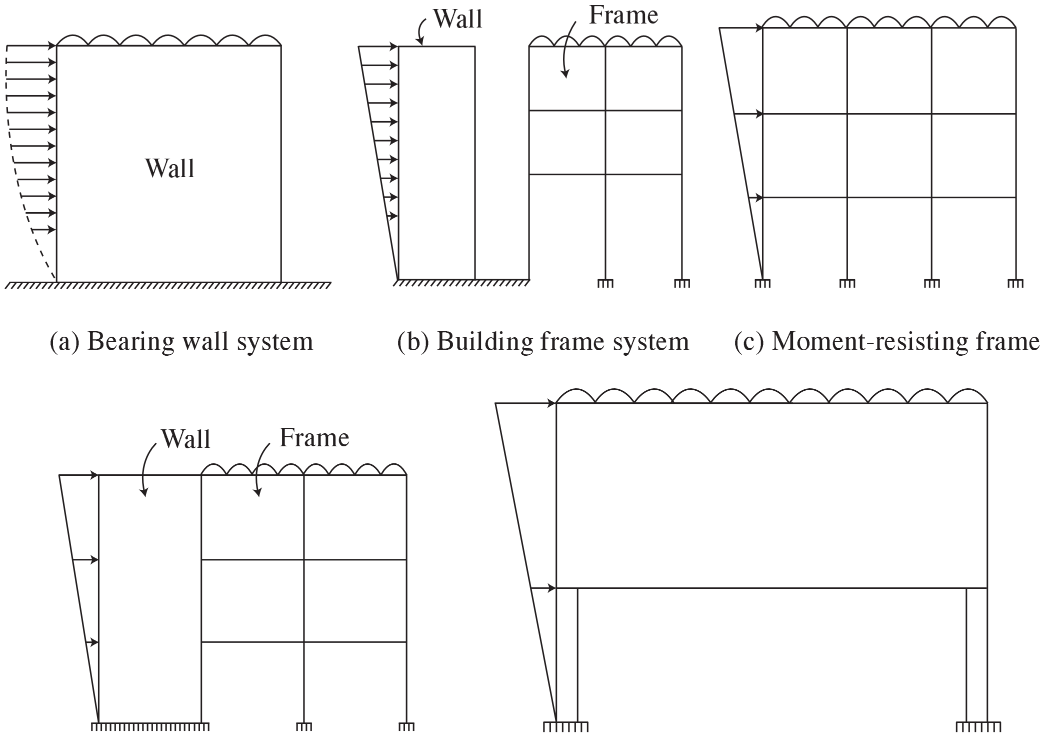
1.3.2 Lateral Load-resisting Systems

In the early structures at the beginning of the twentieth century, they were assumed to carry primarily gravity loads. However, in the modern times the situation has changed drastically. Rapid strides have been made in structural designs/systems and development of high-strength materials. These developments have resulted in significant reduction in building weight as well as its slenderness. This has necessitated the consideration of lateral loads due to wind and earthquake in design. Understandably, buildings, particularly tall buildings, suffer more from lateral loads resulting from wind and earthquake as their slenderness and the associated flexibility increases. As a general rule, other things being equal, the taller the building, the more necessary it is to identify the proper structural system for resisting the lateral loads. Currently, there are many structural systems that can be used for the lateral resistance of tall buildings. These systems have been evolved in the context of resisting seismic forces which are essentially horizontal in direction.

The Uniform Building Code (UBC), USA, has recommended five major categories of building types distinguished by the method used to resist the lateral force. These are illustrated in Fig. 1.5 and they consist of bearing walls, building frames, moment-resisting frames, dual systems, and cantilevered columns. These categories are further subdivided into the types of construction material used as given in Table 1.1.

Table 1.1 Structural Systems and Materials

Structural systems
I. Bearing wall system
(i) Wood light-framed walls with shear panels (up to three storeys or less)
(ii) Concrete or masonry shear walls
(iii) Steel braced frames
(iv) Heavy timber braced frames
II. Building frame system
(i) Steel eccentrically braced frame
(ii) Wood light-framed walls with shear panels (up to three storeys or less)
(iii) Concrete shear walls
(iv) Masonry shear walls
(v) Steel ordinary braced frames
(vi) Heavy timber braced frames
(vii) Steel special concentrically braced frames
III. Moment-resisting frame system
(i) Steel or concrete special moment-resisting frames (SMRF)
(ii) Masonry moment-resisting wall frames (MRWF)
(iii) Steel special truss moment frames
IV. Dual system
(i) Concrete shear walls with SMRF
(ii) Masonry shear walls with SMRF
(iii) Masonry shear walls with masonry MRWF
(iv) Steel eccentrically braced frames with steel SMRF
(v) Steel ordinary braced frames with steel SMRF
(vi) Steel special concentrically braced frames with steel SMRF
V. Inverted pendulum
(i) Cantilevered column elements (including column height)



(a) Bearing wall system (b) Building frame system (c) Moment-resisting frame

(d) Dual system (e) Inverted pendulum system

Fig. 1.5 Different lateral load-resisting systems.

1.3.2.1 Bearing Wall System

In this system, shear walls or braced frames provide support for all or most of the gravity loads and for resisting all lateral loads [Fig. 1.5(a)]. Bearing wall systems do not contain complete vertical load-carrying space frames but may use some columns to support floor or roof vertical loads. This type of system is very common and includes wood-frame buildings, concrete tilt-up buildings, and masonry buildings.

A shear wall is one that resists lateral forces by developing shear in its own plane and cantilevering from its base. It is essentially a very deep cantilever beam that develops flexural stresses besides its own basic shear stress. The wall tends to lift off at one end, and there must be adequate dead load to prevent this, or, it must be tied down to the foundation adequately. Moreover, the connection of the wall to the foundation must be adequate to prevent sliding. Shear walls may be made of sheathed wood-frame walls, reinforced concrete, reinforced masonry, or steel.

1.3.2.2 Building Frame System

This system has two separate entities: one to provide support for lateral forces and the other to bear gravity loads. A frame resists all gravity loads and an independent shear wall or a braced frame resists all lateral loads. This arrangement is shown in Fig. 1.5(b). Failure of the lateral support system will not result in the collapse of the building since the frame continues to support gravity loads. Typically building frame systems use steel braced frames or concrete or masonry shear walls to resist lateral forces. Steel braced frames are often used in combination with concrete shear walls or masonry shear walls.

Braced frames are essentially vertical cantilevered trusses to resist lateral forces by axial tension and compression in the truss members, and may either be concentric or eccentric configuration. Concentric frames have diagonal braces so arranged that the lateral forces act along the direction of their longitudinal axes. Eccentric braced frames use both axial loading of braces and bending of sections of horizontal beam to resist the forces. Figure 1.6 shows typical braced frame configurations.

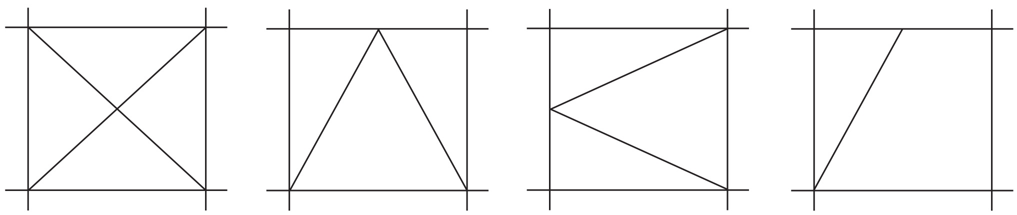


Fig. 1.6 Typical braced frames.

1.3.2.3 Moment-resisting Frames

Moment-resisting frames [Fig. 1.5(c)] can be steel, concrete, or masonry construction. This system provides a complete space frame throughout the building to carry vertical loads, and it uses some of the frame elements to resist lateral forces. Shear walls and braced frames are not used in this system, as shown in Fig. 1.5(c).

Moment-resisting frames, also called *rigid frames,* are specially detailed to provide good ductility and support for both lateral and gravity loads by flexural action. The ability of structural systems and materials to deform and absorb energy, without failure or collapse, is termed as *ductility*. Moment-resisting frames consist of beams and columns in which bending of these members provides the resistance to lateral force. The three categories of moment-resisting frames are as follows:

1. Special moment-resisting frame (SMRF)
2. Intermediate moment-resisting frame (IMRF)
3. Ordinary moment-resisting frame (OMRF)

***Special Moment-resisting Frame* (SMRF)** It is a moment-resisting frame made of structural steel or reinforced concrete that has the ability to absorb a large amount of energy in inelastic range, i.e., when the material is stressed above its yield point without failure and without large and unacceptable deformation. In severe seismic zones, construction of concrete frames must be of SMRF.

***Intermediate Moment-resisting Frame* (IMRF)** It is a concrete frame that has less stringent requirements than that of an SMRF and, in general, used only in moderate seismic zones.

***Ordinary Moment-resisting Frame* (OMRF)** It is a steel or a concrete frame that does not meet the special detailing requirements for ductile behaviour. Materials or systems that are able to absorb energy through deformation are called *ductile*, whereas those able to do so to a lesser extent are called *non-ductile* or *brittle*, e.g., a rubber band is ductile and a cast iron is brittle. The OMRF made up of steel may be used in any seismic zone, whereas that made up of concrete is used in low seismic zones. Since a moment-resisting frame resists lateral loads by bending, it is the most ductile lateral load-resisting system.

1.3.2.4 Dual System

A *dual system* is shown in Fig. 1.5(d). A dual system is one with an essentially complete frame that provides support for vertical loads. Lateral loads are resisted by *both* moment-resisting frames and shear walls or braced frames, in proportion to their relative rigidities. The moment-resisting frame must be designed to resist independently at least 25% of the total required lateral force, and, in addition, the two systems shall be designed to resist the total required lateral force in proportion to their relative rigidities.

1.3.2.5 Inverted Pendulum Structure

This system consists of a structure which is supported on cantilever column elements. These elements provide both lateral load resistance as well as resistance to gravity loads. Therefore, failure of the columns due to lateral forces will also cause failure of the gravity load-carrying capacity.

1.3.2.6 Selection Types

The selection of the appropriate type of lateral force-resisting elements in the construction of buildings is mainly based on the criterion of economics. A single type of resisting element is commonly used in most of the buildings such as in houses where wood-framed shear walls are used, or in concrete tilt-up buildings where concrete shear walls are used. However, other types of buildings may need the use of combination of more than one type of load-resisting system.

The building code permits the use of combinations of these systems but they are also subject to very specific structural design guidelines. When we use different systems, there is bound to be some adjustment in design forces which is inevitable. These adjustments in design forces are required to account for the differences in strength, stiffness, and ductility among the different types of resisting systems when used in combination.

1.3.3 Components of Structural Systems

A structural system essentially consists of three components as shown in Fig. 1.7: (a) structural model; (b) the prescribed actions; and (c) structural responses resulting from structural analysis.

In all cases, a structure must be idealized by a mathematical model so that its behaviour can be evaluated by solving a set of mathematical equations.

A structure is generally transformed into a simple model for analysis purposes. This process is called *idealization of structures*. The idealization consists

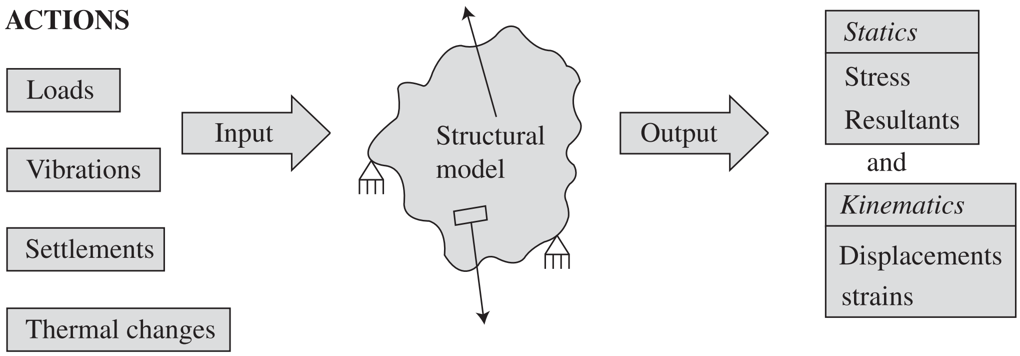


Fig. 1.7 Components of structural systems.

of identifying the parts of a structure as well as individual structural elements. This process requires experience and judgment.

A structural or mathematical model can be defined as an assembly of structural members or elements interconnected at ends or boundaries which may be joints, lines, or surfaces. Thus, a structural model consists of three basic components, namely (a) structural members, (b) joints or nodes or edges or surfaces, and (c) boundary conditions.

A structural system can be 1D, 2D, or 3D depending on the dimensions of loadings and the kind of structural responses that are of interest to the designer. In reality, all structures on the earth, strictly speaking, are 3D in nature. But for the purposes of simplification and easier comprehension, we can recognize a specific pattern of loading under which the key structural responses will remain in just 1D or 2D space. In Fig. 1.8, we have shown some 2D and 3D structural systems.

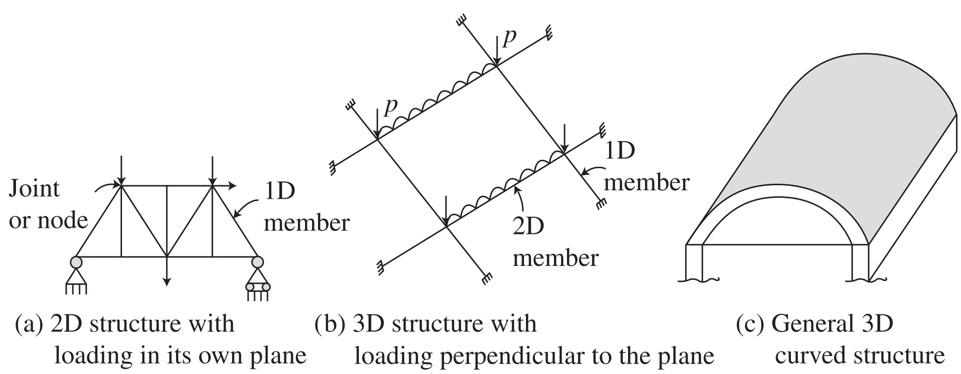


Fig. 1.8 Structures of various geometry.

1.4 LINEAR AND NON-LINEAR STRUCTURES

We use wood, concrete, steel, etc., in the construction of structures. The load resistance and deformation characteristics of structures significantly depend on the properties of these materials. Each of these materials has different properties that should be taken into consideration in the analysis and design of the structure. Typical stress–strain curve for these materials is shown in Fig. 1.9. As is clear from Fig. 1.9, the ultimate tensile strength (UTS) of different materials is different. Therefore, their resistance to loading, which depends on UTS, is also varied. The initial slope of the curve for each material is different. This slope characterizes the modulus of elasticity or Young's modulus E of the material. The modulus of elasticity of each material must be known for the calculation of displacement of structures.

As can be observed in Fig. 1.9 that each material in the initial stage behaves linearly, i.e., the stress and strain are proportional up to a certain limit which is called an *elastic limit*. This is also called a *linear range*. Subsequently, at higher stress the behaviour becomes non-linear, i.e., there is disproportionate increase in strain for a corresponding increase in stress. This zone is called a *non-linear range*. We call a system a *linear structure* when the stresses developed in it are within the elastic limit, i.e., the stresses in the system lie within the linear range. A system is called a *non-linear structure* if the stresses developed in it fall in the plastic or a non-linear range. Such a classification is based on the behaviour of material. The behaviour of a material in the plastic regime is characterized as material non-linearities for representation in structural analysis.

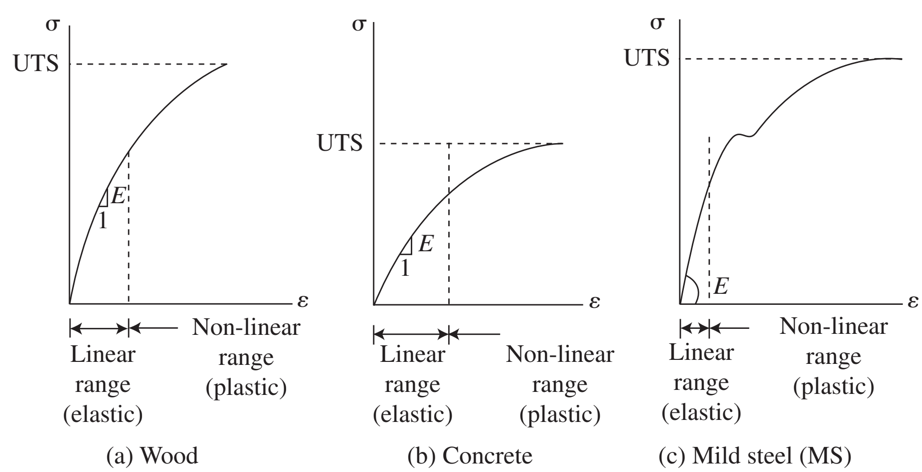


Fig. 1.9 Typical stress–strain relation of various materials.

In addition to material non-linearity, some structures may exhibit non-linear characteristics in its overall behaviour due to changes in its shape under loading. This necessitates that the structure should displace by a significant amount to maintain its overall equilibrium. This kind of behaviour of the structure is called *geometrical non-linearity*. A classical example of this type of non-linearity can be observed in cable structures discussed in Chapter 21. Also, a cantilever structure shown in Fig. 1.10 is another example of geometric non-linearity.

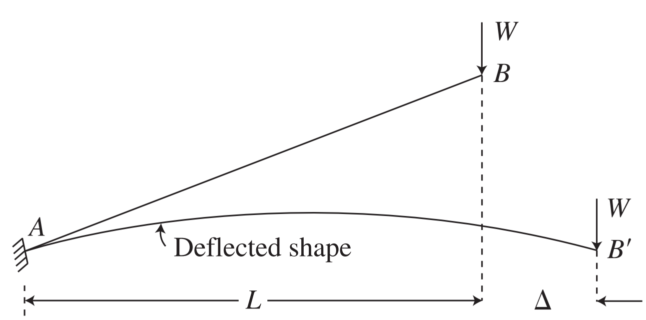
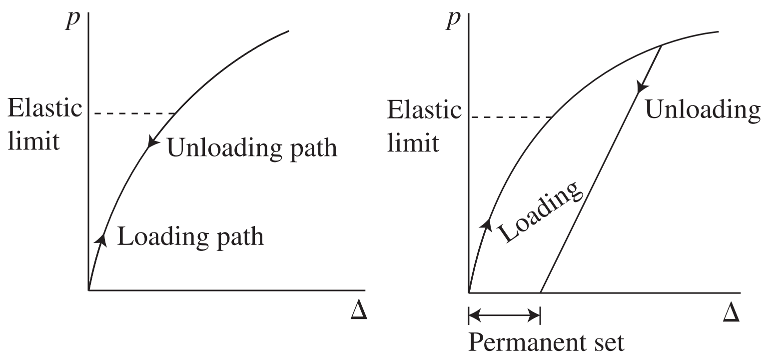


Fig. 1.10 Geometric non-linearity.

An important property of a linear structure is that when it is loaded, the stress in the material increases along a linear path till the elastic limit in the material is reached. Suppose we unload the structure or remove the load on the structure within this stage, the stress diminishes and it retraces the same linear path and



(a) Linear behaviour (b) Non-linear behaviour

Fig. 1.11 Linear and non-linear structures.

the structure returns to its original position without leaving any residual deformation in the structure as shown in Fig. 1.11(a).

In contrast, the stress level in a non-linear structure goes beyond elastic limit and mostly it remains in plastic regime. If the load is removed from the structure once the stresses have crossed elastic limit, then the structure returns to the original position by a different path as shown in Fig. 1.11(b) leaving some residual deformation in the structure. This is called a *permanent set*.

1.4.1 Assumptions Involved in Linearity of Structure

The following assumptions are usually adopted in the case of linear structural system.

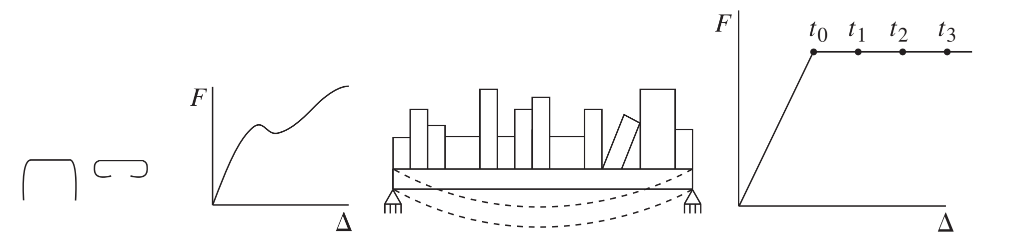
1. Under the applied loads the displacement involved is so small that there is no significant difference between the deformed configuration of the system and its undeformed geometry. The system is still assumed to satisfy the equilibrium conditions.
2. The deformation of the structure is so small that it is able to maintain the linear relation between strain and displacement.
3. The material of the structural system is assumed to be linear elastic, isotropic, and homogeneous and obey Hooke's law.

Because of these above assumptions the overall structural system becomes a linear problem and hence the principle of superposition is applicable.

1.4.2 Structural Non-linearity

The non-linear load–displacement relationship exhibited by the stress–strain relationship with a non-linear function of stress, strain, and/or time, changes in geometry due to large displacements, irreversible structural behaviour upon removal of the external loads; change in boundary conditions such as change in the contact area and the influence of loading sequence on the behaviour of the structure requires a non-linear structural analysis. The structural non-linearity can be classified as geometric non-linearity, material non-linearity, and contact or boundary non-linearity.

We encounter structural non-linearities in all our mundane affairs. For example, a staple pin initially in an inverted channel shape, as shown in Fig. 1.12(a), is permanently bent as shown in Fig. 1.12(a) after it is pinned on the bunch of papers to hold them together. The load–deformation behaviour of the pin is shown in Fig. 1.12(b). When we stack books on wooden shelf [Fig. 1.12(c)], as time passes it sags more and more. The corresponding load–deformation curve is shown in Fig. 1.12(d). If we observe both the load–deformation curves, we can conclude that the non-linear structural behaviour is characterized by the change in structural stiffness, i.e., the capacity of the structure to resist force per unit deformation changes with time because of the plastification of the material.



(a) Stapler pin (b) Force-displacement curve (c) Wooden book shelf (d) Force–displacement curve

Fig. 1.12 Examples of non-linear behaviour.

Thus a non-linear structural behaviour arises from a number of causes, which can be grouped into three principal categories.

1.4.2.1 Changing Status Including Contact

Many common structural features demonstrate non-linear behaviour, i.e., status-dependent. For example, a suspension cable initially slack, may become taut under loading, a roller support is either in contact or not in contact. Status changes might be directly related to load, like in the case of cable, or they might be determined by some external cause. Situations in which *contact* occurs are common to many different non-linear applications. Contact forms a distinctive and important subset to the category of changing-status non-linearities.

1.4.2.2 Geometric Non-linearities

If a structure experiences large deformations, its changing configuration can cause the structure to respond non-linearly. A mundane example is the pole vault. Initially, it is straight; when the sports person runs and hits in groove, it bends excessively as a result of which the sports person jumps high and clears the horizontal bar. The force–deformation curve for this case can be depicted as in Fig. 1.13.

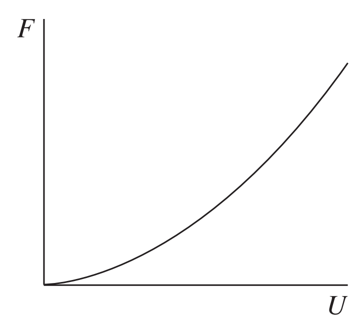


Fig. 1.13 Pole-vault behaviour.

1.4.2.3 Material Non-linearities

Non-linear stress–strain relationships are a common cause of non-linear structural behaviour. Several factors can influence the stress–strain properties of a material, including load-history, environmental conditions, and the amount of time that a load is applied.

1.4.2.4 Solution Technique for Non-linear Problems

To solve non-linear problems we usually employ Newton–Raphson approach. In this approach the load is subdivided into a series of load increments. The load increment can be applied over several load steps. Figure 1.14 illustrates the use of Newton–Raphson equilibrium iterations.

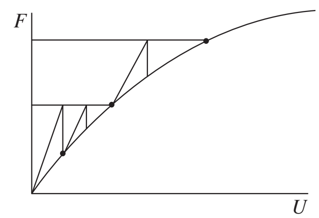


Fig. 1.14 Newton–Raphson approach.

Before each solution, the Newton–Raphson method evaluates the out-of-balance vector, which is the difference between the loads corresponding to the element stresses and the applied loads. The approach performs a linear solution, using the out-of-balance loads, and check for convergence. If the convergence criteria are not satisfied, the out-of-balance load vector is re-evaluated, the stiffness matrix (Chapter 13) is updated, and a new solution is obtained. This iterative procedure continues until the problem converges.

1.4.3 Principle of Superposition

This principle forms the basis for much of the theory of structural analysis. We can state this principle as

> *the total displacement or the forces induced internally at a point in a structure acted upon by several externally applied loadings can be computed by summing up of all the displacements or internal forces caused by each of the external loads acting individually, one at a time.*

This statement is valid only when a linear relationship exists among loads, internal forces, and displacements.

The essential requirements of the principle of superposition are

1. The stress–strain relationship of the material must be linear and the material should obey Hooke's law. This implies that the load is proportional to displacement.
2. The resulting displacement of the structure must be small so that there is no significant change in the geometry of the structure under loading. Large displacement will significantly change the position and orientation of the loads.

1.5 CONDITIONS OF EQUILIBRIUM

In order to apply the principle of statics to a structural system, the structure must be at rest. This is possible only when the sum of the applied loads and support reactions is zero and there is no resultant couple at any point in the structure. For this situation, all component parts of the structural system are also in equilibrium.

A structure is in equilibrium with a system of applied loads when the resultant moment about any point is zero. For a system of coplanar forces this may be expressed by the three equations of static equilibrium.

$$\sum F_{\mathrm{H}} = 0; \qquad \sum F_{\mathrm{V}} = 0; \qquad \sum M = 0 \tag{1.1}$$

where F_{H} and F_{V} are the resolved components in the horizontal and vertical directions of a force and M is the moment of a force about any point.

In the case of 3D structures, the forces can be resolved into three orthogonal directions, namely, X, Y, and Z coordinate axes. Moreover, if the resultant force vector is zero then its components in three mutually perpendicular directions also disappear. Hence, Eq. (1.1) may be written in three coordinate directions as follows:

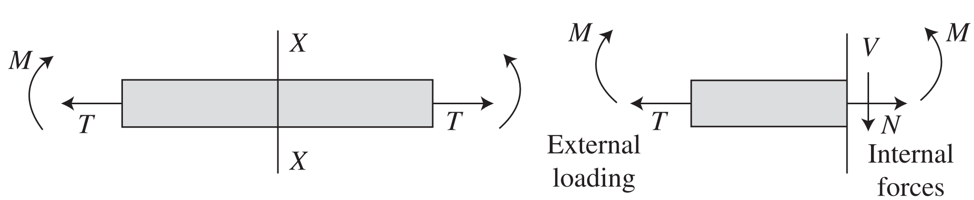
$$\sum F_x = 0; \qquad \sum F_y = 0; \quad \sum F_z = 0 \tag{1.2a}$$

$$\sum M_x = 0; \qquad \sum M_y = 0; \qquad \sum M_z = 0 \tag{1.2b}$$

These six equations are called the *equations of equilibrium* of space structures and are necessary and sufficient conditions for equilibrium. All the equilibrium equations must be satisfied simultaneously for the structure to be in equilibrium.

Using Eqs (1.1) and (1.2) we could find out the reactions at the supports in a structure. Once these reactions are evaluated, we could determine the internal stress resultants (reactions, axial forces, moments, etc.) in the structure. Correct solution for reaction and internal stresses must satisfy the equations of static equilibrium for the entire structure. They must also satisfy equilibrium equations for any part of the structure as a *free body*. A sketch depicting the free body with its associated forces and internal stresses is called a *free-body diagram* (FBD). If the number of unknown reactions is more than the number of equilibrium equations, then we cannot determine the reactions with only equilibrium equations. Such structures are known as the *statically indeterminate structures*. In such cases, we need to obtain extra equations based on deformation of the structure. These equations obtained from displacement consideration are called *compatibility equsations*.

We can make use of the equations of equilibrium to determine the internal forces in a member or structure. For this, we cut the member and isolate it from other parts. This is called a *free body*. We draw an FBD representing the internal forces on it as shown in Fig. 1.15. In general, the internal forces acting at the cut section of the member consist of normal force N, shear force V, and bending moment M as shown in Fig. 1.15.



(a) Member (b) FBD—left segment

Fig. 1.15 Internal forces.

While representing the forces on a body we use a straight arrow with a single head as $\rightarrow$. A moment is represented as a curved arrow with head either as $\curvearrowright$ for clockwise sense or $\circlearrowleft$ for anticlockwise sense. Sometimes we represent a moment as a straight arrow with double heads as $\rightarrow\!\rightarrow$.

1.5.1 Sign Convention

As the forces and displacements are direction-dependent we need to adopt a sign convention to sum up the results of various actions. We adopt here the sign convention as shown in Fig. 1.16 for a 3D structure. Here *X*, *Y*, and *Z* are the coordinate axes and are shown in positive directions. When we move from *X* to *Y* in the horizontal plane, the *Z*-axis must advance in its positive direction. This is called *left-hand system*. We assume that forces directed along the positive direction of axes are positive. For a left-hand system the couple should be a left-hand screw progressing in the direction of the coordinate axes. So, an anticlockwise moment is taken as positive here.

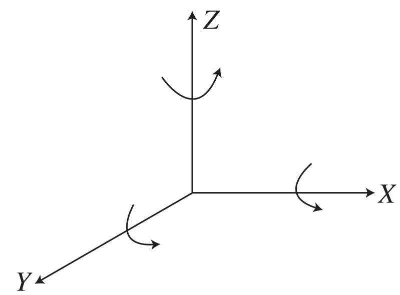


Fig. 1.16 Sign convention.

1.6 DEGREES OF FREEDOM, DETERMINATE, AND INDETERMINATE STRUCTURES

In this section, we now explain these terms that are frequently referred in structural analysis.

1.6.1 Degrees of Freedom

The *degrees of freedom* (DOF) can be defined as a set of independent displacements that specify completely the deformed position and orientation of the body or system under loading. Here, displacements include deflections and rotations as well. A rigid body that moves in 3D space in linear directions has three *translational* displacement components as DOFs. The rigid body can also undergo angular motion, which is called *rotation*. So, the body has three *rotational* DOFs. Altogether a rigid body can have at most six DOFS, three translations, and three rotations. Translation refers to the ability of a body to move without rotating whereas rotation refers to its angular motion about some *axis*.

When a structure is loaded, the joints, also called *nodes*, will undergo unknown displacements. These displacements are referred to as the *DOF* for the structures.

1.6.2 Determinate Structures

The conditions of equilibrium discussed in Section 1.5 are *necessary* and *sufficient* conditions to establish the equilibrium of structures. When structures are loaded, they pass on these loads to the support as reactions. The applied forces and the resulting reactions keep the structure in equilibrium. However, these reactions are mostly unknowns. We normally evaluate these reactions by using the equations of equilibrium. If all the reactions in a structure can be determined strictly only by the application of equilibrium equations, the structure is referred to as *statically determinate*. In other words, we can define a determinate structure as the one which can be fully analysed and all internal forces and stresses determined through the use of one or more of the six equations of equilibrium without recourse to stiffness, deflection, or other criteria for analysis.

Given a set of forces and reactions in equilibrium, the structural geometry of determinate structures takes care of itself. In other words, force–deformation compatibility for such structures is automatically satisfied for any set of forces and reactions in equilibrium. For example, the support reactions and hence, the moments and shears in a simple beam (Chapter 3) or a three-hinged arch (Chapter 20) can be found statically without paying any attention to their deformed shapes. As may be verified easily, a determinate structure has only as many support reactions as absolutely necessary for its stability. The removal of even a single reaction makes the structure unstable.

Figure 1.17 shows the determinate structures. In Fig. 1.17(a), the frame has three support reactions which can be calculated easily by Eq. (1.1). The arch in Fig. 1.17(b) has four support reactions against the three equations of equilibrium available for solution. So, it seems that reactions cannot be computed statically. However, the condition that the moment at the hinge C be zero provides the additional fourth equation for finding the four unknown reactions. Such additional equations are called *condition equations*. A statically determinate structure may also be defined alternatively as the one in which the number of unknown reactions R equals the sum of the number of *applicable* equations of equilibrium n and that of the condition equations c, i.e.,

$$R = n + c \tag{1.3}$$

Equation (1.3) is called the *equations of statics*.

The qualification 'applicable' is important because equilibrium equations which are applicable to a problem need only be counted in assessing its determinacy. For example, in the continuous beam as shown in Fig. 1.17(c), as the loading

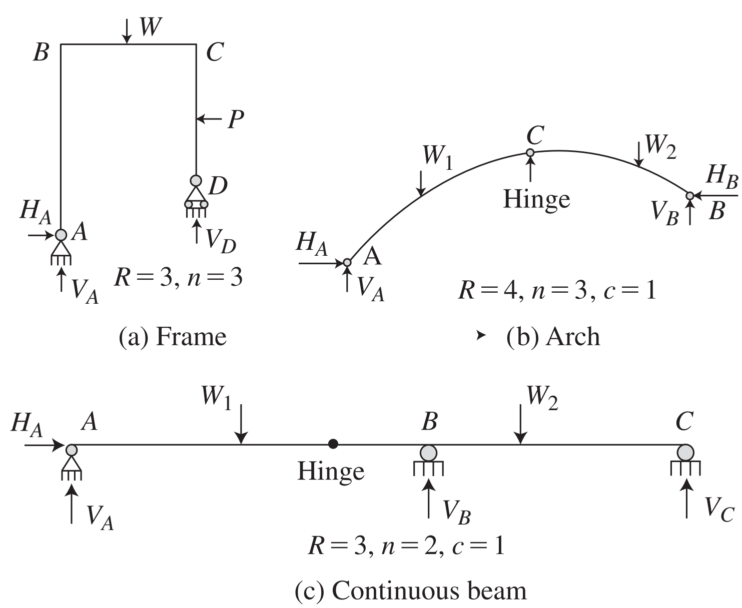


Fig. 1.17 Determinate structures.

is only vertical, only two conditions, namely, $\Sigma F_V = 0$ and $\Sigma M = 0$ are applicable. Therefore, $\Sigma F_H = 0$ is meaningless in the absence of horizontal loads on the beam.

1.6.3 Indeterminate Structures

Structures in which the reactions cannot be evaluated by the application of static equilibrium equations alone are defined a *statically indeterminate* or *hyperstatic structures*. They are also known as *redundant structures*. In these structures, the number of unknown reactions is greater than the number of available equations of static equilibrium. However, sometimes it is quite possible that the support reactions are statically determinate, but internal forces remain indeterminate. For example, we consider a truss shown in Fig. 1.18(a). We will discuss in Chapter 2 as how to evaluate the forces and reactions in a truss. Accordingly, the truss in Fig. 1.18(a) is statically determinate both for support reactions and forces in the members. In contrast, the truss shown in Fig. 1.18(b) is statically determinate only with reference to the calculations of support reactions.

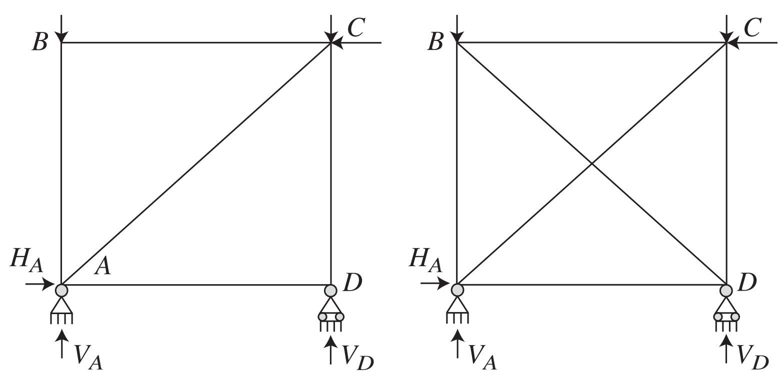


Fig. 1.18 Determinate and indeterminate structures.

We now consider, for example, a continuous beam (discussed in Chapter 9) shown in Fig. 1.19. It has six unknown support reactions as shown in Fig. 1.18 as against three equilibrium equations, namely, $\Sigma F_V = 0$, $\Sigma F_H = 0$, and $\Sigma M = 0$ available for the determination of these six reactions. Unless we determine these six reactions, it is not possible to evaluate the internal forces in the beam. Three extra equations should be set up to circumvent this difficulty. We can develop these equations from the geometrical conditions. For example, we can specify that the vertical deflections at B, C, and D are zero. These additional equations are called *equations of compatibility* and their number determines the *degree of indeterminacy D* of the structure. The reactions, for the solution of which the compatibility equations are developed, are termed as *redundants R*.

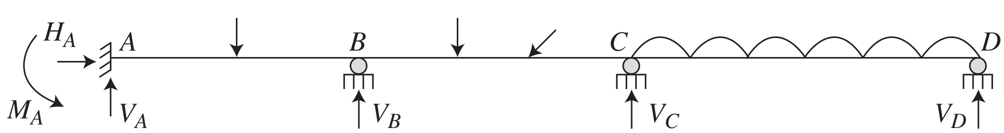


Fig. 1.19 Continuous beam.

It may be observed in Fig. 1.19 that the supports B, C, and D may be removed without affecting the stability of the beam. This action reduces the beam into a determinate one which is called a *primary* or *released structure*. So, we can conclude here that an indeterminate structure has more support reactions than are absolutely necessary for its stability. This characteristic may be used to determine the degree of indeterminacy of structures. We can divide indeterminate structures into three categories as follows:

1. Externally indeterminate structures
2. Internally indeterminate structures
3. Structures with combined indeterminacies.

We describe each one in detail below.

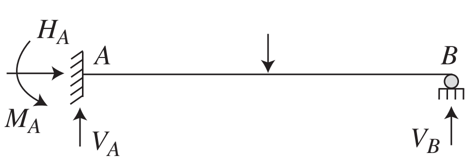
1.6.3.1 Externally Indeterminate Structures

When the total number of external reactions in a structure is greater than the number of equations of statics applicable to the entire structure, then such a structure is called *externally indeterminate* one. The number of external reactions over and above the number of equations of statics defines the *degree of external indeterminacy E* of the structure. The numbers of additional equations which are equal in number to the external redundants matching with the geometrical conditions at supports are required to supplement the equations of statics for a solution of the problem.

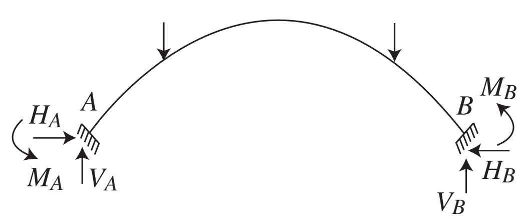
The degree of external indeterminacy E of a structure may be evaluated by any one of the methods listed below.

1. We can determine the difference between the total number of external reactions in the structure and the number of equations of statics.
2. The redundant reactions are not essential to the stability of the structure. So, we can count the number of such support reactions as can be safely removed in order to render the structure externally determinate without affecting its stability.

A few examples of externally indeterminate structures are shown in Fig. 1.20. In Fig. 1.20(a), we have shown a propped beam as discussed in Chapter 7 in detail. We have three reactions at support A. The additional support at B causes indeterminacy to the structure. If we remove this support the beam is reduced to a cantilever which we know is stable. Therefore, the degree of external indeterminacy E for this beam is 1. The arch shown in Fig. 1.20(b) has both ends fixed (Chapter 20). It has six reactions, three at support A and three at support B. We can remove



(a) Propped beam



(b) Fixed arch

Fig. 1.20 Degree of external indeterminacy.

the support B, thus releasing all the three reactions at B. This action converts the arch into a stable and determinate structure. The three released reactions at B constitute the degree of external indeterminacy. So, for the arch in Fig. 1.20(b) the degree of external indeterminacy is 3.

1.6.3.2 Internally Indeterminate Structures

Sometimes structures may be externally determinate but internal forces cannot be determined by equations of equilibrium alone, e.g., Fig. 1.18(b). Such structures are called *internally indeterminate* ones. A structure which is externally indeterminate should also be necessarily internally indeterminate because internal forces cannot be determined without knowing the external reactions. Hence, we cannot determine the internal indeterminacy of an externally indeterminate structure without accounting in someway for the external indeterminacy. For this purpose, we remove either the external redundants to render the structure externally indeterminate without in any way affecting its overall stability before finding its degree of internal indeterminacy I, or their number accounted for in the equations that determine I. These aspects are explained below with examples.

For instance, we consider here the internal indeterminacy of plane building frames. These frames depend on the assumed rigidity of their joints for their ability to resist loads. Internal forces in the constituent members, namely, beams and columns of a frame normally consist of a moment, a shear, and a normal force. We consider here a frame shown in Fig. 1.21(a). It has six external reactions shown in Fig. 1.21(a). It can be observed that it is externally indeterminate to degree 3. If we remove the fixity at support H thus releasing all three reactions at H and convert the frame into a determinate one as well as stable one. However, the internal forces in beams and columns remain indeterminate. Therefore, it is a clear case of internal indeterminacy. Now, we cut any two beams and study the internal force at these sections. We take sections XX and YY as shown in Fig. 1.21(a). At each section, we have three internal forces, namely,

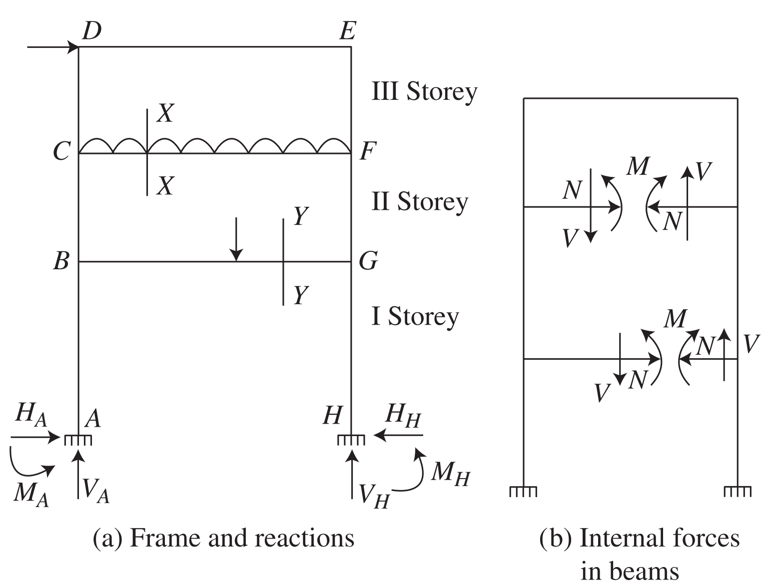


Fig. 1.21 Degree of internal indeterminacy.

moment M, shear force V, and normal force N as shown in Fig. 1.21(b). A total of six redundants is now exposed. Once these redundants are determined, we can compute forces in other beam and in all columns. Thus, all internal forces in all the members of the frame are clearly known. Therefore, the determinacy of the frame depends on the degree of internal determinacy I of the frame. In the frame shown in Fig. 1.21, the degree of internal indeterminacy is 6.

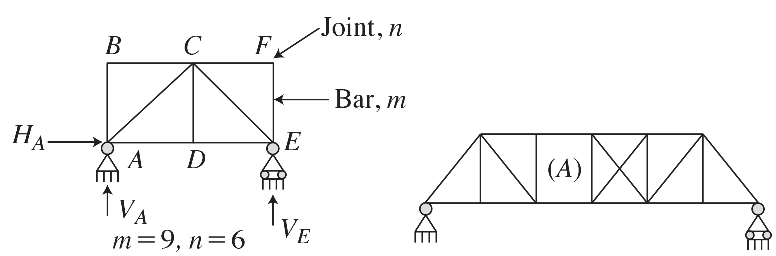
For evaluating I of a frame we can adopt the following rule. *We should count the number of p closed or completed cells in the plane frame, other than that of the first storey. We can get I for the frame by multiplying the number of cells p by 3, i.e.,*

$$I = 3p \tag{1.4}$$

Next, we evaluate the internal indeterminacy of plane trusses. We will discuss at length in Chapter 2 about the kind of trusses, their stability, and the method of determination of internal forces in the members of the trusses. However, to explain the concept of internal indeterminacy we discuss here about some aspects of trusses. A truss consists of bars connected at their ends by joints. Therefore, a truss has m number of members and n number of joints or nodes [Fig. 1.22(a)]. We can determine the member forces in a truss from the equilibrium of the truss joints. At each joint we have two equilibrium equations, namely, $\Sigma F_V = 0$ and $\Sigma F_H = 0$ for the solution of member forces. With n number of joints, we have *2n* number of equations for the whole truss. If we define r as the number of support reactions *necessary and sufficient for the external determinacy and stability* of truss, then we can propose the following equation for internal determinacy of the truss as

$$2n = (m + r) \tag{1.5}$$

However, mere satisfaction of Eq. (1.5) does not ensure internal indeterminacy of a truss. For example, the truss in Fig. 1.22(b) satisfies Eq. (1.5) but is not a stable structure since the panel marked (A) forms an unstable mechanism capable of undergoing excessive deformation under loads. Therefore, Eq. (1.5) must be



(a) Determinate truss
(b) Unstable truss

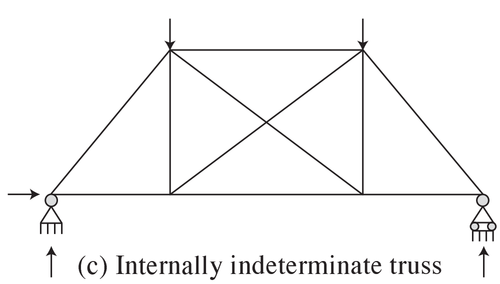

(c) Internally indeterminate truss

Fig. 1.22 Plane truss.

satisfied by the whole truss as well as by its different parts like (A) individually. In other words, a truss that satisfies Eq. (1.5) is internally determinate only if it is stable.

When the number of equilibrium equations $2n$ is less than the number of combined unknown member forces and reactions, i.e., $(m + r)$ in a truss, then it is called *internally indeterminate*, i.e.,

$$2n < (m + r) \tag{1.6}$$

The difference between the sum of the unknown member forces and reactions and the number equations, i.e., $(m + r - 2n)$ defines the degree of internal indeterminacy of the truss, i.e.,

$$I = (m + r - 2n) \tag{1.7}$$

If, however, $(m + r)$ is less than $2n$ the truss is unstable, i.e.,

$$(m + r) < 2n \tag{1.8}$$

A typical truss with one degree of internal indeterminacy is shown in Fig. 1.22(c).

1.6.3.3 Structure With Combined Indeterminacies

For a complete solution of a structure, the total indeterminacy, i.e., external and internal combined together, is essentially required. The *degree of total indeterminacy D* is determined either by adding the degree of external and internal indeterminacy, i.e., $E + I$ or by directly as explained below.

In the case of a truss, we can use Eq. (1.5) to evaluate the indeterminacy of the truss, provided r is replaced by the total number of external reactions R. Thus, for a determinate truss both externally and internally, we can say that

$$2n = (m + R) \tag{1.9}$$

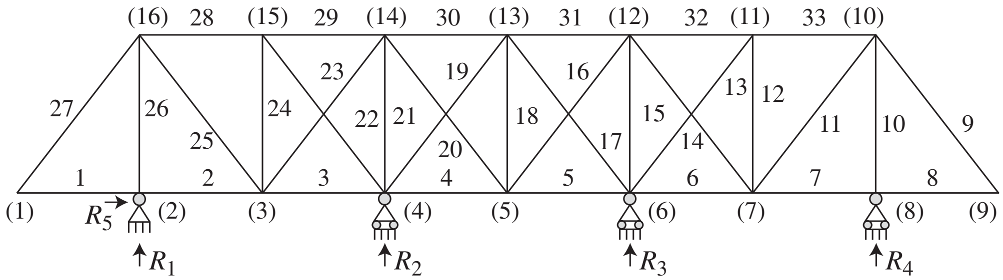
should be satisfied. Therefore, the total degree of internal indeterminacy is given by

$$D = (m + R - 2n) \tag{1.10}$$

We can develop similar equation for rigid-jointed frames. In the case of a frame we have three equations of equilibrium, namely, $\Sigma F_{\mathrm{V}} = 0$, $\Sigma F_{\mathrm{H}} = 0$, and $\Sigma M = 0$ at each joint. Therefore, we can write $3n$ equations for n number of joints in a frame. If m is the number of elements in a frame, and R is the total number of external reactions, the degree of total indeterminacy D is given by

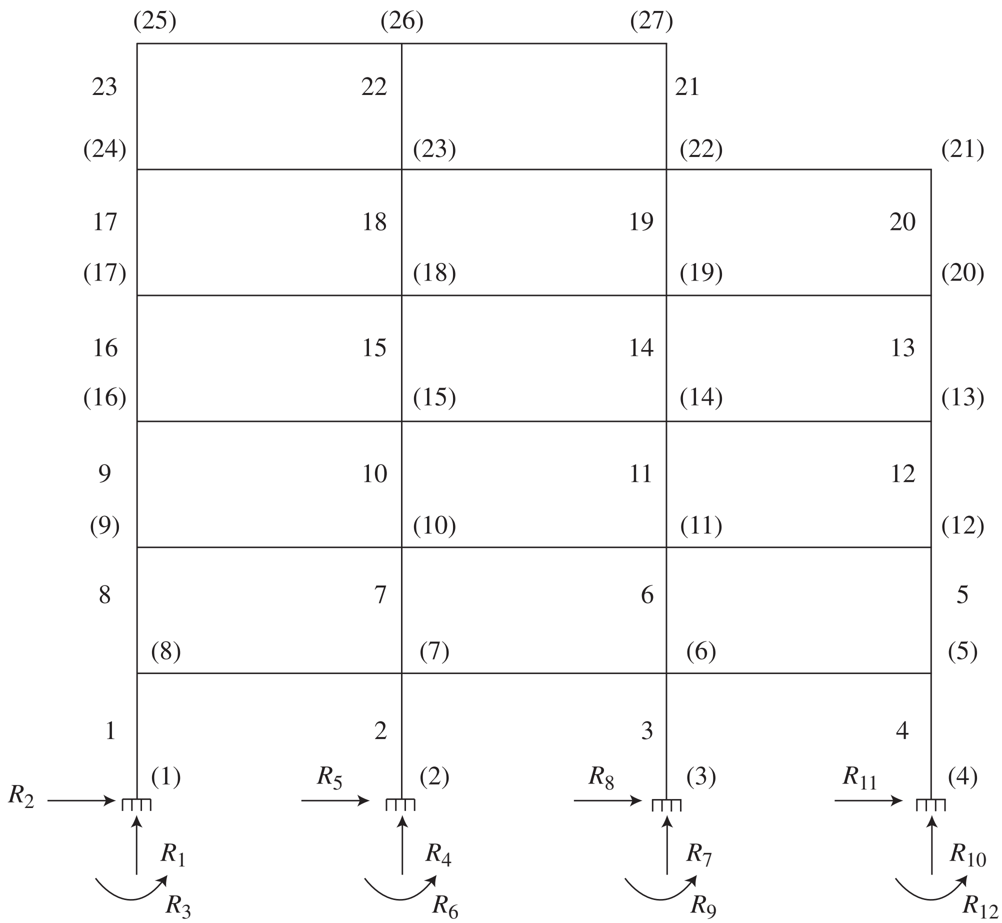
$$D = (3m + R - 3n) \tag{1.11}$$

We can easily comprehend by remembering that every member has three internal redundants so that $(3m + R)$ determines the total number of unknowns to be calculated against $3n$ numbers of joint equilibrium equations. Therefore, additional equations to be obtained from the conditions of compatibility at judiciously selected point in the structure are $(3m + R - 3n)$. We have shown the application of Eq. (1.11) in Fig. 1.23(a) and Eq. (1.5) in Fig. 1.23(b), respectively.



$m = 33$, $n = 16$, $R = 5$, $D = 6$

(a) Indeterminate truss



$m = 40$, $n = 27$, $R = 12$, $D = 51$

(b) Indeterminate frame

Fig. 1.23 Total degree of indeterminacy.

1.7 STATICS AND KINEMATICS

We have discussed in the previous sections that loads applied on structural systems in turn induce internal forces in the system. As a consequence of this, the system undergoes deformation which generically is called as *motion*. The study relating to forces and motions constitutes an applied science which is a branch of *mechanics*. The cardinal principle underlying this study is the equilibrium which we discussed earlier. It is a condition which describes a state of balance of a system when forces applied on it. As the structural system is initially at rest and in equilibrium too under a system of forces acting in it, we call that part of mechanics concerned with relations between these forces as *statics*.

1.7.1 Statics

We now introduce certain fundamental ideas in statics.

1.7.1.1 Applied and Reactive Forces

Fundamental to the field of mechanics is the concept of forces, and the composition and resolution of forces. A *force* is a directed interaction between bodies. Force interactions have the effect of causing change in the shape of or motion, or both, of the bodies involved. In SI unit, a force is expressed in Newton (N) or kilo Newton (kN). A force applied to a body tends to cause the body to translate in the direction of the force. Depending on the point of application of the force on the body, the force may tend to cause the body to rotate too. This tendency to produce rotation is called the *moment of force*. A moment is also called a *couple*. With respect to a point or line, the magnitude of this turning or rotational tendency is equal to the product of the magnitude of the force and the perpendicular distance from the *line of action* of the force to the point or line under consideration. The moment M of a force F about a point A is simply given as $M_A = F \times a$, where a is the perpendicular distance from the line of action of F to the point A. The parameter a is called the *moment arm of force*. A moment has the unit of force times distance. In SI unit, a moment is expressed in Nm or kNm.

Forces and moments that act on a system can be divided into two primary types, namely, *applied* and *reactive*. In engineering applications, *applied forces* are those that act directly on a structure, e.g., dead load, wind load, etc. *Reactive forces* are those that are generated by the action of one body on another and hence typically occur at connections or supports. They are also termed as reactions. The concept of reaction emanates from the *Newton's third law* of motion which states that for every action there is an equal and opposite reaction. More precisely, the law states that whenever one body exerts a force on another, the second body in turn always exerts on the first a force which is equal in magnitude and opposite in direction, and has the same line of action.

In Fig. 1.24(a), we have shown a block resting on a foundation. The block exerts its weight (W), which is a force, on the foundation. We consider the free body of the block as shown in Fig. 1.24(b) and analyse the forces acting on it. The weight W of the body exerts a force on the foundation in the downward direction as shown in Fig. 1.24(b). The foundation in turn exerts a reaction R on the block in the upward direction and in the same line of action of the weight.

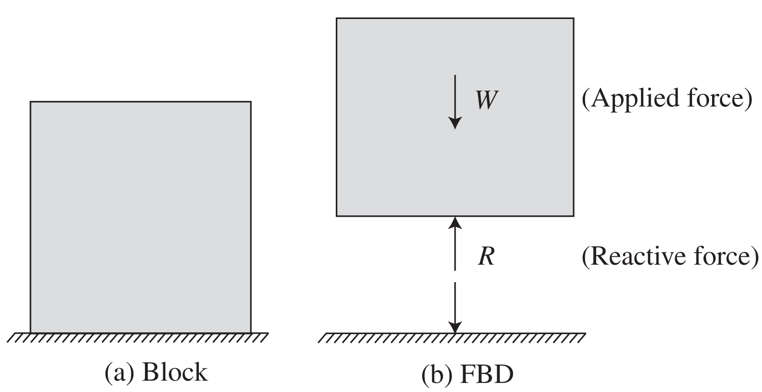


Fig. 1.24 Block resting on foundation.

In another example, we consider a beam placed on supports at its ends as shown in Fig. 1.25(a). Such an arrangement is called a *simple beam*. A load is applied on the beam in the *transverse direction*. In Fig. 1.25(a), the load W on the beam causes downward forces on the foundation and upward reactive forces R at supports are consequently developed. A pair of action and reaction forces thus exists at each interface between the beam and its foundations.

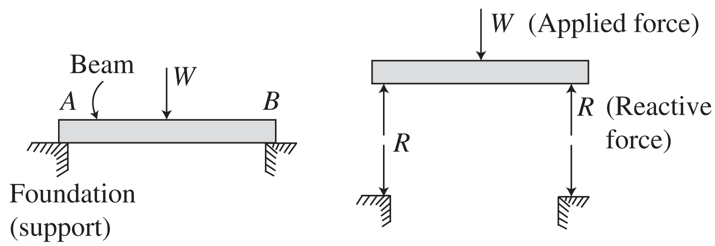


Fig. 1.25 A simple beam.

In some cases, moments form part of a reaction system as well. In Fig. 1.26(a), we have shown a beam projecting from a wall and loaded transversely as shown. This arrangement is called a *cantilever*. The FBD is shown in Fig. 1.26(b). Accordingly, the load on the beam causes a downward force on the beam as well as a moment too in the clockwise sense. The wall in turn exerts an upward reactive force on the beam and a moment in the counterclockwise sense.

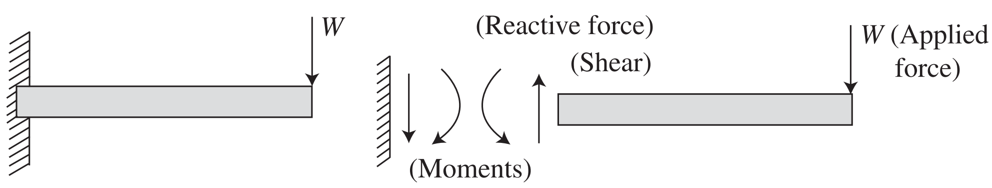


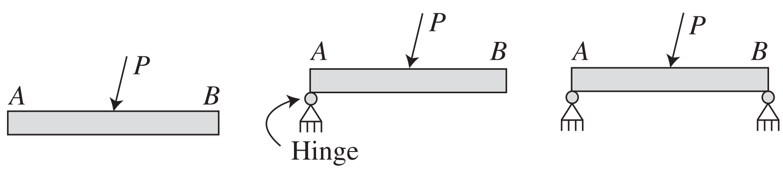
Fig. 1.26 Reactive system—forces and moments.

If a system as illustrated in the above examples is indeed in a state of equilibrium, it is quite obvious that the general conditions of equilibrium for a rigid body that were stated in Section 1.5 must be satisfied. The magnitude and direction of any reactive forces developed must be such that equilibrium is maintained and are thus necessarily dependent on the characteristics of the applied force system. Therefore, the entire system of applied and reactive forces acting on a body must be in a state of equilibrium. This is checked by using FBDs, also called *equilibrium diagrams*. Construction of FBDs and finding reactions for loaded structural elements are a common first step in a complete structural analysis.

1.7.1.2 Support Conditions

The support conditions are also called *boundary conditions*. Most structures are either partly or completely restrained so that they cannot move freely in space. Such restrictions on the free motion of a body are called *restraints* and are supplied by supports that connect the structure to some stationary body. The nature of the reactive forces developed on a loaded body depends on the exact way in which the body is either supported or connected to other bodies.

For example, consider a planar structure such as the bar *AB* shown in Fig. 1.27(a). If this bar were a free body and were acted upon by a force *P*, it would move freely in space with some combined translation and rotation. If, however, a restraint were introduced in the form of a hinge that connected the bar to some stationary body at point *A* as shown in Fig. 1.27(b), then the motion of the bar would be partly restricted and could consist only of a rotation about the hinge. During such a rotation, point *B* would move along an arc with point *A* as the centre. Essentially, the movement of point *B* is in the vertical direction. On the other hand if point *B* is restrained from moving in the vertical direction, the rotation about the hinge at point *A* would be prevented and thus the free motion of the bar would be completely restricted.



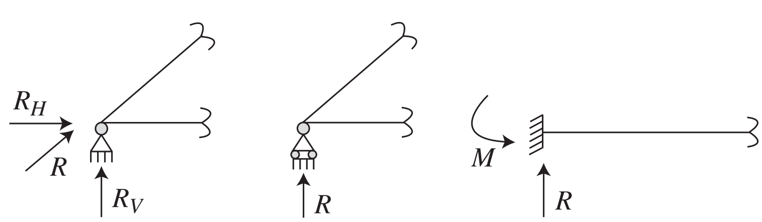
(a) Free body (b) Hinged at one end (c) Both ends hinged

Fig. 1.27 Boundary conditions.

Several basic types of support conditions are available. Of primary importance are pinned connections, roller connections, and fixed connections.

In *pinned connection*, also called *hinged support*, shown in Fig. 1.28(a), the joint allows the connected members to rotate freely but does not allow translations to occur in any direction. Consequently, the joint cannot provide moment resistance but can provide resistance to force in any direction. A hinge support supplies a reactive force, the line of action of which is known to pass through the centre of the pin but the magnitude and direction of which are unknown. These two unknowns of such a reaction could also be represented by the unknown magnitudes of its horizontal and vertical components, R_H and R_V, respectively, both acting through the centre of the pin.

A *roller connection* or *support* [Fig. 1.28(b)] also permits rotations to occur freely. However, it resists translations only in the direction perpendicular to the face of the support. It does not provide any force resistance parallel to the surface of the support. A roller support provides a reactive force that is applied at a known point and acts in a known direction but the magnitude is unknown.



(a) Pinned connection (b) Roller connection (c) Fixed support

Fig. 1.28 Different types of supports.

A fixed support [Fig. 1.28(c)] encases the member and hence completely restraints rotations and translations in any direction. Consequently, it can provide moment resistance and force resistance in any direction. A fixed support, therefore, supplies a reaction, the magnitude, point of application, and direction of which are all unknown. These three unknowns can also be considered to be a force that acts through a specific point but has an unknown magnitude and direction and a couple of unknown magnitude.

Generally, a reactive force is represented by an arrow with a single head as ↑ or with a dash across an arrow as ⍿.

1.7.1.3 Evaluation of Reactions

So far we have been discussing that a system of forces applied on a structural system generates reactive forces at the supports. If the supports are replaced by the reactions that they supply to the system, it will be acted upon by a general system of forces consisting of the known loads and the unknown reactions. If the structure is in static equilibrium under these forces, the three equations of static equilibrium stated in Eq. (1.1) for planar structure can be written in terms of the known loads and the unknown elements defining the reactions. The simultaneous solution of these three equations will, in certain cases, determine the magnitude of the unknown reactions.

1.7.1.4 Stability of Structural System

A fundamental consideration in designing a structure is that of assuring its *overall stability* under any type of possible loading condition. A structure, as a whole unit, might overturn, slide, or twist about its base especially when subjected to wind or seismic force applied in the horizontal direction as shown in Fig. 1.29. Tall structures with small bases are prone to overturning effects. Seismic forces tend to cause overturning or sliding actions, but they are dependent in magnitude on the weight of the structure because of the inertial effect of earthquake force. If a system is planned unsymmetrically the horizontal

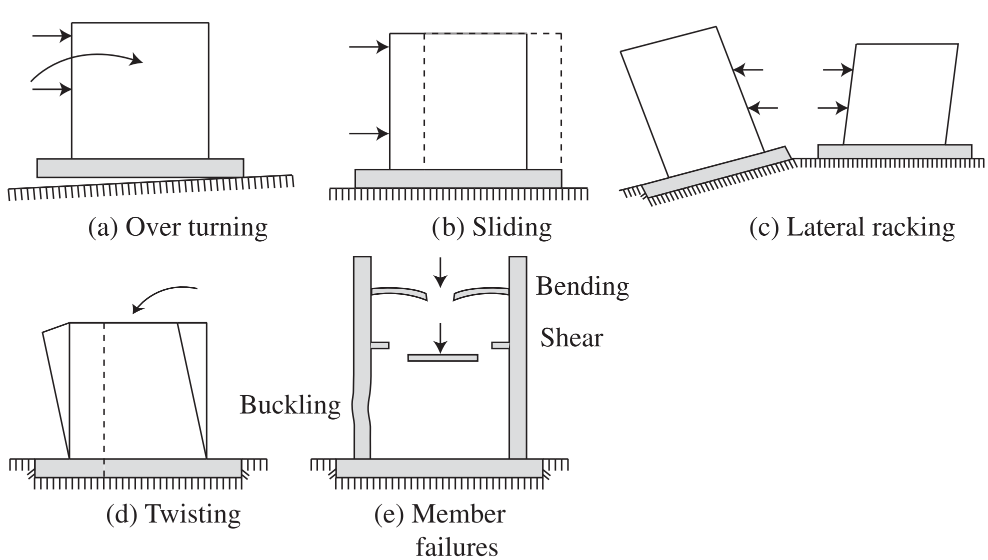


Fig. 1.29 Failure of overall structural system.

forces may cause its twisting because of the eccentricity between the centre of mass and centre of stiffness.

Overturning or twisting need not be caused only by horizontally acting forces. Because of eccentricity in construction a system can overturn under its own self-weight.

All structures undergo some changes in shape under applied loads. In a stable structure the deformations induced by the load are typically small, and internal forces are generated in the structure by the action of the load that tend to restore the structure to its original shape after the removal of the load. In an unstable structure, the deformations induced by a load are typically massive and often tend to increase continuously as long as the load exists. An unstable structure does not generate internal forces that tend to restore the structure to its original configuration. As a load is applied to unstable structures, they collapse instantaneously and totally. Therefore, a structural designer should consider his primary responsibility to ensure that a proposed structure does, in fact, form a stable configuration because stability is a crucial issue in the design of structures.

1.7.2 Kinematics

In Section 1.7.1, we had discussed about one part of *mechanics* called *statics* which deals mainly with forces and moments applied on a structural system. There is another part of mechanics called *dynamics* which refers to the other part of mechanics dealing with rigid bodies in motion. Dynamics is divided into two parts, namely, *kinematics* and *kinetics.* Kinematics is the study of the geometry of motion; it is used to relate displacement, velocity, acceleration, and time, without any reference to the forces causing the motion. Kinetics is the study of the relation existing between the forces acting on a body, the mass of the body, and the motion of the body. It is used to predict the motion caused by the given forces or to determine the forces required to produce a given motion. However, we restrict our discussion here to only about kinematics.

In structural analysis, kinematics refers to quantities associated with geometry, the position changes, or the deformation of geometry. This term is used in opposition to the term 'statics'.

Displacement refers to a *translation* or a *rotation* of a specific point in a structure. For example, we consider a simple beam as shown in Fig. 1.30. It is free to undergo displacement in the form of translation in the direction perpendicular to its own axis as shown in Fig. 1.30 which is called *deflection* as well as rotate at its supports. The quantity δ is the vertical translation of the beam and is called *deflection of the beam*. The rotation at support A is θ_A and at support B is θ_B. These rotations are called *slopes*.

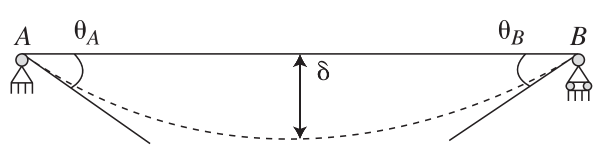


Fig. 1.30 Displacement of simple beam.

A joint in a truss can translate in two mutually perpendicular directions as shown in Fig. 1.31. The joint C can displace along x and y directions only. The joint cannot rotate.

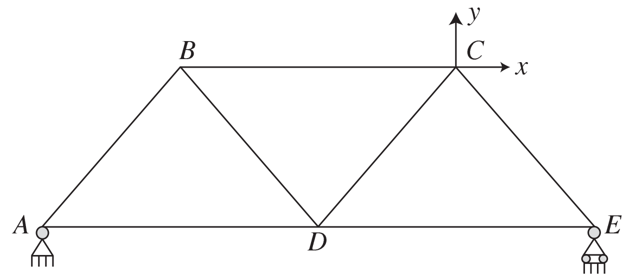


Fig. 1.31 Displacement in a truss.

A rigid frame can undergo translation and rotation at joints as shown in Fig. 1.32. The joint B in Fig. 1.32 undergoes horizontal translation Δ_B and a rotation θ_B.

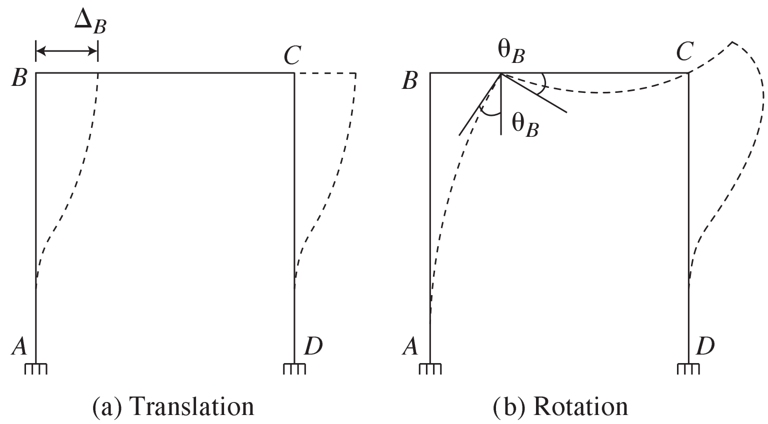


Fig. 1.32 Displacement in a frame.

These translations and rotations constitute the degrees of freedom of a structural system discussed in Section 1.6.1. In structural analysis these displacements other than that at the supports are, in general, not known. Therefore, the objective of the analysis is to determine their values. The number of the independent joint displacement in a structure is called the *degree of kinematic indeterminacy* or the *number of degrees of freedom*. This number is a sum of the degree of freedom in rotation and in translation. For example, in a two span beam shown in Fig. 1.33 the degree of kinematic indeterminacy is 2 since the structure can undergo rotations at joints B and C and these are indeterminates. Rotation D_1 at joint B and rotation D_2 at joint C are the two unknowns. Because support A is fixed, the rotation D_3 is zero which is a known quantity and hence determinate. More details on kinematic indeterminacy specific to indeterminate beams and frames are given in Chapter 10 and specific to indeterminate trusses in Chapter 22.

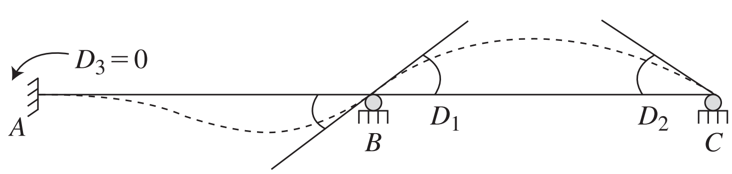
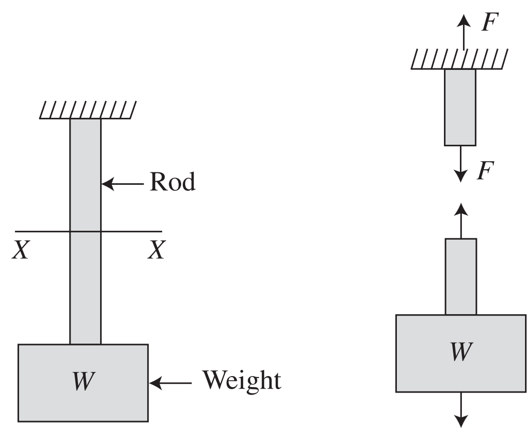


Fig. 1.33 Degree of kinematic indeterminacy.

1.8 STRESS RESULTANTS

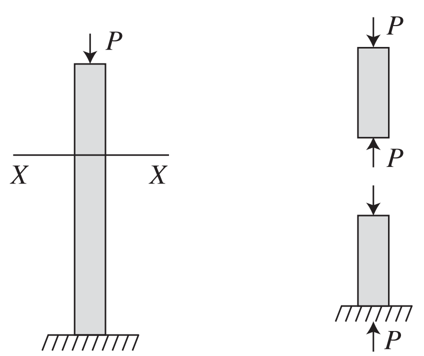
Forces and moments can either be external or internal. Forces and moments or moments that are applied to a structure are described as *external*, e.g., gravity and wind loads applied on a structure. The action of an external force on a structure due to its environment or use produces *internal* forces within a structure. These are called *stress resultants*. The most common stress resultants are tension, compression, bending, shear, torsion, and bearing. Tension and compression result from the axial loading of a member whereas bending and shear result from transverse loading. Torsion arises in the context of twisting of a member. Bearing is developed at the interface between a member and its support.

We can determine the stress resultants by passing a section across the member, cutting it into two segments, and analysing the free body. For example, we consider an axially loaded rod by way of hanging a weight at one end as shown in Fig. 1.34(a). We take a section XX as shown in Fig. 1.34(a). The FBD of the cut section is shown in Fig. 1.34(b). At cut section we have a resultant F which will balance the weight W and acts in the same line that of W and hence $F = W$. As W acts downward, F acts upward. It is tension and is the stress resultant. It is quite easy to visualize from Fig. 1.34(b) that W and F are in equilibrium. As no other force is applied on the rod, at every section, the stress resultant F remains the same.



(a) Hanging weight (b) FBD

Fig. 1.34 Axial tension member.



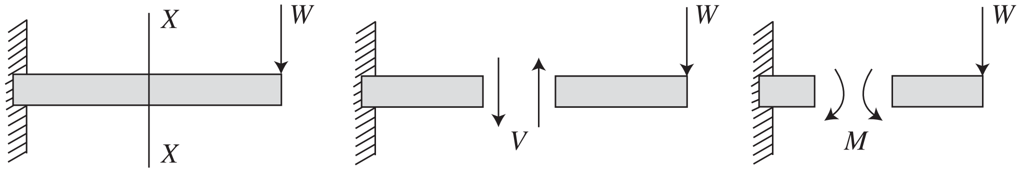
(a) Compressive force (b) FBD

Fig. 1.35 Axial compressive member.

Next, we consider an axial bar subjected to a compressive force as shown in Fig. 1.35(a). We take section XX and draw FBD as shown in Fig. 1.35(b). At the cut section the stress resultant is P. From Fig. 1.35(b), we can observe that the applied force and the stress resultant are in equilibrium. The stress resultant in this case is compression.

We now consider a cantilever as shown in Fig. 1.36(a). At section XX we have a shear V [Fig. 1.36(b)] and a bending moment M [Fig. 1.36(c)] as stress resultants.

Tension, compression, shear, and bending moment are the common stress resultants that we come across frequently in the structural analysis of different types of systems that we will discuss in subsequent chapters in this book. A structural system may have one or more of the stress resultants depending on the type



(a) Cantilever (b) Shear (c) Bending

Fig. 1.36 Cantilever beam.

of loading and its geometrical configuration. The methodology of determination of these stress resultants for various structures is covered in the following chapters.

1.9 ANALYSIS OF DETERMINATE AND INDETERMINATE STRUCTURES

The objective of a structural analysis is to check whether the given structural system under a set of applied loading is safe enough by sustaining the loads and it has adequate load path to transfer the loads to the foundation without collapsing. In order to ensure proper functioning or serviceability of the structure, it is necessary to consider all possible combination of loads. In the analysis, we should consider the worst combination of loads that is going to be applied on the structure during its life time. Also, we should verify that the structure with external forces and internal stress resultants is in a state of static equilibrium and that the displacements resulting from applied loading are within permissible limits. These requirements are common to both determinate and indeterminate structures.

1.9.1 Determinate Structures

In the determinate structure, the analysis is carried out, first, by determining the reactions at supports. This is accomplished by applying the static equilibrium equations to the whole structure and thus determining the unknown reactions at supports. Once the reactions are known, it is easier to consider free body of various parts of the structure and evaluate the different stress resultants. With the known stress resultants, we can compute the deformation in various members of the structure using the basic principles of mechanics and material characteristics. This method of analysis will be applied for solving problems of determinate plane trusses in Chapter 2 for the evaluation of forces and in Chapter 6 for the determination of displacements. Similarly, in the case of determinate simple beams, in Chapter 3, we will demonstrate this method in the evaluation of various stress resultants and in Chapter 6 various procedures for the determination of displacements in simple beams will be illustrated.

1.9.2 Indeterminate Structures

In the case of indeterminate structures, it is difficult to determine the reactive forces and stress resultants mere with the application of the conditions of equilibrium alone. We still require setting up of additional equations. These additional equations are usually derived from the geometrical conditions of the structure or from the displacements of the structures. In indeterminate structures we have more number of unknown reactions than that could be determined by the application of conditions of equilibrium. These excess unknown reactions are called *redundants*. In one method of analysis called *force method* or *flexibility method*, we remove the selected redundants and thus render the structure as determinate one. This determinate structure is called a *primary structure* or a *released structure*. We, then, calculate the displacement of the primary structure corresponding to the redundant under the applied loading. In the released structure, we apply a unit load corresponding to the redundant and evaluate the corresponding displacement. This is called the *flexibility coefficient*. Now, we write an equation on the geometrical conditions at the supports from where the redundants have been chosen. This equation consists of a sum of the displacement of the released structure at the location of the redundant and the product of the flexibility coefficient times the redundant and the sum in turn equated to the boundary condition prevalent in the original structure. This equation is called the *compatibility equation* because it is based on the boundary conditions prescribed in the original structure. The number of equations is equal to the number of redundants. This method of analysis will be adopted in Chapter 15 for solving problems involving indeterminate beams, plane frames, grids, and trusses.

The second method of analysis for the solution of indeterminate structures is called the *displacement method*. In this method of analysis, the independent unknown displacement components involved in the structure are first identified. They are considered as the basic unknowns involved in the problem. The internal forces in the structure are then expressed in terms of these unknown displacements, using force–displacement relations. For each unknown displacement component, a corresponding equilibrium equation is written in terms of known external forces and the unknown internal forces, which are expressed in terms of displacements. The number of these equations is equal to the unknown displacements. We solve these equations simultaneously to determine these values. Once these displacements are known, we can back substitute and compute the internal forces. The *slope-deflection method* discussed in Chapter 10 and the *moment distribution method* discussed in Chapter 11 belong to this category of analysis.

The force method and displacement method of analysis are called *classical methods of analysis*. In both these methods if the number of equations is large, then calculation by hand becomes tedious and cumbersome. Therefore, we resort to matrix algebra for solution of equations. With the advent of computer, the matrix method of analysis has gained popularity and the solution of structural problem involving complex structures has been made easy. In Chapter 15, we will discuss the *matrix force method of analysis*. Similarly, the *matrix stiffness method of analysis* is presented in Chapter 16.

In the modern *finite element analysis* (FEA), we discretize the given structure into *finite elements* and write their flexibility or stiffness matrices. A flexibility matrix consists of a set of flexibility coefficients which are nothing but the displacement per unit force. A stiffness matrix consists of a set of stiffness coefficients which are nothing but the force per unit displacement. Then we assemble the element stiffness matrices into a structural stiffness matrix. We write the equations of equilibrium using the stiffness matrix and the external force vector and solve for the unknowns. A brief introduction about FEA is presented in

Chapter 17. Because of the difficulties associated with the selection of redundants and the corresponding released structures, flexibility method is not easily amenable for computerization. Therefore, computer codes are developed based mainly on stiffness method of analysis.

Nowadays a number of FEA software packages such as ANSYS, ABACUS, NISA, NASTRAN, and so on, are commercially available to assist structural analysts for modelling as well as for solving problems involving thousands of degrees of freedoms. However, these issues are beyond the scope of this book.

Recapitulation of Important Formulae

- $\sum F_H = ;\quad \sum F_V = 0;\quad \sum M = 0$
- $R = n + c$
- $I = 3p$
- $2n = m + r$
- $2n < m + r$
- $m + R < 2n$
- $D = m + R - 2n$
- $D = 3m + R - 3n$

EXERCISES

1. What is a load path?
2. List the different types of loads.
3. List the types of materials used in construction.
4. What are the different forms of structures? Explain them.
5. What are the different structural systems available?
6. Write notes on (a) beam and slab in situ concrete and (b) 'flat-slab' in situ concrete.
7. Describe the lateral load-resisting systems.
8. Explain in detail the building frame systems.
9. What is ductility?
10. What is a moment-resisting frame? Explain its different types.
11. Describe the various components of structural system.
12. What is a linear system and state the assumptions involved in it?
13. Explain structural non-linearity.
14. Describe the conditions of equilibrium.
15. What is a free body? Explain how is it useful in the analysis?
16. What are determinate and indeterminate structures?
17. What are the different types of indeterminacy present in structures and how to calculate the degree of indeterminacy in each case?
18. Explain what is meant by statics.
19. What are the stress resultants and how to determine them?
20. Explain briefly the different methods of analysis of structures.