Circuits and Networks

SECOND EDITION

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Oxford University Press is a department of the University of Oxford.
It furthers the University's objective of excellence in research, scholarship,
and education by publishing worldwide. Oxford is a registered trade mark of
Oxford University Press in the UK and in certain other countries.

Published in India by
Oxford University Press
YMCA Library Building, 1 Jai Singh Road, New Delhi 110001, India

First Edition published in 2010
Second Edition published in 2016

ISBN-13: 978-0-19-946092-2
ISBN-10: 0-19-946092-2

Typeset in Times New Roman
by Tranistics Data Technologies Pvt. Ltd, Kolkata 700091
Printed in India by Replika Press Pvt. Ltd, Haryana 131028

To our families, friends, mentors, and guides
for being there

M.S. Sukhija
T.K. Nagsarkar

Features of the Book

Coverage

Simplified lucid explanation in chapters with newly introduced topics such as Terminating Half Sections, Composite filters, and Bode Plots. Additional solved examples for practice and better understanding of concepts in every chapter. MATLAB-based solved and unsolved problems.

Key Concepts

- Classification of voltage and current signals into periodic, non-periodic, and random signa
- Define the various terms related to voltage anc frequency, period, instantaneous value, aver (RMS), or effective value
- Representation of periodic and non-periodic si functions
- Combination of different functions by addition to obtain complex waveforms.

Solved Examples

Solved examples with screenshots to demonstrate the use of MATLAB for solving problems based on circuit analysis, design, and synthesis.

```
>> t = linspace(0, 1, 10000);          % divides the time axis between 0
                                                 to 1 sec. into 10000 parts
>> v = 2*(1–4*t)·*exp(–4*t);          % input voltage across the inductor
>> plot(t, v)                                  % plot v (t) versus t
>> grid on                                       % grid is turned on
>> xlabel('Time t in sec')                          % label x-axis
>> ylabel('Inductor voltage v in volts')            % label y-axis
```


Key Concepts

- Describing the method of nodal analysis for computation of node voltages in a circuit
- Describing the method of mesh analysis technique for computation of mesh curre
- Form
- Use c

Key Concepts

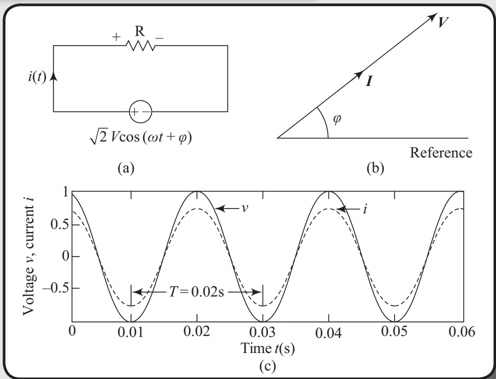
- Classification of voltage and current signals that occur in elect into periodic, non-periodic, and random signals
- Define the various terms related to voltage and current signals suc frequency, period, instantaneous value, average value, root m (RMS), or effective value

Key Concepts

Key Concepts at the beginning of every chapter give a brief description of the topics covered in the chapter and the concepts taught.

Illustrations

Neatly Illustrations circuits for easy visualization of circuits using standard symbols for electrical and electronic components. Graphs depicting a comparative study of electrical quantities and phasor diagrams for better understanding of the phase relationships.



Solved Examples

Numerous solved examples in every chapter provide step-wise solutions for practice and better understanding.

Example 7.7 Find the instantaneous and average power absorbed by the network in Fig. 7.1 if $v(t) = 220\cos(314t + 30°)$ and $i(t) = 15\cos(314t - 12°)$A.

Solution From Eq. (7.1)

$$p(t) = 220\cos(314t + 30°) \times 15\cos(314t - 12°) \qquad (7.7.1)$$

Rewrite Eq. (7.7.1) by making use of the trigonometric identity as follows:

$$p(t) = 220 \times 15 \times \frac{1}{2}\left[\cos(628t + 18°) + \cos(42°)\right]$$
$$= \left[1226.19 + 1650\cos(628t + 18°)\right]\text{W} \qquad (7.7.2)$$

Phase angle difference between voltage and current $\varphi = 30° - (-12)° = 42°$

From Eq. (7.9) $P_{avg} = \frac{220}{\sqrt{2}} \times \frac{15}{\sqrt{2}}\cos(42°) = 1226\,\text{W}$, which is the same as the constant term in Eq. (7.7.2)

Exercises

A recap of the important concepts is provided at the end of every chapter for easy reference of faculty as well as students. The recap is followed by some typical MCQs and Numerical Problems in the 'Exercises' section which further assist in revision. These features prove to be a rich resource for self-evaluation for the students as well as for class evaluation by the faculty.

Recapitulation

- Instantaneous power in an AC circuit:

$$p(t) = v(t)i(t) = VI\left\{\cos\varphi\left[1 + \cos 2(\omega t + \theta_i)\right] - \sin\varphi \sin 2(\omega t + \theta_i)\right\}\text{VA}$$

Exercises

Review Questions

1. What is instantaneous AC power?
2. Define and explain (a) average, (b) active, and (c) reactive powers.
3. Derive expressions for active and reactive powers.
4. Discuss the variation of power in a pure (a) resistor, (b) inductor, and (c) capacitor.
5. What is a power triangle? Explain its significance.

Appendices

MATLAB and Pspice utilities in Appendices A and B demonstrate their versatility in coding and solving problems in electric circuits. In Appendix C answers to end chapter unsolved problems have been included. The self-appraisal test in Appendix D provides a tool to the students to quickly identify their strengths and weaknesses and take remedial action.

Appendix A

MATLAB ... In Line...

A.1 Introduction

MATLAB is an acronym for **MAT**...

Appendix B

Linear Circuit Analysis with PSpice

...he simplest of numerical solutions such ...ous equations or ordinary differential ...

Appendix C

Unsolved Problems

Chapter 1

1.5 [18 C]

1.6 [60,000sin(4000t)C]

1.7 [(i) 21.6 kC (ii) 36 W (iii) 36 Wh

1.17 [(i) 90 Ω (ii) 27 V (iii) 8.1 W]

Appendix D

Self Appraisal Test

D.1 Introduction

The self appraisal test (SAT) consists of selected multiple choice objective questions. The test has been designed for the reader to quickly evaluate the level of understanding and grasp the principles of analysis, design and synthesis, along with their applications in linear circuits. The appraisal grid in D.4 enables the reader to identify areas which require strengthening.

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A list of reference material at the end of the book provides details of the resources that can be accessed by faculty as well as students for further reading.

Preface to the Second Edition

A survey undertaken by Oxford University Press (India) to increase the effectiveness of our *Circuits and Networks*: *Analysis*, *Design*, *and Synthesis* book, inputs received from its readers and our own numerous references forms the basis of this revision. Following are the important changes:

- The number of solved examples, to further enhance the understanding of the theoretical principles and their applications, has been doubled.
- Techniques to use MATLAB applications in problem solving have been clearly demonstrated.
- Language has been made crisper without compromising upon its flow and understanding.
- New topics have been included to complete the coverage of the circuits' curricula.
  Chapter 3: Saw Tooth Analyses and Doublet
  Chapter 8: Four-wire Systems
  Chapter 13: Bode Plots
  Chapter 14: Terminating Half Section'
  Chapter 16: Functioning of Composite Filters
- A note 'FOR THE STUDENT.....' has been added to guide him/her to master this very important course.

The authors feel that these changes will enhance the utility of the textbook.

The authors would like to thank The Math Works Inc. for permitting the use of MATLAB in development of applications for problem solving. They would also wish to express their gratitude to the editorial team at OUP and the myriad readers of their work for maintaining a constant flow of inputs which has formed the basis of this revision.

For the Student...

Electric circuits form the basis of all engineering disciplines involving voltage/current. This book has been designed to make the learning of circuit analysis, design, and synthesis exciting and enjoyable. At the same time it lays a strong foundation for the subjects to be taught in the ensuing semesters such as 'Network Theory' and 'Control Engineering'. While the book will serve to strengthen the physical concepts and applications taught by the faculty, the task of learning and understanding the techniques and methods will have to be taken up by you. To help you to enjoy the experience and do well in the circuit analysis, design, and synthesis course we put down ideas which you should bear in mind.

- Regularity in the study of this course is essential.
- Large number of solved examples having varying difficulty levels has been included. To understand the intricacies of the solutions, solve the examples yourself. Do not read the solution.
- For strengthening the principles learnt, attempt the chapter-end problems and answer the objective type questions.
- Keeping in view the usefulness of MATLAB in problem solving in circuits and other courses, several examples have been included showing how to develop MATLAB applications for problem solving. Appendix A quickly introduces you to the MATLAB commands. The surest way of learning MATLAB is to start developing your applications after you have learnt a few commands.
- Recapitulation summarizes what has been learnt and also serves as a reference check point.
- Attempt to answer the Review Questions. This will add to your skills learnt in the classroom.
- Considerable effort has been put in to make the material and language of the text student friendly and also learning the application of the principles of circuit engineering a fruitful experience. The related physics and mathematics help in understanding the theory and lays the foundation for other voltage/current engineering courses.

While we know that you will thoroughly enjoy doing the course do not hesitate to contact us if you feel we can help you.

M.S. Sukhija
T.K. Nagsarkar

Preface to the First Edition

And, when you want something, all the universe conspires in helping you to achieve it

Paulo Coelho in 'The Alchemist'

Electric circuits make up an inseparable part of the gadgets and equipment of modern-day living. A universe without circuits is unimaginable. An in-depth understanding of the theoretical concepts and practical applications of circuits and their analyses and design are imperative to grasp the fundamentals of other disciplines of engineering, such as power systems, computers, telecommunication, etc.

The aim of the book is to present in an organized manner the fundamentals and principles of circuits, and to enthuse the readers to recreate, rediscover, and experience the excitement of analysis, design, and synthesis of circuits as practised. The book provides an exhaustive study of the response of linear networks in time and frequency domains to a wide variety of excitations, including the impulse excitation. Overall, the book helps to build an irrevocable bond between conceptualizing the theoretical principles and applications through problem solving while at the same time helping students prepare for the rigours of meeting the course requirements.

The contents of the book have been formulated, based on a study of syllabi of foremost national and international universities. Although, at the national level, in most institutes a basic course in electrical engineering is required to be studied at the undergraduate level, yet several basic concepts of electricity are reviewed from the perspective of circuit equations. Knowledge of differential and integral calculus is the only prerequisite for this book.

Pedagogical Features

Comprehensive coverage of topics with equal emphasis on theory and practice Commencing with an inspirational quote, each chapter then familiarizes the students with the objectives of the contents of the respective chapters. The language has been kept simple to ensure that students are easily able to grasp the fundamentals of circuit analysis.

Numerous solved examples interspersed with the text that apply theoretical concepts learnt All new terms and techniques are lucidly defined and their applications described through solved examples which immediately follow the derivation of the circuit equations.

Recap of key formulae and numerous problems for practice at the end of every chapter The chapter-end problems, which expose the reader to a increasing level of difficulty, follow the general pattern of the text in the chapter. To further hone the skills of the readers, multiple-choice questions have been included at the end of each chapter.

Separate appendices on MATLAB and PSpice Keeping in view the advancements in computer software tools and the need to keep the text abreast with the present-day requirements, Appendices A and B have been added to describe the utilization of MATLAB and PSpice, respectively, in circuit analysis. It may be emphasized that both MATLAB and PSpice should be seen as tools for doing away with the drudgery of computations. They cannot be substituted for the physical interpretation of the results which exclusively fall within the domain of the reader.

Self appraisal test at the end of the book for a holistic chapter-wise evaluation Consisting of 170 multiple-choice objective questions, with ten questions randomly selected from each chapter*, the self appraisal test help quickly assess the reader's strengths and weaknesses.*

Contents and Coverage

The book is divided into 17 chapters. A brief description of each chapter is given below:

Chapter 1 delineates between different electric materials, defines the basic electric terms, circuit elements and their characteristics, various types of independent and dependent energy sources, along with their transformations are described. Ohm's law and Kirchhoff's laws, methodology to write circuit equations and manipulation of series-parallel networks are also covered.

Chapter 2 describes the formulation of nodal and mesh equations of circuits and their solution techniques. Use of supernode and supermesh in circuit analysis is detailed. Choosing between the nodal and mesh analyses techniques, for a given circuit configuration is also described.

Chapter 3 classifies the different types of voltage and current signals and defines the associated terms, such as frequency, period, instantaneous, average, and RMS values. Representation of periodic and non-periodic signals as mathematical functions and their combinations into complex waveforms has been described.

Chapter 4 presents natural and forced responses (step-, pulse-, and impulse function) of *RL*, *RC*, and *RLC* circuits.

Chapter 5 introduces sinusoidal voltage and current functions and their phasor representation. Phasor diagrams, conceptualization of impedance, admittance, application of nodal and mesh analyses techniques and source transformation, using phasors to determine the forced response of different configurations of circuits is included. Series and parallel *R*, *L*, and *C* resonant circuits, determination of *Q*-factor, and bandwidth is also described.

Chapter 6 defines and explains the use of network theorems such as superposition, compensation, Tellegen's, and Millman's theorems. Determination of Thevenin and Norton equivalent circuits and employing the same for maximum load transfer is also included. Concept of reciprocity for linear, time-invariant networks is described.

Chapter 7 defines instantaneous and average power, conceptualizes complex, active, reactive, and apparent power; and introduces load power factor and its importance in power factor improvement.

Chapter 8 introduces AC poly-phase systems and advantages of three-phase systems over single-phase systems in power generation and transmission. Phasor representation of three-phase generated voltages and techniques for analysing Y-connected and Δ-connected three-phase circuits are presented. Methods for measurement of active and reactive power in three-phase systems are also outlined.

Chapter 9 includes concepts of self inductance, mutual inductance, coefficient of coupling, and highlights procedures for writing response equations, including energy computations, in the time domain for circuits containing mutual inductances, using the dot convention. It also explains the development of analog of coupled circuits and principle of working of linear, auto, and ideal transformers. The chapter also outlines the calculation of reflected impedance and its application in impedance matching.

Chapter 10 covers graph vocabulary and application of graph theory to circuit analysis. Formulation of incidence matrices and their use in developing nodal and mesh network equations for response analysis is explained in detail. Duality in networks is also outlined.

Chapter 11 defines Laplace and inverse Laplace transforms and builds a basic understanding of their properties. Laplace transforms of commonly employed forcing functions are derived. Application of Laplace and inverse Laplace transforms is explained to study the response of circuits whose simulation leads to differential equations. Initial and final value theorems are defined and their applications are described.

Chapter 12 includes conceptualization and development of equivalent circuits, by including the initial conditions, in the complex frequency domain. Development of impedance and admittance functions of differently configured circuits and their analyses is also outlined.

Chapter 13 describes mathematical conceptualization, development, and characteristics (in terms of their poles, zeroes, and gain constants) of transfer functions in impedance/admittance forms. Restrictions on locations of poles and zeroes and the calculation of amplitude and phase responses, in time domain, from the transfer function of a network are detailed. The Routh-Hurwitz stability criterion and its application are also included.

Chapter 14 categorises two-port networks in z, y, h, g, t, t'-parameters. Co-relation between the parameters and their applications in circuit analysis is outlined. Formulae for input admittance, voltage gain, current gain; and Thevenin equivalent at output ports are derived. Series, parallel, and cascade connections of two port networks have been discussed.

Chapter 15 explains the Fourier series transformation, in trigonometric and exponential forms, and determination of effective values and power of non-sinusoidal signals/functions. It builds a rigorous understanding of even, odd, and half-wave symmetry along with the determination of the Fourier coefficients. Line spectra computation of harmonic component of periodic waves is outlined. Finally, skills to apply Fourier transforms to analyse circuits in the frequency domain are described.

Chapter 16 introduces basic concepts, characteristics, and classification of filters and attenuators. It explains the working of LP, HP, constant *K*, and *m*-derived filters and develops techniques to analyse *T* and π-filter networks. The methodology to analyse various types of attenuators and compute insertion loss has also been discussed.

Chapter 17 describes the basic tools for synthesising passive networks from known input and response. Realizability of physical passive networks (positive real functions and Hurwitz polynomial), necessary and sufficient conditions for a function to be positive real, and methodology for synthesizing simple linear passive one port networks, consisting of *L* and *C* elements only, are discussed at length.

Appendix A presents a discourse on MATLAB software and its applications to solve circuit analysis related problems. **Appendix B** is a tutorial on linear circuit analysis with PSpice. **Appendix C** includes answers to the end-chapter unsolved problems. **Appendix D** contains a self appraisal test constituted of multiple choice objective type questions and an appraisal grid for evaluation. The results of the test will help a student to identify his/her topics of weakness.

The underlying methodology of one author preparing the text first and the other looking at the same from the perspective of teaching and learning requirements of the readers, as in the earlier two textbooks, *Basic Electrical Engineering* and *Power Systems Analysis*, remains unaltered. We sincerely hope that students and faculty members will find this book useful and enriching.

Acknowledgements

This work on *Circuits and Networks: Analysis, Design, and Synthesis*, in essence, is the metamorphosis of the knowledge and experience gained through teaching electrical engineering subjects, both at the graduate and postgraduate levels. A project of such width and depth cannot be completed without interaction and inputs from faculty and peers. Therefore, the authors acknowledge with a profound sense of gratitude the contributions made by the faculty during their tenure of teaching activity.

The authors wish to also gratefully recognize Dr (Mrs) Harjit Kaur Sukhija for her inputs in tackling difficult mathematical problems. The comments and invaluable ideas offered by the students are gratefully acknowledged, since they have helped the authors focus on their needs in preparing the text. No acknowledgement can be completed without thankfully recording the inputs made, particularly in finalizing the contents and structure of the book, by the content developers and the editorial staff of the Oxford University Press (OUP) India.

M.S. Sukhija

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Brief Contents

Detailed Contents

List of Symbols

$a = (N_1/N_2)$	turns ratio of the transformer, area of cross section, m^2, real constant
a_0, a_n, b_n	Fourier coefficients
a_{ij}	element of the incidence matrix
A	constant, unit of current in amperes, bus incidence matrix
$\hat{A}$	the element node incidence matrix
A, B, C, D	transmission parameters or *ABCD* or *t*-parameters
A', B', C', D'	inverse transmission parameters or *t*'-parameters
A_1	residue
A_n	Fourier coefficients in exponential form
A_P	attenuation in power signal
A^t	the transpose of the bus incidence matrix A
A_V, A_I,	attenuations in voltage and current signals, respectively
b	number of branches, real constant
BE	band elimination filter
BP	band pass filter
c	the intercept of the line on the ordinate (vertical axis)
c_{ij}	the element of the cut-set incidence matrix
$\hat{C}$	the enhanced cut-set matrix
C	capacitance of a capacitor in Farad (F), basic cut-set incidence matrix
C_{eq}	equivalent capacitance
d	diameter, m
dB	Decibel
di	change in the current, A
di_1	increase of current in coil 1 in *dt* seconds
$d\Phi$	change in flux due to change in current *di*, Weber
$d\Phi_{12}$	increase of mutual flux in coil 2 due to the increase of di_1 A in the coil 1, Weber
D	determinant of the matrix
$D(s)$	denominator polynomial is *s*, Laplace transform of the input signal
e	negative charge of an electron, induced voltage in volts, number of elements
$e(t)$	induced voltage (V)
$e_2(t)$	emf induced in coil 2
e_A, e_B, e_C	instantaneous values of the induced emfs in phases A, B, and C, respectively
$\mathcal{E}$	electric field intensity
E	RMS value of the voltage
$E(s)$	excitation function
E_A, E_B, E_C	RMS voltages of phases A, B, and C, respectively
f	frequency or the number of cycles per second or Hertz (Hz)
$f(t)$	function of time, periodic function in time *t*
f_C	cut-off frequency
f_{rs}, f_{rp}	resonant frequency of the series arm and the parallel resonance (or anti-resonance) frequency of the shunt arm
$\mathcal{F}$	magnitude of electrostatic force in Newton (N)
$\mathcal{F}^{-1}$	used to indicate the inverse Fourier transform operation
$\mathcal{F}$	used to indicate Fourier transform operation
$F(s)$	function of *s* and independent of time *t*, transform network function, driving point immittance function
$F(\omega)$	Fourier transform of the function $f(t)$
F_{avg}	average value of a periodic function $f(t)$ with a period T
F_{eff}	effective or root mean square value of a periodic function $f(t)$ with a period T, the RMS value of a periodic function $f(t)$

F_n — amplitude of the nth harmonic

$g_{11}, g_{12}, g_{21}, g_{22}$ — inverse hybrid or g-parameters

$g_T(t)$ — gate function of duration T

$[G]$ — conductance matrix of the circuit

G — conductance of the conductor, mho or siemens (S)

$G_{21}(s), G_{12}(s)$ — voltage transfer functions, or voltage gain ratios

G_{ii} — self-conductance of ith node

G_{ij} — transfer conductance between the ith node and jth node

$h_{11}, h_{12}, h_{21}, h_{22}$ — hybrid or h-parameters

HP — high pass filter

$[i]$ — the current vector containing currents through the elements

$[i_S]$ — current vector containing source currents in parallel with the elements

i — instantaneous current

$i(0)$ — initial current at $t = 0$

$i_C(t)$ — instantaneous current

$i_k(t)$ — current through the branch k of a circuit at time t

$i(t)$ — the instantaneous value of current at any instant of time t, time varying current

$i_1(t)$ — current in coil 1

i_{avg} — average current in ampere (A)

i_f — forced response of current

$i_{Impulse}(t)$ — the circuit current response due to the impulse voltage

i_{jk} — the current through the element j-k

$i_L(t)$ — current through the load resistor, the current through inductor L

i_m — current in the mth element, impedance transfer functions

i_n — natural response of current

$i_S\ (t)$ — time dependent current source

i_S — output of the current source

$i_S(t)$ — current delivered by the source

i_{S}, j_k — the source current in parallel with the element j-k

$i_{Step}(t)$ — the circuit current response due to a step voltage

$[I]$ — column vector of algebraic sum of currents of all sources entering the node, the column vector of unknown mesh currents

$[\boldsymbol{I}_{bus}]$ — the injected bus currents

$\boldsymbol{I}^*_k$ — conjugate of current $\boldsymbol{I}_k$

I_m — maximum value of current

$I(s)$ — Laplace transform of $i(t)$

I_0 — peak or maximum value of the current wave

I_1 — the current entering terminal-1 equals the current leaving the terminal-1'

$\boldsymbol{I}_1, \boldsymbol{I}_2$ — the complex effective values of the currents in the coils 1 and 2, respectively, primary and secondary phasor currents respectively, A

I_2 — the current entering terminal-2 equals the current leaving the terminal-2'

$\boldsymbol{I}_A, \boldsymbol{I}_B, \boldsymbol{I}_C$ — RMS currents in phases A, B, and C, respectively

I_{avg} — average value of a periodic voltage wave $i(t)$

I_{eff} — effective or root mean square (RMS) value of a periodic current $i(t)$

I_k — injected current at node k

I_L — line current phasor

I_n — magnitude of the nth harmonic of the resultant current

I_N — Norton current, current in the neutral

I_P — phasor current

I_S — magnitude of current of DC source

$I_{S\ max}$ — maximum current supplied by the voltage source

$\mathcal{I}S(t)$ — the projection of the complex frequency phasor $S(t)$ on the imaginary-axis

j — the operator causes a phasor to rotate through 90° in the anti-clockwise direction without affecting its magnitude

$j\omega$ — angular frequency

k — coefficient of coupling

k_{ij} — element of the matrix K

K — a real number independent of frequency for constant K filters, bus path incidence matrix, constant, scale factor

K, K_1, K_2	constants
l	length in metre (m)
L	inductance in Henry (H)
L_1, L_2	self-inductances of coil 1 and coil 2, respectively, self-inductance of the primary and secondary winding respectively, H
L_{eq}	equivalent inductance
LP	low pass filter
m	number of independent mesh equations in a circuit, the slope of a line
m, n	positive whole numbers
m_{ij}	element of the loop incidence matrix
$\hat{M}$	enhanced loops incidence matrix
M	loop-incidence matrix
M_{12}	mutual inductance between coil 1 and coil 2
M_{21}	mutual inductance between coil 2 and coil 1
M_{rq}, ϕ_{rq}	the magnitude and phase angle respectively of the phasor (p_q p_r)
n	number of nodes including the reference node, load neutral
N	source neutral, number of turns of the coil, ratio of the natural logarithm of the output signal (voltage or current) to the input signal (correspondingly voltage or current), called Neper
$N(s)$	Laplace transform of the output response, polynomials in s
N_1	number of turns of coil 1/ primary winding
N_2	number of turns of coil 2/ secondary winding
N_{MC}	number of equations from mesh current method
N_{node}	total number of nodes
N_{NV}	number of equations from node-voltage method
NS	necessary and sufficient conditions
$N_{V\,S}$, $N_{C\,S}$	numbers of voltage and current sources, respectively
p	instantaneous power in watts (W), power in J/s or watts (W)
$p(t)$	the instantaneous power at any instant of time t
$p_1, p_2, \ldots, p_n$	poles of the network function
pf	power factor = (cos φ)
p_k	distinct pole
p_L	power transferred to the load
$p_{L\,max}$	maximum power delivered to the load
p_{loss}	power loss
p_R	the power dissipated in the resistor
p_T	total instantaneous power
P	average power over the period T, number of poles of the machine
PRF	positive real functions
$P(s)$, $Q(s)$	even and odd components of Hurwitz polynomial $D(s)$, respectively
P_{avg}	average power
PFC	passive power factor correctors
P_r	reflected power
P_S, P_R,	intensities of the power signals at the transmission (source) and destination (load) ends, respectively,
q	charge in coulomb (C)
Q	reactive power, also known as reactive volt ampere and is equal to VI sin φ
Q_{fi}	final value of inductive load
Q_{in}	initial inductive load
r	distance in metre (m)
$r(t)$	unit-ramp function or the delta function
r_S	the source resistance of a practical voltage source
rf	reactive factor = sin φ
$[R]$	resistance matrix of the circuit
R	conductor resistance in ohm (Ω)
$\mathcal{R}$	reluctance of the magnetic path
$R(s)$, $E(s)$	denote function the response
$R_1(s)$, $R_2(s)$	remainders
R_1, R_2	resistance of the primary and the secondary winding respectively, Ω
R_{12}, R_{23}, and R_{31}	resistors connected in delta between the nodes 1, 2, and 3
R_{1n}, R_{2n}, and R_{3n}	resistors connected in star between nodes 1, 2, and 3
R_{eq}	equivalent resistance
R_{ii}	self-resistance of ith node

R_{ij} transfer resistance between the ith node and jth node
R_{in} input resistance
R_{L} load resistor, Ω
R_{N} Norton's equivalent resistance
R_{O} output resistance
R_{OC}, R_{SC} open circuit and short circuit resistance looking into the port at 1-1'
$\mathcal{R}S(t)$ the projection of the complex frequency phasor $S(t)$ on the real-axis
R_{TH} Thevenin equivalent resistance
s the natural frequency, complex frequency
$s = \sigma + j\omega$ complex frequency variable
S complex power
$S(t) = A\, e\, j^{\omega t}$ the rotating phasor signal
S_k complex power in branch k
t time in seconds (s)
T time period in seconds
T_0 the duration of the pulse in seconds
u velocity in m/s
$u(t - T_{\mathrm{S}})$ shifted unit-step function
$u(t + T_{\mathrm{S}})$ flipped step function
$u(t)$ unit-step function
v voltage in volts (V)
$v'_c(t)$ natural response of voltage across capacitor
$v''_c(t)$ forced response of voltage across capacitor
$v(0)$ initial voltage across an element at time $t = 0$
$v(t)$ the instantaneous value of voltage at any instant of time t
$v_{\mathrm{C}}(0)$ the initial voltage across capacitor C at $t = 0$
$v_C(t)$ instantaneous voltage
v_i the open circuit input terminal voltage
v_{jk} voltage across the element j-k
v_k voltage across the k-th element
$v_k(t)$ voltage through the branch k of a circuit at time t
$v_L(t)$, $i_L(t)$ instantaneous voltage and current respectively of inductance L
$v_R(t)$, $i_R(t)$ instantaneous voltage and current respectively of resistance R
$[v_{\mathrm{S}}]$ the voltage vector containing source voltages in series with the elements
v_{S} output voltage of voltage source
$v_{\mathrm{S}}(t)$ time-dependent voltage source
$v_{\mathrm{S},\,jk}$ source voltage in series with the element j-k
v_{TH} Thevenin voltage
$[v]$ the voltage vector containing voltages across the elements
$[V]$ column vector of node voltages, the column vector of algebraic sum of all the source voltages around the mesh
$[\boldsymbol{V}_{\mathrm{br}}]$, $[\boldsymbol{I}_{\mathrm{br}}]$ vector of branch voltages and branch currents, respectively
$[\boldsymbol{V}_{\mathrm{bus}}]$ bus voltage with respect to a reference node
$[\boldsymbol{V}_{\mathrm{L}}]$, $[\boldsymbol{I}_{\mathrm{L}}]$ vector of loop voltages and loop currents, respectively
$\boldsymbol{V}$ the phasor voltage which is a complex number
$V(s)$ transform of $v(t)$
V effective or RMS values of the voltage
V_1 voltage across the terminals1-1'
$V_1, V_2, \ldots, V_n$ voltages at free points 1, 2, 3, ..., n with respect to the ground
$\boldsymbol{V}_{\mathbf{AB}}$, $\boldsymbol{V}_{\mathbf{BC}}$, $\boldsymbol{V}_{\mathbf{CA}}$ line voltages
V_{avg} average value of a periodic voltage wave $v(t)$
V_C voltage across capacitor C
V_{DC} DC component of the applied voltage
V_{eff} effective or root mean square value of a periodic voltage $v(t)$
V_g magnitude of voltage source, volts
V_{L} magnitude of line voltage, voltage across inductance L
$\boldsymbol{V}_{\mathrm{LA}}$, $\boldsymbol{V}_{\mathrm{LB}}$, $\boldsymbol{V}_{\mathrm{LC}}$ voltage across the loads in phases A, B, and C, respectively
V_{m} the maximum value of the sinusoidal voltage
V_n the magnitude of the nth harmonic of the applied voltage
$\boldsymbol{V}_N$, $\boldsymbol{V}_{\mathbf{n}}$ voltages of the load neutral and the source neutral, respectively

V_P	magnitude of phase voltage
$V_S\,u(t)$	step voltage of strength V_S
$V_S\,\delta\,(t)$	voltage impulse of strength V_S
V_S	magnitude of voltage of DC source
$\boldsymbol{V_S}$	complex effective values of the voltage source
w	energy in Joules (J)
$\mathcal{W}$	work done in N-m or joule (J); the electric energy in W-s
$\mathcal{W}(t)$	net energy input to the coupled circuit at any instant of time t
$W(s)$	multiplicative factor
W_1, W_2, W_R	reading of wattmeters
$W_{1\Omega}$	energy expended in a 1-Ω resistor
$\mathcal{W}_C$	energy stored in capacitor
$\mathcal{W}_L$	energy stored in inductor (J)
X_C	the reactance of the capacitor C in ohm
X_L	the reactance of the inductor L in ohm
$y_{11}, y_{12}, y_{22}, y_{21}$	admittance or y-parameters
y_{jk}	self admittance of the element j-k
$[y]$	primitive self impedances matrix
$[Y_{br}]$	branch admittance matrix
$[Y_{bus}]$	bus admittance matrix
$[Y_L]$	loop admittance matrix
$Y(s)$	transform admittance
$Y_1, Y_2, \ldots, Y_n$	linear admittances
$Y_{11}(s)$, $Y_{22}(s)$	the driving point admittances at the respective ports 1-1' and 2-2'
$Y_{21}(s)$, $Y_{12}(s)$	admittance transfer functions
Y_{eq}	the equivalent admittance in ohm
Y_{in}	input admittance, mho
Y_{out}	output admittance, mho
$Y_{R-L}\,(s)$	admittance function
$z_1, z_2, \ldots, z_m$	zeroes of the network function
$z_{11}, z_{12}, z_{22}, z_{21}$	the impedance, or z-parameters
z_{jk}	self impedance of the element j-k
$[z]$	primitive self admittance matrix
$[Z_{br}]$	branch impedance matrix
$[Z_{bus}]$	bus impedance matrix
$[Z_L]$	loop impedance matrix
Z	the impedance of the circuit, a complex quantity and has the unit of ohm
$Z(s)$	the impedance of a passive network, transform impedance
Z_1	series arm impedance, driving point impedance
$Z_{11}(s)$	the driving point impedance at port 1-1'
Z_{12}, Z_{23}, Z_{31}	the resistors connected in delta between the nodes 1, 2, and 3
Z_{1n}, Z_{2n}, Z_{3n}	the resistors connected in star between nodes 1, 2, and 3
Z_2	shunt arm impedance, driving point impedance
$Z_{22}(s)$	driving point impedance at port 2-2'
Z_{eq}	equivalent impedance in ohm
Z_g	internal impedance, Ω
Z_{i1}	image impedance
Z_{in}	input impedance
Z_{int}	equivalent impedance of the circuit
Z_L	load impedance, Ω
Z_l, Z_N, Z_L	per phase impedance in ohms of the line conductor, the neutral conductor, and the load, respectively
$Z_{LC}(s)$ or $Y_{LC}(s)$	driving point L-C immittance
$Z_{L\Delta}$, Z_{LY}	impedance of each phase of the balance Δ-connected load, and the impedance of each arm of the equivalent Y-connected load
Z_N	Norton's equivalent impedance
Z_o	characteristic impedance
Z_{OC}, Z_{SC}	open and short circuit parameters
Z_{oT}	characteristic impedance of T-network
$Z_{o\pi}$	characteristic impedance of π-network
Z_r	reflected impedance
$Z_{RC}(s)$	R-C impedance function
Z_S	internal source impedance
Z_{TH}	conjugate of Thevenin impedance
α	attenuation constant, damping constant
$\alpha_1, \alpha_2, \ldots, \alpha_n$	dimensionless constants
$\alpha_{12}\,(s)$ and $\alpha_{21}\,(s)$	current transfer ratio or current gain
β	phase constant

$\beta_1, \beta_2, ..., \beta_m$ constants having the units of ohm

$\delta(t - T_0)$ delayed unit impulse function

$\delta(t)$ unit impulse function

Δq quantity of charge in coulombs that flow in Δt seconds

ΔR variation in the resistance of a branch

Δt time interval, seconds (s)

ΔZ variation in the impedance of a branch

ε absolute permittivity of the medium; $\varepsilon = \varepsilon_0\varepsilon_r$

ε_0 absolute permittivity of free space ($\varepsilon_0 = 8.85$ ´ 10^{12}, F/m)

ε_r relative permittivity of the medium

ϕ_{ni} phase angle of the *n*th harmonic of the resultant current

Φ_1, Φ_2 fluxes in coil 1 and in coil 2, respectively

Φ_{21}, ψ_{21} flux and flux linkages respectively of the coil 1 due to a current $i_2(t)$ in the coil 2

γ propagation or image transfer constant

$\gamma_1, \gamma_2, ..., \gamma_m$ dimensionless constants

φ phase angle

φ power factor angle which equals $(\theta_v - \theta_i)$

φ_{fi} final pf angle

φ_{in} initial pf angle

$\lambda_1, \lambda_2, ..., \lambda_n$ constants having the unit of siemen

$\mathcal{L}$ Laplace transform operation

$\mathcal{L}^{-1}$ inverse Laplace transform

θ_n phase angle of the *n*th harmonic

θ_v, θ_i phase angles of voltage and current waves, respectively

ρ specific resistance or resistivity of the conductor, Ω-m, reflection coefficient

σ real number, specific conductance or conductivity of the material, mho/m or S/m, real part of *s*

τ dummy variable for *t*, the transmission coefficient for the sinusoidal power supply

$\tau = L/R$ time constant of *RL* circuit

$\tau = RC$ time constant of *RC* circuit

ω angular speed of the rotor in electrical angle per second, angular velocity in rad/s

ω_0 angular velocity corresponding to fundamental frequency f_0, rad/s, resonant or natural frequency in rad/s

ω_{d} damped resonant frequency

$\psi = N_\Phi$ flux linkage in Weber-turns

ψ_{12} flux linkages of the coil 2 due to a current i_1 in the coil 1

Chapter 1
Definitions and Basic Circuit Concepts

Do not wait; the time will never be "just right". Start where you stand, and work with whatever tools you may have at your command, and better tools will be found as you go along.

—George Herbert

Key Concepts

- Introduction of electrical materials—conductors, semiconductors, and insulators
- Defining the basic electrical terms—charge, current, voltage, power, and energy
- Defining circuit components, linear, bilateral and unilateral elements, lumped and distributed parameter elements, passive and active branches, node, loop, and mesh
- Understanding the characteristics of the basic circuit elements, such as resistors, inductors, and capacitors
- Defining independent and dependent voltage and current sources
- Ability to transform a voltage source into a current source and vice versa without modifying the response in the network
- Defining Ohm's law, Kirchhoff's current law (KCL), and Kirchhoff's voltage law (KVL) and their applications in the determination of voltages and currents in circuits.
- Developing the ability to calculate equivalent resistance of series–parallel combinations of resistances
- Reducing series–parallel combination of inductances
- Understanding of voltage and current division
- Application of star–delta conversion for simplifying resistive circuits

1.1 Preamble

Generally speaking, network analysis is any structured technique used to mathematically analyse a circuit. A physical 'electrical network' or 'electrical circuit' is a system of interconnected energy sources such as voltage sources or current sources; electrical elements such as resistors, inductors, and capacitors; electronic devices such as diodes, transistors, etc.; switches, loads, and connecting wires for interconnection of the components. The component can be as small as an integrated circuit on a silicon chip or as large as an electricity distribution network.

Based on well-defined electrical laws, an electrical circuit can be analysed to compute voltages and current flows for all the elements of the network, and if desired other quantities such as charge, field distribution, energy, power can be computed. Conversely, by employing the same electrical laws, a circuit may be synthesized to produce a given output from a known input. Fundamental laws, concepts, and terms associated with electricity are introduced here under.

1.2 Electrical Materials

Electrical materials are classified into conductors, semiconductors, and insulators depending upon the energy gap between the valence and conduction bands.

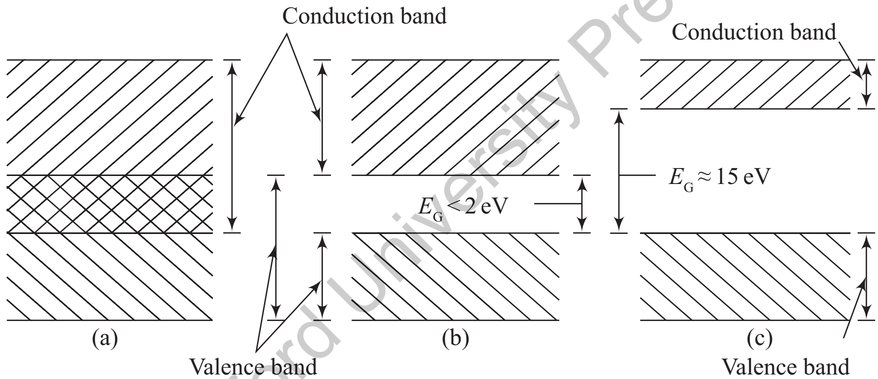


Fig. 1.1 Energy band in electrical materials

It may be noted from Fig. 1.1(a) that the conductors possess overlapping valence and conduction bands whereas in insulators (also called dielectrics), the gap between the two bands is large [Fig. 1.1(c)]. Table 1.1 provides a classification of the common types of electrical materials.

Table 1.1 Classification of electrical materials

Classification	Materials
Conductors	Aluminium, copper, iron, silver
Semiconductors	Carbon, germanium, silicon
Dielectrics	Air, glass, mica, plastic, rubber

1.3 Atomic Structure and Electric Charge

An atom is constituted of a nucleus with negatively charged electrons revolving around it in elliptical orbits. The nucleus is made up of protons and neutrons. Table 1.2 enlists the properties of the constituents of an atom.

Table 1.2 Properties of the constituents of an atom

Property ↓ / Constituent →	Electron	Proton	Neutron
Charge	-1.602×10^{-19} C	1.602×10^{-19} C	Nil
Mass	9.109×10^{-31} kg	1.672×10^{-27} kg	1.675×10^{-27} kg

In a neutral atom, the number of electrons is equal to the number of protons.

1.4 Voltage and Current

Both movement and separation of charges exhibit electrical characteristics. This section explains the concepts of voltage and current which are essential to understanding circuit theory.

1.4.1 Voltage

According to Coulomb's law, forces of attraction (between unlike charges) and repulsion (between like charges) are set up when charges are separated. Energy is required to be spent to overcome the force of attraction to move the charges through a specific distance. The energy per unit charge required to overcome the force is called *voltage* and in differential form is written as

$$v = \frac{dw}{dq} \tag{1.1}$$

where v is the voltage in volts, w is the energy in joules, and q is the charge in coulombs.

All opposite charges possess specified potential energy and the difference in potential energy of the charges is defined as *potential difference*, which is measured in volt (V). A potential difference of 1 V = 1 J/C. The polarity reference for the voltage is indicated by plus (+) and minus (−) signs.

1.4.2 Current

When the randomly moving electrons (or charge) are made to move in a given direction by the application of a voltage, the resultant movement of charge leads to the flow of current. Thus, electric current is defined as the flow of charge per unit time and mathematically is expressed as

$$i = \frac{dq}{dt} \text{ C/sec (or Ampere)} \tag{1.2}$$

where i is the current in amperes, t is the time in seconds.

The unit of current is ampere (A) and a current of 1 A means that the rate of flow of charge is 1 C/s. Since current is due to the flow of electrons, it has direction. Conventionally a positive direction of flow of current is marked by a reference arrow and is assumed to be in the opposite direction of the flow of electrons.

Example 1.1 The rate of flow of electrons in a conductor is 10^{20} electrons per second. What is the magnitude of current flow in amperes in the conductor?

Solution Charge flow per second $= 1.602 \times 10^{-19} \times 10^{20} = 16.02$ C

Remembering that by definition 1 C/s = 1 A, magnitude of current flow = 16.02 A

Example 1.2 A current of 0.75 A transfers 100 C across a conductor. Determine the time of flow of current.

Solution Recalling that current in amperes is charge transferred per second (Q/t), the time of flow of current is

$$t = \frac{Q}{I} = \frac{100}{0.75} = 133.33 \text{ s}$$

Example 1.3 (a) Derive an expression for charge build up due to current flow. (b) In an electric wire, at $t = 0$ s a current of 7.5 A begins to flow. What is the total charge flow in t sec. If the current flow is stopped at $t = 10$ s, calculate the charge which has flown in the wire.

Solution

(a) Assume that $i(t)$ represents current flow. The expression for charge build-up is computed from Eq. (1.2) as follows:

$$q(t) = \int i(t)\,dt \text{ C} \tag{1.3.1}$$

(b) Use of Eq. (1.3.1) gives

$$q(t) = \int_0^t 7.5dt = 7.5\int_0^t dt = 7.5[t]_0^t = 7.5\,t \text{ C} \tag{1.3.2}$$

Substituting $t = 10$ s in Eq. (1.3.2) leads to

$$q(t) = 7.5 \times 10 = 75 \text{ C}$$

Example 1.4 The charge flowing through a conductor is given by $q = t\sin(15.70\,t)$ mC. Derive an expression for the flow of current and calculate its magnitude at 0.4 s.

Solution From Eq. (1.2), the current flow is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}\left[t\sin\left(15.70\right)t\right] = \left[\sin(15.70t) + 15.70\,t\cos(15.70t)\right]\text{mA} \tag{1.4.1}$$

Substitution of $t = 0.4$ in Eq. (1.4.1) leads to the magnitude of the current as under

$$i = \left[\sin\left(15.70 \times 0.4\right) + 15.70 \times 0.4 \times \cos\left(15.70 \times 0.4\right)\right] = 6.28 \text{ mA}$$

1.5 Power and Energy

In circuit analysis, computation of current and voltage, by themselves, may not be sufficient due to the following reasons:

(i) The output of a system could often be non-electrical such as chemical, mechanical.
(ii) Electrical devices, such as generators, motors, are designed to handle specific power.

Therefore, it is necessary to correlate voltage and current to power and energy. Power is defined as energy per unit time, that is,

$$p = \frac{dw}{dt} \tag{1.3}$$

where p is the power in watts, w is the energy in joules, t is the time in seconds

It can be easily shown that power is associated with the flow of charge. Rewriting Eq. (1.3) gives

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right) \times \left(\frac{dq}{dt}\right) = v \times i \tag{1.4}$$

The unit of power is J/sec or watt (named after the Scottish engineer, James Watt). The unit of energy is joule or watt-second. Hence, 1 W is equivalent to 1 J/s. Alternately, 1 W of power is generated when 1 J of energy is consumed.

Passive sign convention: As in the case of voltage, power is a signed quantity. Electrical engineers adopt the 'passive' sign convention which states that if positive current flows into the positive terminal of an element the power dissipated is positive, that is, the element absorbs power; whereas if the current leaves the positive terminal of an element, the power dissipated is negative, that is, the element delivers power.

Table 1.3 Summary of basic electrical quantities

Quantity	Symbol	Unit	Notation
Charge	Q	C	
Voltage	v	V	dw/dq
Current	i	A	dq/dt
Power	p	W	$dw/dt = v \times i$
Energy	E	J or Watt-sec	$w \times t$

Example 1.5 Apply passive sign convention to the two terminal circuits in Fig. 1.2 and identify, with justification, whether the circuit element is generating or absorbing power.

Solution

Fig. 1.2(a): Since current is entering the +ve terminal of the circuit element, power is absorbed ($p = vi$).

Fig. 1.2(b): Since current is leaving the +ve terminal of the circuit element, power is generated ($p = -vi$).

Fig. 1.2(c): Since current is entering the −ve terminal of the circuit element, power is generated ($p = -vi$).

Fig. 1.2(d): Since current is leaving the −ve terminal of the circuit element, power is absorbed ($p = vi$).

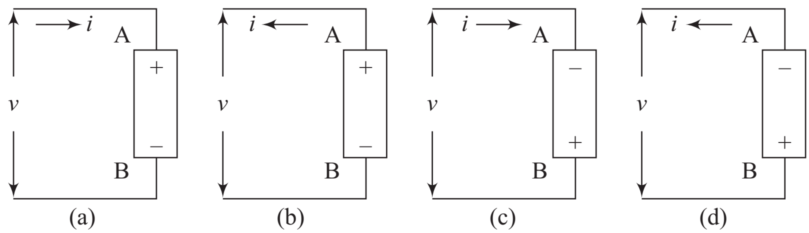


Fig. 1.2

Example 1.6 The current flowing in the circuit element in Fig. 1.2(a) is $i = 30e^{-6000t}$ A for $t \geq 0$. Compute the total charge flowing into the circuit element. Assume $i = 0$ at $t < 0$.

Solution Using of Eq. (1.2) leads to

$$q = \int_0^t dq = \int_0^t i\,dt = \int_0^\infty 30e^{-6000\,t}dt = -0.005\left[e^{-6000\,t}\right]_0^\infty = 5000\ \mu\text{C}$$

Example 1.7 A current $i(t) = 10e^{-4000\,t}$ A flows across the circuit element in Fig. 1.2(a) when a voltage $v(t) = 15e^{-4000\,t}$kV is applied for $t \geq 0$s across its terminals. If v = 0 at $t < 0$ s, compute (i) power supplied to the element at 2 ms and (ii) total energy supplied to the circuit element.

Solution From Eq. (1.4), it is seen that power supplied to the circuit element can be written as

$$p = vi = \left(15000e^{-4000t}\right) \times \left(10e^{-4000t}\right) = 150000e^{-8000t}\ \text{W}$$

(i) Power supplied to the element at 2 ms is given by

$$p(0.002) = 150000e^{-8000\times 0.002} = 150000e^{-16} = 0.017\ \text{W}$$

(ii) Equation (1.3) is used to derive an expression for energy as under

$$w(t) = \int_0^t p(t)\,dt = 150{,}000\int_0^t e^{-8000t}dt = \left[\frac{150{,}000}{-8000}e^{-8000t}\right]_0^t = -18.75\left[e^{-8000t}\right]_0^t \tag{1.7.1}$$

To compute the total energy supplied, put $t = \infty$ in Eq. (1.7.1).

Thus, $w(t) = -18.75\left[e^{-8000t}\right]_0^\infty = 18.75$ J

Example 1.8 (a) Prove $v = dw/dq$. (b) An electric circuit delivers 48 W when the current flow is 12 A. Calculate the energy per coulomb of charge.

Solution From Eq. (1.4), it is seen that $v = p/i$ V.
Substituting Eqs (1.3) and (1.2) for p and i, respectively, in the above expression gives

$$v = \frac{p}{i} = \frac{dw/dt}{dq/dt} = \frac{dw}{dt}\times\frac{dt}{dq} = \frac{dw}{dq} \text{ or } dw = v \times dq \tag{1.8.1}$$

From the given data, v = 48/12 = 4 V
Since i = 12 A, by definition, $dq = i \times dt = 12 \times 1 = 12$ C
Hence, from Eq. (1.8.1) $dw = 4 \times 12 = 48$ J
Energy per coulomb of charge = 48/12 = 4 J

1.6 Basic Circuit Elements

Prior to proceeding with the discussion of the characteristics of basic circuit elements, it would be appropriate to define terms frequently employed in circuits.

Circuit element: An individual component such as a resistor, inductor, capacitor, diode, transistor, energy source, which constitutes a circuit, is known as a circuit element.

Network and circuit: A network is a connection of two or more circuit elements. A circuit is a network that has at least one closed path. Every circuit is a network, but all networks may not be circuits.

Branch: A branch is an element of the network having only two terminals.

Passive and active branch: A branch is said to be active when it contains one or more energy sources. A passive branch does not contain an energy source.

Linear element: When the current and voltage relationship in an element can be simulated by a linear equation either algebraic, differential, or integral type, the element is said to be a linear element.

Bilateral and unilateral element: A bilateral element conducts equally well in either direction. Resistors and inductors are examples of bilateral elements. When the current–voltage relations are different for the two directions of current flow, the element is said to be unilateral. Diode is a unilateral element.

Lumped and distributed parameter elements: Lumped parameter elements are those, which for the purpose of analysis may be treated as physically separate elements such as resistance, inductance, capacitance. The distributed parameter element cannot be modelled as a combination of physically identifiable separate resistor, inductor, or, capacitor.

Node: A junction point of two or more branches is known as a node.

Loop and mesh: Any closed path, formed by the branches in a network, is known as a loop. A mesh is a loop, which does not enclose any other loop within it.

A circuit is constituted of five basic elements of which (a) three are passive and (b) two are active.

(a) *Passive* elements represent devices which do not generate electrical energy and are categorized as
   (i) Resistors (ii) Inductors (iii) Capacitors

(b) *Active* elements model devices which generate electrical energy and are subdivided into
   (i) Voltage sources (ii) Current sources

Energy source may be constant (DC) or they may be a function of time (AC).

It would be no exaggeration to state that it is feasible to model systems in most disciplines of electrical engineering (power, electronic, control, instrumentation, and so on) with these five elements and analyse them. In this section, representation of these five basic elements is described.

1.6.1 Resistor

A resistor is a physical device whose principal characteristic is to offer resistance to the flow of current and it consumes electrical energy. Its symbolic representation is shown in Fig. 1.3.

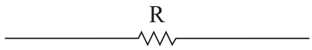


Fig. 1.3 Symbolic representation of a resistor

A linear resistor is one which obeys Ohm's law, that is, current through a resistor is proportional to the potential difference across it. The resistance R of a conductor is directly proportional to the length l in metre (m) of the conductor and inversely proportional to its area of cross section a in m^2, that is,

$$R = \rho \times \frac{l}{a} \tag{1.5}$$

where ρ is called the specific resistance or, resistivity of the conductor and has the unit of Ω−m.

All resistors dissipate heat.

Inverse of resistance (R) is called conductance (G) and has the unit of mho or siemen (S).

$$G = \frac{1}{R} = \frac{a}{\rho\, l} = \frac{1}{\rho} \times \frac{a}{l} = \sigma \times \frac{a}{l}\ \mathrm{S} \tag{1.6}$$

where σ is called the specific conductivity or, conductivity of the material and has the unit of siemen per metre or, mho per metre.

The resistance of most conductors and all metals increases with increase in temperature. The change in resistance varies linearly with a change in temperature and is mathematically expressed as shown below

$$R_2 = R_1\left[1 + \alpha_1\left(t_2 - t_1\right)\right]\Omega \tag{1.7}$$

where R_1 and R_2 are resistances at temperatures t_1°C and t_2°C, respectively, α_1 is the resistance temperature coefficient in per °C.

Typical values of resistivity and temperature coefficients of resistance of different materials at 20°C are given in Table 1.4.

Table 1.4 Resistivity and temperature coefficient at 20°C of common conducting materials

Material	**Resistivity (ρ) at 20°C in Ω−m**	**Temperature coefficient (α) in per °C**
Annealed copper	1.69×10^{-8} to 1.74×10^{-8}	0.00393
Hard-drawn aluminium	2.80×10^{-8}	0.0039
Carbon	6500×10^{-8}	−0.000476
Tungsten	5.6×10^{-8}	0.0045
Manganin	48×10^{-8}	0
Constantan (Eureka)	48×10^{-8}	0

In a physical resistor, when v volts is applied across its two terminals and the current flowing through it is i amperes, then as per Ohm's law $v = iR$. Using Eq. (1.4), the expression for power takes the form

$$p = vi = (iR)\, i = i^2 R\ \mathrm{W} \tag{1.8}$$

Further, when $i = v/R$ is substituted in Eq. (1.4), the power is given by

$$p = vi = v(v/R) = v^2/R\ \mathrm{W} \tag{1.8a}$$

Energy dissipated in a resistor in t sec is written as

$$W = \int_0^t p\,dt = p \times t = \left(i^2 R\right) \times t = \left(v^2/R\right) \times t\ \text{W-sec} \tag{1.9}$$

Example 1.9 A circular conductor has a resistance of R_1 Ω when its diameter and length are d and l m, respectively. (a) Find the change in resistance when its diameter is halved and its length is increased four times. (b) By how much should the length of the conductor be changed in order to keep the resistance value at R_1 when the diameter of the conductor is reduced to d/3?

Solution Area of the conductor $a = (\pi/4) \times d^2\ \text{m}^2$

From Eq. (1.5), $R_1 = \rho \dfrac{l}{a} = \rho \dfrac{4l}{\pi d^2} \Omega$ (1.9.1)

(a) When the diameter is halved, area of the conductor $a_1 = (\pi/4) \times (d/2)^2 = \pi d^2/16\,\text{m}^2$

The length of the conductor is $4l$, resistance of the conductor R_2 is obtained from Eq. (1.5) as

$$R_2 = \rho \frac{4l}{\left[\pi d^2/16\right]} = \rho \frac{64l}{\pi d^2} \Omega \tag{1.9.2}$$

Dividing Eqs (1.9.2) by (1.9.1) gives $R_2 = 16R_1$. Thus, the resistance increases by 16 times.

(b) When diameter of the conductor is reduced to $d/3$, then the area a_2 is obtained as

$$a = (\pi/4) \times (d/3)^2 = \pi d^2/36\,\text{m}^2$$

Assuming the length of the conductor to be x m, the resistance from Eq. (1.5),

$$R_1 = \rho \frac{36x}{\pi d^2} \Omega \tag{1.9.3}$$

Equating Eqs (1.9.1) and (1.9.3) gives

$$\rho \frac{4l}{\pi d^2} = \rho \frac{36x}{\pi d^2}, \text{ or, } \quad x = l/9$$

Thus, the length of the conductor will be reduced by $(1/9)l$.

Example 1.10 Given that $R_2 = R_1\left[1 + \alpha_1(t_2 - t_1)\right]\Omega$, prove that the temperature coefficient at the reference temperature of 0° can be expressed as $\alpha_0 = \dfrac{R_t - R_0}{R_0 t}$ / °C, where t is the rise in temperature in °C and R_t Ω represents the change in resistance from R_0 Ω at 0°C.

Solution The given relation can be written as

$$\alpha_1 = \frac{(R_2 - R_1)}{R_1(t_2 - t_1)} / ^\circ\text{C}$$

Similarly, if α_2 represents the temperature coefficient at t_2°C, it can be easily seen that

$$\alpha_2 = \frac{(R_2 - R_1)}{R_2(t_2 - t_1)} / ^\circ\text{C}$$

The relation for α_0 is obtained by substituting $R_2 = R_t$, $R_1 = R_0$, $t_1 = 0$, and $t_2 = t$. Thus,

$$\alpha_0 = \frac{(R_t - R_0)}{R_0 t} / ^\circ\text{C} \tag{1.10.1}$$

Example 1.11 The resistance of a metal conductor at 0°C is 15 Ω. If the resistance increases to 16.5 Ω at 25°C, determine the temperature coefficient of resistance at 25°C. In addition, calculate α_0.

Solution Simplification of Eq. (1.10.1) yields

$$R_0 = R_t(1 - \alpha_0 t)$$

or $$\alpha_0 = \frac{(1 - R_0/R_t)}{t} = \frac{(1 - 15/16\ 5)}{25} = 0\ 0036/^\circ\text{C}$$

Again using (1.10.1), it is seen that

$$\alpha_0 = \frac{(R_t - R_0)}{R_0 t} = \frac{(16.5 - 15)}{15 \times 25} = 0.004 \text{ per}^\circ\text{C}$$

Example 1.12 A DC voltage of 220 V is applied across a tungsten filament whose resistance varies as a function of time given by $R(t) = 4e^{2.5t}\Omega$. Compute the heat energy dissipated after 10 s.

Solution As per Eq. (1.9), the heat energy dissipated is given by

$$W = \int_0^{10} p dt = \int_0^{10} \frac{v^2}{R(t)} dt = \int_0^{10} \frac{(220)^2}{4e^{2.5t}} dt = 12100 \int_0^{10} e^{-2.5t} dt$$

$$-\frac{12000}{2.5}\left[e^{-2.5t}\right]_0^{10} = 4840\left[1 - e^{-2.5\times 10}\right] = 4840 \text{ W} - \text{sec/J}$$

1.6.2 Inductor

An inductor is a physical device which stores electrical energy when a current flows through it. A practical inductor is made of several turns of wire wound on a magnetic or an air core. Figure 1.4 shows a schematic representation of an inductor.

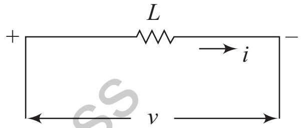


Fig. 1.4 Schematic representation of an inductor

If the resistance of the inductor wire is negligible, the voltage v volts developed across it is proportional to the rate of change of current $\frac{di}{dt}$ in amperes/sec.

Thus, $$v \propto \frac{di}{dt}, \tag{1.10}$$

or $$v = L\frac{di}{dt}$$

The proportionality constant L is called the inductance and is a result of the coiled conductor linking a magnetic field. It has the unit of henry (H) named after the American physicist Joseph Henry. Integration of Eq. (1.10) results in

$$i = \frac{1}{L}\int_0^t v dt + i(0) \text{ A} \tag{1.11}$$

where i (0) is the initial current at $t = 0$.

By making use of Eq. (1.4), the instantaneous power in an inductor, at any instant, is written as

$$p = vi = L\, i\frac{di}{dt} \text{ W} \tag{1.12}$$

Similarly, the energy stored in an inductor is given by

$$W_L = \int_0^t p\, dt = \int_0^t vi\, dt = \int_0^t \left(L\frac{di}{dt}\right) idt = L\int_0^t i\, di = \frac{1}{2}L\, i^2 \text{ J} \tag{1.13}$$

From the foregoing, following observations may be made in respect of the behaviour of an inductor:

- The current in an inductor cannot change instantaneously [see Eq. (1.11)].
- When constant or direct current flows through an inductor ($di/dt = 0$) and the induced voltage is zero [see Eq. (1.10)], that is, the inductor behaves like a short circuit (SC).
- When the current is increasing, di/dt is positive, and energy is received from the source and stored in the magnetic field of the inductor. Similarly, when

current is decreasing di/dt is negative; the energy stored in the magnetic field of the inductor is returned to the source.

Example 1.13 A current having a variation shown in Fig. 1.5 is applied to a pure inductor having a value of 2 H. Calculate the voltage across the inductor at time $t = 1$ and $t = 3$ sec.

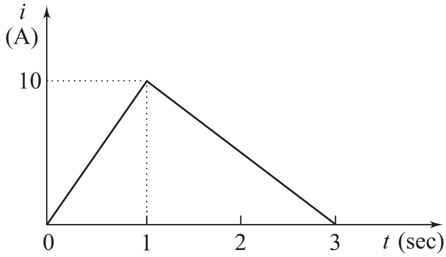


Fig. 1.5

Solution *For the period $0 \le t \le 1$ sec*

Current, $i = 10\,t$ A

Rate of change of current $\frac{di}{dt} = 10$ A/sec

Therefore, at $t = 1$ sec, voltage across the inductor is

$$L\frac{di}{dt} = 2 \times 10 = 20\,\text{V}$$

For the period $1 \le t \le 3$ sec

Rate of change of current $\frac{di}{dt} = -5$ A/sec

Therefore, at $t = 3$ sec, voltage across the inductor is

$$L\frac{di}{dt} = 2 \times -5 = -10\,\text{V}$$

Example 1.14 A voltage wave having the time variation shown in Fig. 1.6 is applied to a pure inductor having a value of 0.5 H. Calculate the current through the inductor at times $t = 1, 2, 3, 4, 5$ sec. Sketch the variation of current through the inductor over 5 sec.

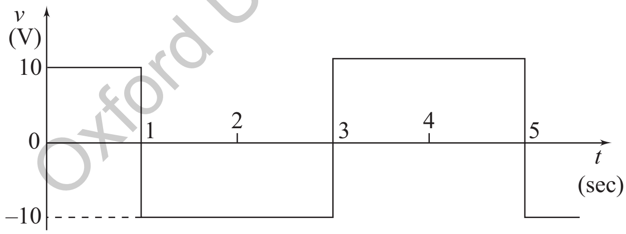


Fig. 1.6

Solution For the period $0 \le t \le 1$sec, $v = 10$ V; $i(0) = 0$. The current i may be expressed using Eq. (1.11) as

$$i = \frac{1}{L}\int_0^t v\,dt + i(0) = \frac{1}{0.5}\int_0^t 10\,dt = 20\int_0^t dt = 20t$$

Then at $t = 1$ sec, $i = 20 \times 1 = 20$ A

For the period $1 \le t \le 3$ sec, $v = -10$ V; $i(1) = 20$ A, then current

$$i = \frac{1}{L}\int_1^t v dt + i(1) = \frac{1}{0.5}\int_1^t -10 dt + 20 = -20\int_1^t dt + 20 = -20(t-1) + 20$$

Then at $t = 2$ sec, $i = -\ 20 \times (2-1) + 20 = -20 + 20 = 0\,\text{A}$

And at $t = 3$ sec, $i = -20 \times (3-1) + 20 = -40 + 20 = -20\,\text{A}$

For the period $3 \le t \le 5$ sec, $v = 10$ V; $i(3) = -20$ A,

$$i = \frac{1}{L}\int_3^t v dt + i(3) = \frac{1}{0.5}\int_3^t 10 dt - 20 = \int_3^t dt - 20 = 20(t-3) - 20$$

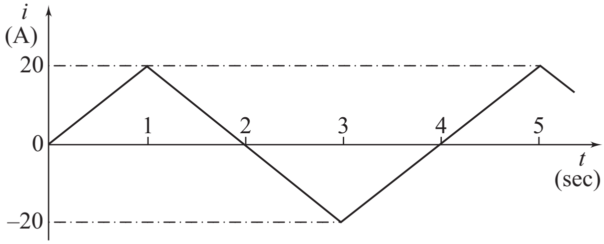


Fig. 1.7 Variation of current through the inductor

Then at $t = 4$ sec, $i = 20\times(4-3)-20 = 20-20 = 0\,\text{A}$

And at $t = 5$ sec, $i = 20\times(5-3)-20 = 40-20 = 20\,\text{A}$

Example 1.15 A voltage pulse $v = 2e^{-4t}(1-4t)\,\text{V}$ for $t > 0$ s is applied across a 200 mH pure inductor. Assume $v = 0$ V for $t < 0$ s and derive expressions as functions of time for (a) the flow of current in the inductor, (b) power, and (c) energy.

Solution

(a) If it is assumed that $i = 0$ for $t < 0$, then $i(0) = 0$ in Eq. (1.11). Hence, the expression for the flow of current through the inductor is used as follows:

$$i(t) = \frac{1}{0.2}\int_0^t 2e^{-4t}(1-4t)\,dt = 10\int_0^t e^{-4t}(1-4t)\,dt \tag{1.15.1}$$

Integration of Eq. (1.15.1) leads to

$$i(t) = 10\left\{\left[-\frac{e^{-4t}}{4}\right]_0^t - 4\left[-\frac{te^{-4t}}{4}\bigg|_0^t - \int_0^t -\frac{e^{-4t}}{4}\,dt\right]\right\}$$

$$= 10\left\{\left[-\frac{e^{-4t}}{4}\right]_0^t - 4\left[-\frac{te^{-4t}}{4}\bigg|_0^t - \frac{e^{-4t}}{16}\bigg|_0^t\right]\right\} = 10te^{-4t}$$

(b) Using Eq. (1.12), expression for power is obtained as

$$p = vi = \left[2e^{-4t}(1-4t)\right]\times 10te^{-4t} = 20te^{-8t} - 80t^2e^{-8t}\ \text{W}$$

(c) Expression for energy is obtained by employing Eq. (1.13) as

$$W = \frac{1}{2}\times 0.2\times\left(10te^{-4t}\right)^2 = 10t^2e^{-8t}\ \text{J}$$

Example 1.16 Use the expressions in Example 1.15 for v, i, p, and W and plot their variations against time t. Use MATLAB facility to plot the curves. From the plots determine the time interval in which the inductor is (a) absorbing, (b) returning energy to the source, and (c) maximum energy stored.

Solution Plot of inductor voltage versus time

```
>> t = linspace(0, 1, 10000);          % divides the time axis between 0
                                         to 1 sec. into 10000 parts
>> v = 2*(1–4*t)·*exp(–4*t);           % input voltage across the inductor
>> plot(t, v)                          % plot v (t) versus t
>> grid on                             % grid is turned on
>> xlabel('Time t in sec')             % label x-axis
>> ylabel('Inductor voltage v in volts')   % label y-axis
```

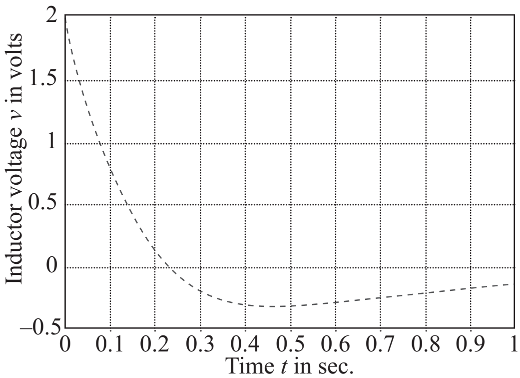


Fig. 1.8

Plot of inductor current versus time

```
>> t = linspace(0, 1, 10000);            % divides the time axis between
                                           0 to 1 sec. into 10000 parts
>> i = 10*t*exp(-4*t);                   % current through the inductor
>> plot(t, i)                            % plot i (t) versus t
>> grid on                               % grid is turned on
>> xlabel('Time t in sec')               % label x-axis
>> ylabel('Inductor current i in amperes')   % label y-axis
```

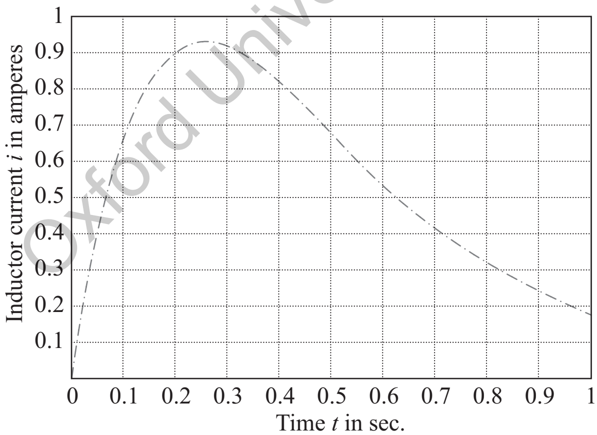


Fig. 1.9

Plot of power versus time

```
>> t = linspace(0, 1, 10000);            % divides the time axis between
                                           0 to 1 sec. into 10000 parts
>> p = 20*t·*exp(-8*t)-80*(t·^2)·*exp(-8*t);   % power in the inductor
>> plot(t, p)                            % plot p (t) versus t
>> grid on                               % grid is turned on
>> xlabel('Time t in sec')               % label x-axis
>> ylabel('Power p in watts')            % label y-axis
```

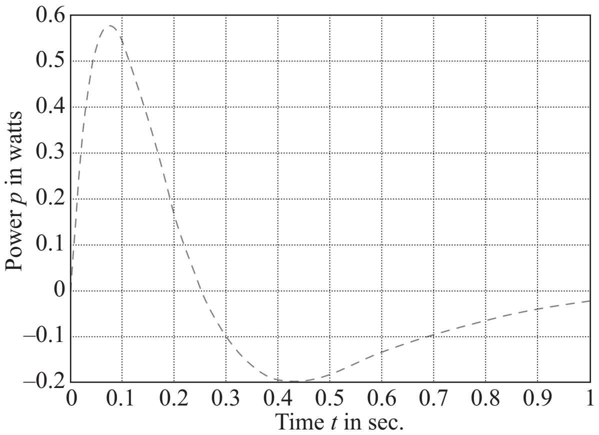


Fig. 1.10

Plot of energy versus time

```
>> t = linspace(0, 1, 10000);          % divides the time axis between 0
                                         to 1 sec. into 10000 parts
>> W = 10*(t·^2)·*exp(–8*t);           % energy in the inductor
>> plot(t, W)                          % plot w (t) versus t
>> grid on                             % grid is turned on
>> xlabel('Time t in sec')             % label x-axis
>> ylabel('Inductor energy W in joules')  % label y-axis
```

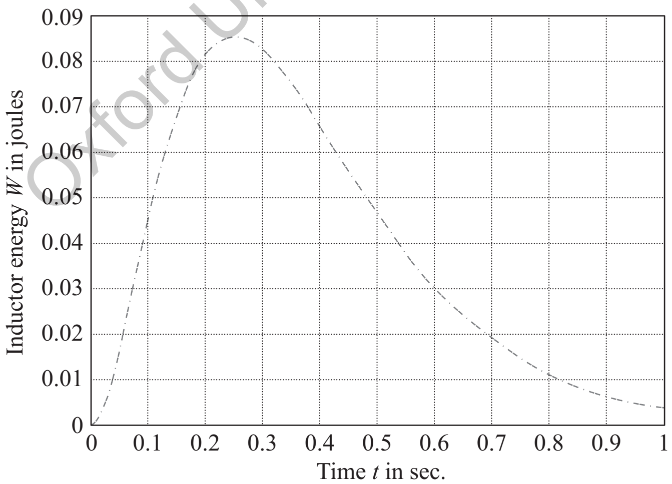


Fig. 1.11

(a) From the energy versus time plot (Fig. 1.11), it may be seen that energy is increasing from the interval 0 to 0.25 s. Hence this is the period when the inductor is absorbing energy. In addition, it may be seen from the power versus time plot that during this period $p > 0$.

(b) From the time 0.25 s to ∞, energy is decreasing in the inductor. Thus, the inductor is returning energy to the source during this period.

(c) From the energy versus time plot, the maximum energy stored in the inductor occurs at 0.25 s and its magnitude is 0.085 J.

1.6.3 Capacitor

A capacitor is a physical device that stores electric energy in the form of charge separation when it is polarized by applying a suitable voltage. A practical capacitor is made up of two parallel conducting plates separated by an insulating material or air called a dielectric. A schematic representation of a capacitor is shown in Fig. 1.12.

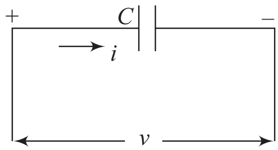


Fig. 1.12 Schematic representation of a capacitor

In the presence of a time-varying voltage v across the capacitor, the charge within the dielectric is displaced leading to a flow of current i, called the *displacement current*. At the terminals of the capacitor, the current appearing is similar to the conduction current and is mathematically written as

$$i = C\frac{dv}{dt}\text{ A} \tag{1.14}$$

In Eq. (1.14), i is the current in amperes through the capacitor; C is the proportionality constant and is called the capacitance of the capacitor, v is the applied voltage in volts, and t is time in s. Capacitance reflects the ability of the capacitor to store charge and has the unit of farad (F). In practice, the unit employed is microfarad (μF) since farad is too large a unit.

Integration of Eq. (1.14) with respect to time determines the voltage across capacitor as under

$$v = \frac{1}{C}\int_0^t i\,dt + v(0)\text{ volts} \tag{1.15}$$

where $v(0)$ is initial voltage at $t = 0$.

Power p in the capacitor is written as

$$p = vi = C\,v\frac{dv}{dt}\text{ W} \tag{1.16}$$

The energy W_C, in the capacitor at any time t, is given by

$$W_C = \int_0^t p\,dt\text{ J} \tag{1.17}$$

Substituting for p from Eq. (1.16) in Eq. (1.17), and assuming that at $t = 0$, $v = 0$, and at any time t s, the voltage across the capacitor is v volts gives

$$W_C = \int_0^v \left(C\,v\frac{dv}{dt}\right)dt = C\int_0^v v\,dv = \frac{1}{2}C\,v^2\text{ J} \tag{1.18}$$

From the foregoing, following observations are made in respect of the behaviour of a capacitor:

- The voltage in a capacitor cannot change instantaneously [see Eq. (1.15)].

- When constant or direct voltage is applied across a capacitor $\left(\frac{dv}{dt}=0\right)$ no conduction current can flow through the capacitor, that is, the capacitor behaves like an open circuit (OC).
- When the voltage is increasing $\frac{dv}{dt}$ is positive; energy is received from the source and stored in the electric field of the capacitor. Similarly, when voltage is decreasing, $\frac{dv}{dt}$ is negative; the energy stored in the electric field of the capacitor is returned to the source.

Hence, similar to an inductor, a capacitor is also a storage device which manifests itself in a circuit when the voltage is varying.

Example 1.17 A voltage wave having a time variation of 20 V/sec is applied to a pure capacitor having a value of 25 mF. Find (a) the current during the period $0 \le t \le 1$sec, (b) charge accumulated across the capacitor at $t = 1$sec, (c) power in the capacitor at $t = 1$sec, and (d) energy stored in the capacitor at $t = 1$sec.

Solution

(a) Current through the capacitor i may be obtained using Eq. (1.14) as

$$i = C\frac{dv}{dt} = 25\times10^{-6}\times20 = 500\ \mu\text{A}$$

(b) From Eq. (1.14) the charge q across a capacitor of C F can be written as $q = Cv$ where v volts is the voltage across it. At $t = 1$ sec, $v = 20$ V, thus,

$$q = Cv = 25\times10^{-6}\times20 = 500\ \mu\text{C}$$

(c) At $t = 1$sec, power $p = v \times i = 20 \times 500 \times 10^{-6} = 1 \times 10^{-2}$ W

(d) At $t = 1$sec, energy stored in the capacitor, W_C, can be obtained using Eq. (1.18) as

$$W_C = \frac{1}{2}Cv^2 = \frac{1}{2}\times25\times10^{-6}\times(20)^2 = 5\times10^{-3}\ \text{J}$$

Example 1.18 A current having variation shown in Fig. 1.13 is applied to a pure capacitor having a value of 5 mF. Calculate the charge, voltage, power, and energy at time $t = 2$ sec.

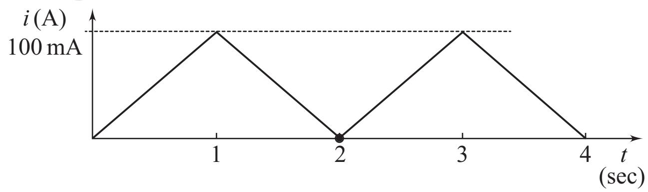


Fig. 1.13

Solution For the period $0 \le t \le 1$sec, $i = 100 \times 10^{-3}\ t = 0.1\ t$ A

At $t = 1$sec,

$$q = \int_0^t i\,dt = \int_0^t 0.1t\,dt = 0.1\times\left[\frac{t^2}{2}\right]_{t=0}^{t=1} = 0.05[t^2]_{t=0}^{t=1} = 0.05[1-0] = 0.05\text{C}$$

$$v = \frac{q}{C} = \frac{1}{C}\int_0^t 0.1t\,dt = \frac{0.05t^2}{500\times10^{-6}} = 100t^2 = 100\text{V}$$

Where $t = 1\text{sec},$

$$p = v \times i = 100 \times 0.1 = 10\,\text{W}$$

$$W_C = \int_0^t vidt = \int_0^t 100t^2 \times 0.01t\,dt = \int_0^1 t^3 dt = \left[\frac{t^4}{4}\right]_{t=0}^{t=1} = \frac{1}{4}[1-0] = 0.25\,\text{J}$$

For the peroid $0 \le t \le 1$ sec, $i = 0.2 - 0.1t$ A

At $t = 2$ sec,

$$\text{Charge } q = q_{t=1} + \int_1^t idt = 0.05 + \int_1^2 (0.2 - 0.1t)dt = 0.05 + \left[0.2t - 0.1 \times \frac{t^2}{2}\right]_{t=1}^{t=2}$$

$$= 0.05 + [0.2(2-1) - 0.05(2^2 - 1^2)] = 0.05 + 0.05 = 0.1\text{C}$$

$$\text{Voltage } v = \frac{q}{C} = \frac{1}{C}\left[0.05 + \int_1^2 (0.2 - 0.1t)\,dt\right]$$

$$= \frac{1}{500 \times 10^{-6}}[0.05 + 0.2t - 0.05t^2]_{t=1}^{t=2}$$

$$= \frac{10^6}{500}[0.05 + 0.2(2-1) - 0.05(2^2 - 1^2)] = \frac{10^6}{500} \times 0.1 = 200\text{ V}$$

$$\text{Power } \; p = v \times i = 200 \times 0 = 0\,\text{W}$$

$$\text{Energy } W_C = W_{C.\text{t}=1} + \int_1^2 vidt$$

$$= 0.25 + \int_1^2 \left[\frac{1}{500 \times 10^{-6}}[0.05 + 0.2t - 0.05t^2] \times (0.2 - 1t)\right] dt$$

$$= 0.25 + \frac{10^6}{500}\int_1^2 [0.01 + 0.035t - 0.03t^2 + 0.005t^3]dt$$

$$\text{Energy } W_C = 0.25 + \frac{10^6}{500}\left[0.01t + 0.035 \times \frac{t^2}{2} - 0.03 \times \frac{t^3}{3} + 0.005 \times \frac{t^4}{4}\right]_{t=1}^{t=2}$$

$$= 0.25 + \frac{10^6}{500} \times 0.01125 = 0.25 + 22.50 = 22.75\text{ J}$$

Example 1.19 A varying current represented by the curve shown in Fig. 1.14 is flowing through an ideal capacitor having a capacitance of 500 μF. Derive expressions for the voltage and energy developed across the capacitor. Plot the variation of voltage versus time. Assume that the capacitor is initially uncharged.

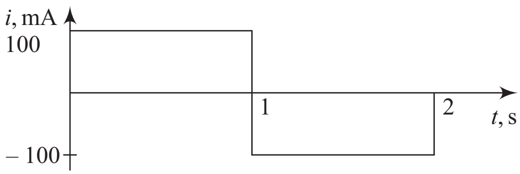


Fig. 1.14

Solution For the period $0 \ge t \ge 1.0$, the current flowing through the capacitor is written as $i = 0.1$ A. The voltage developed across the capacitor is given by

$$v = \frac{1}{C}\int idt = \frac{1}{500 \times 10^{-6}}\int (0.10)\,dt + v(0) = 200t + v(0)\,\text{V}$$

where $v(0)$ is an integration constant. Since the capacitor is uncharged at $t = 0$, $v(0) = 0$. Hence,

$$v = 200t \text{ V} \tag{1.19.1}$$

Energy stored $W_{C-0 \le t \le 1.0} = \dfrac{1}{2}Cv^2 = \dfrac{1}{2} \times \left(500 \times 10^{-6}\right) \times \left(200t\right)^2 = 10t^2$ J

For the period $1.0 \le t \le 2.0$, the current flowing through the capacitor is written as $i = -0.1$ A. Thus voltage developed is written as

$$v = \frac{1}{500 \times 10^{-6}} \times \int (-0.10)\, dt = -200t + v(1)\,\text{V}$$

where $v(1)$ is an integration constant. From Eq. (1.19.1), at $t = 1.0$ s, $v = 200$ V.

Hence, $200 = -200 \times (1.0) + v(1)$ or $v(1) = 400\,\text{V}$

Thus the expression for the voltage, for the period $1.0 \ge t \ge 2.0$, is as under

$$v = -200t + 400 = 200\,(2 - t)\,\text{V}$$

Energy stored $Wc_{-1.0 \le t \le 2.0}$

$$= \frac{1}{2} Cv^2$$

$$= \frac{1}{2} \times (500 \times 10^{-6}) \times [200(2 - t)]^2$$

$$= 10(2 - t)^2\ \text{J}$$

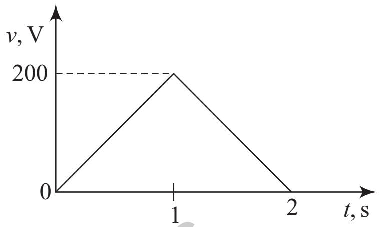


Fig. 1.15

The variation of voltage, across the capacitor, with time is shown in Fig. 1.15.

Example 1.20 A voltage signal $v = \sin(2t)$ V is applied across a 500 μF capacitor. If $v = 0$ for $t \le 0\,\text{s}$, compute expressions for capacitor (a) current, (b) power, and (c) energy for $t \ge 0$s. Sketch the various curves versus time in seconds.

Solution

(a) The capacitor current is computed using Eq. (1.14) as follows:

$$i = \frac{10^6}{500} \times \frac{d}{dt}[\sin(2t)] = 4000\cos(2t)\,\text{A}$$

(b) The power is computed from Eq. (1.16) as given below.

$$p = vi = 4000\sin(2t)\cos(2t) = 2000\sin(4t)\,\text{W}$$

(c) Energy in the capacitor is determined by employing Eq. (1.18) as under

$$W_C = \frac{1}{2} \times (500 \times 10^{-6}) \sin^2(2t) = 2.5 \times 10^{-4} \sin^2(2t)\,\text{J}$$

Figure 1.16 shows a plot of the various quantities.

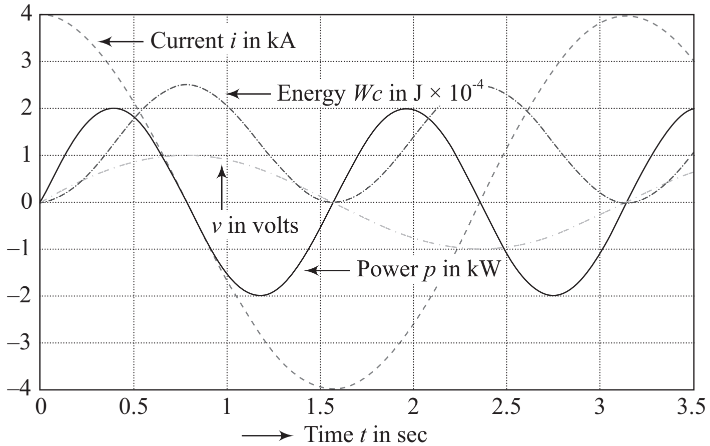


Fig. 1.16

Example 1.21 An ideal capacitor of 500 μF is excited by a current signal which is mathematically expressed as under

$$i(t) = 3\text{A for } 0 \le t \le 2\text{s}$$

and $$i(t) = 3e^{-(t-2)}\text{A for } 2 \le t \le \infty\text{s}$$

If the capacitor is initially uncharged, derive an expression for the voltage versus time and sketch the current and voltage signals.

Solution Since the capacitor carries no charge initially, $v(0) = 0$ V in Eq. (1.15). Thus, for $0 \le t \le 2$s, the voltage across the capacitor can be written as

$$v(t) = \frac{10^6}{500}\int 3dt = 6000t\text{ V} \tag{1.21.1}$$

For the period $2 \le t \le \infty$s, the voltage across the capacitor is expressed as

$$v = \frac{10^6}{500}\int 3e^{-(t-2)}dt + v(2) = \left[-6000e^{-(t-2)} + v(2)\right]\text{V} \tag{1.21.2}$$

where $v(2)$ is an integration constant.

From Eq. (1.21.1), at $t = 2$ s, $v(t) = 12{,}000$ V, and substitution of $t = 2$ and $v(t) = 12{,}000$ in Eq. (1.21.2) gives $v(2) = 18{,}000$ and Eq. (1.21.2) modifies to

$$v = 6000\left[3 - e^{-(t-2)}\right]\text{V} \tag{1.21.3}$$

The plot of the capacitor current and voltage signals is shown in Fig. 1.17.

It may be noted that the capacitor reaches a constant voltage value of 18 kV over a long period of time.

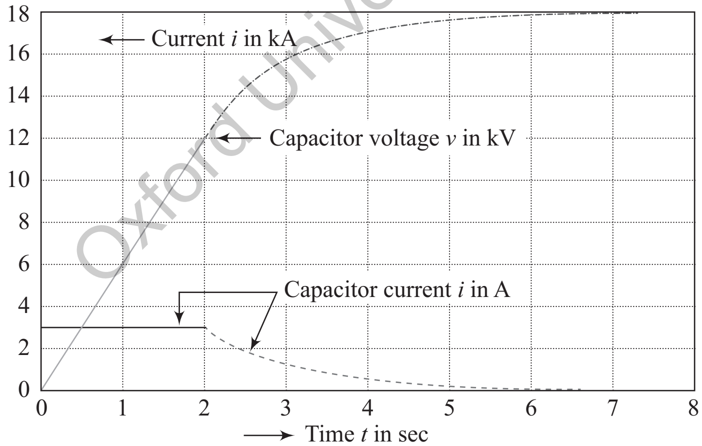


Fig. 1.17

Example 1.22 Derive expressions for power and energy for the capacitor in Example 1.21 and sketch the corresponding curves. Calculate the energy stored during the period (a) $0 \le t \le 2$s and (b) $2 \le t \le \infty$s. What is the total energy stored in the capacitor?

Solution For convenience, the capacitor current and voltage expressions are reproduced as follows.

$$i(t) = 3;\ v(t) = 6000t\text{ V for } 0 \le t \le 2\text{s}$$

and $$i(t) = 3e^{-(t-2)}\text{A};\ v = 6000\left[3 - e^{-(t-2)}\right]\text{V for } 2 \le t \le \infty\text{s}$$

Equation (1.16) is used to calculate the expressions for power as follows.

For $0 \le t \le 2$s, power $p = 3 \times 6000t = 18\,t$ kW

For $2 \le t \le \infty$s, power $p = 3e^{-(t-2)} \times 6000\left[3 - e^{-(t-2)}\right] = 18e^{-(t-2)}\left[3 - e^{-(t-2)}\right]$kW

Similarly Eq. (1.18) is used to calculate the expressions for energy as shown below.

For $0 \le t \le 2$ s, energy $W_C = \frac{1}{2} \times 500 \times 10^{-6} \times (6000t)^2 = 9t^2$ kJ

For $2 \le t \le \infty$ s, energy $W_C = \frac{1}{2} \times 500 \times 10^{-6} \times \left\{6000\left[3 - e^{-(t-2)}\right]\right\}^2 = 9\left[3 - e^{-(t-2)}\right]^2$ kJ

The power and energy curves are shown in Fig. 1.18.

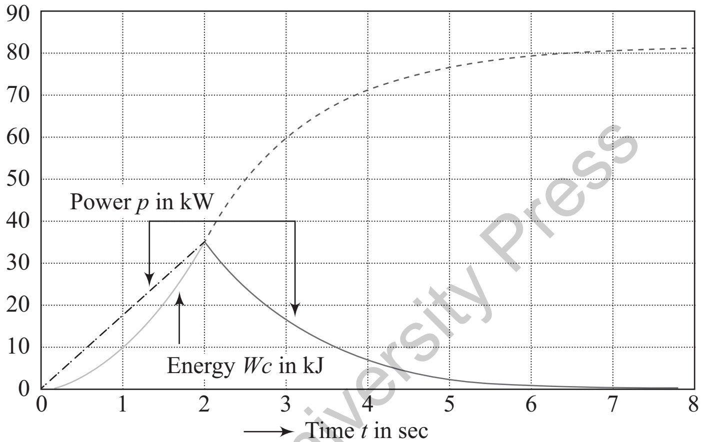


Fig. 1.18

(a) Energy stored during $0 \le t \le 2$ s, $W_{C_{0-2}} = \int_0^2 18t\,dt = \left|9t^2\right|_0^2 = 36$ kJ

(b) Energy stored during $2 \le t \le \infty$ s, $W_{C_{2-\infty}}$

$$= 18\left\{3\int_2^\infty e^{-(t-2)}dt - \int_2^\infty e^{-2(t-2)}dt\right\}$$

$$= 18\left\{\left|-3e^{-(t-2)} + \frac{e^{-2(t-2)}}{2}\right|_2^\infty\right\} = 45 \text{ kJ}$$

Therefore, total energy stored in the capacitor = 36 + 45 = 81 kJ.

Example 1.23 Across an ideal 0.4 μF capacitor, the following voltage signal is applied:

$$v = \begin{cases} 0 \text{ V for } t \le 0 \text{ sec} \\ 5t \text{ V for } 0 \le t \le 1 \text{ sec} \\ (10 - 5t) \text{ V for } 1 \le t \le 2 \text{ sec} \end{cases}$$

Derive expressions for capacitor (a) current, (b) power, and (c) energy as a function of time.

Solution

(a) Equation (1.14) is employed to determine capacitor current.

For $t \le 0$s, $i = 0$ since $v = 0$

For $0 \le t \le 1$ s, $i = 0.4 \times 10^{-6} \times \dfrac{d}{dt}(5t) = 2\,\mu A$

For $1 \le t \le 2$ s, $i = 0.4 \times 10^{-6} \times \dfrac{d}{dt}(10 - 5t) = -2\,\mu A$

(b) Equation (1.16) is used to determine capacitor power.

For $t \le 0$ s, $p = v \times i = 0$

For $0 \le t \le 1$ s, $p = 5t \times 2 = 10\,t\;\mu W$

For $1 \le t \le 2$ s, $p = (10 - 5t) \times (-2) = 10(t - 2)\;\mu W$

(c) Equation (1.18) is applied to determine capacitor energy.

For $t \le 0$ s, $W_C = \dfrac{1}{2} \times 0.4 \times 10^{-6} \times v^2 = 0$

For $0 \le t \le 1$ s, $W_C = \dfrac{1}{2} \times \left(0.4 \times 10^{-6}\right) \times (5t)^2 = 5t^2\;\mu J$

For $1 \le t \le 2$ s, $W_C = \dfrac{1}{2} \times \left(0.4 \times 10^{-6}\right) \times (10 - 5t)^2 = 5\left(t^2 - 4t + 4\right)\mu J$

Example 1.24 Use MATLAB to plot capacitor curves for (a) voltage, (b) current, (c) power, and (d) energy. Identify the periods during which energy is being stored and returned by the capacitor. Show that the energies stored and returned by the capacitor are equal.

Solution The MATLAB program for plotting the various curves versus time is written as follows:

```
>> line([0, 1], [0, 5])                  % Line command for
                                    plotting the voltage signal
>> hold on
>> grid on
>> line([1, 2], [5, 0])                  % Line command for
                                    plotting the voltage signal
>> line([0, 1], [2, 2])                  % Line command for
                                    plotting the current signal
>> line([1, 2], [-2, -2])                % Line command for
                                    plotting the current signal
>> t=linspace(0, 1, 5000);
>> p=10*t;                      % Computation of capacitor power
>> plot(t, p)                          % Plot of capacitor power
>> t = linspace(1, 2, 5000);
>> p = 10*(t – 2);              % Computation of capacitor power
>> plot(t, p)                          % Plot of capacitor power
>> t = linspace(0, 1, 5000);
>> Wc = 5*t^2;                 % Computation of capacitor energy
>> plot(t, Wc)                        % Plot of capacitor energy
>> t = linspace(1, 2, 5000);
>> Wc=5*(t^2 – 4*t + 4);       % Computation of capacitor energy
>> plot(t, Wc)                        % Plot of capacitor energy
>> xlabel('Time t in sec')                     % x-axis labelling
```

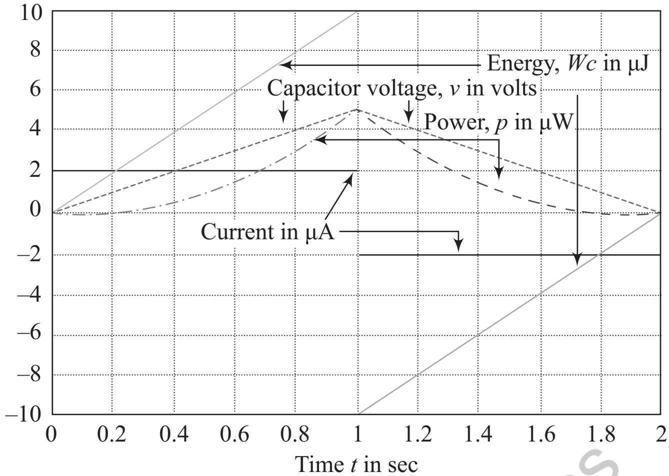


Fig. 1.19

The period during which the capacitor is storing energy is the period when power is increasing, that is, from 0 to 1 s. Similarly, the period during which the capacitor is returning energy is the period when power is decreasing, that is, from 1 to 2 s.

Energy stored by the capacitor $= \int_0^1 p\,dt = 10\int_0^1 t\,dt = 10\left[t^2/2\right]_0^1 = 5\,\mu\text{J}$

Similarly, energy returned by the capacitor $= \int_1^2 p\,dt = 10\int_1^2 (t-2)\,dt = 10\left[t^2/2 - 2t\right]_1^2 = 5\,\mu\text{J}$.

Thus, energies stored and returned by the capacitor are equal.

1.6.4 Ideal Independent Voltage Sources

An ideal independent voltage source is an element which can supply from its terminals any magnitude of current, in any direction, at a specified constant voltage. Figure 1.20 shows the symbolic representation and the *v–i* characteristics of an ideal independent voltage source.

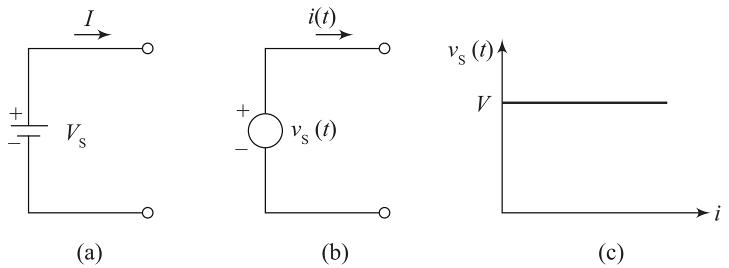


Fig. 1.20 Symbolic representation of an ideal independent voltage source: (a) DC voltage, (b) time-dependent voltage, and (c) *v–i* characteristics

From the *v–i* characteristics in Fig. 1.20(c), it is seen that the magnitude *V* of the voltage source is independent of the magnitude of the current supplied by it. Open circuit (OC) and short circuit (SC) conditions in an ideal voltage source are, respectively, represented by $i_S = 0$ and $v_S = 0$.

1.6.5 Ideal Independent Current Sources

An ideal independent current source is an element which can maintain any magnitude of voltage while supplying specified constant current from its terminals. Figure 1.21 shows the symbolic representation and the *v–i* characteristic of an ideal independent current source.

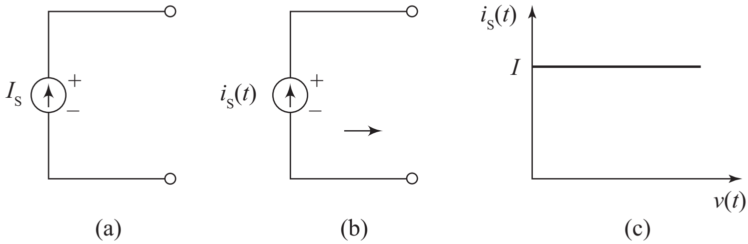


Fig. 1.21 Symbolic representation of an ideal independent current source: (a) DC current, (b) time-dependent current, and (c) *v–i* characteristics

From the *v–i* characteristics in Fig. 1.21(c), it is seen that the magnitude I of the current source is independent of the magnitude of the voltage maintained by it. Open circuit and SC conditions in an ideal current source are, respectively, represented by $v_S = 0$ and $i_S = 0$.

In practice, it has been found that it is more convenient to use voltage sources for analysing electric circuits whereas the use of current sources has been found to be handy for electronic circuit analyses.

Example 1.25 An 8 Ω resistance is connected across a 24 V ideal voltage source. Determine (i) circuit current and (ii) voltage drop across the resistance. If a 4 Ω resistance is connected in series with the 8 Ω resistance, calculate (iii) circuit current, (iv) voltage drop across each resistor, and (v) power delivered by the source in each case.

Solution

(i) Circuit current $= 24/8 = 3\,\text{A}$

(ii) Since the full voltage of the voltage source is applied across the 8 Ω resistance, voltage across it is 24 V.

(iii) In this case, full voltage of the source is applied across the (8 + 4) Ω resistances in series. Thus, current $I = 24/(8+4) = 2\,\text{A}$

(iv) Voltage across 8 Ω resistance 2 × 8 = 16 V
Voltage across 4 Ω resistance 2 × 4 = 8 V

(v) Power delivered to the 8 Ω resistor = $V \times I$ = 24 × 3 = 72 W

Power delivered to the 8 Ω resistor in series combination = $V \times I$ = 16 × 2 = 32 W

Power delivered to the 4 Ω resistor in series combination = $V \times I$ = 8 × 2 = 16 W

Total power delivered by the source = 32 + 16 = 48 W.

Example 1.26 Repeat Example 1.25 when the voltage source is replaced by an ideal current source of 24 A.

Solution In this case, the ideal current source supplies a constant current of 24 A under both types of connections.

(i) Voltage drop across the 8 Ω resistance $= 24 \times 8 = 192\,\text{V}$
(ii) When a 4 Ω resistance is connected in series, the constant current of 24 A from the current source flows through both the resistances.
(iii) Voltage drop across 8 Ω resistance $= 24 \times 8 = 192\,\text{V}$
Voltage drop across 4 Ω resistance $= 24 \times 4 = 96$ V
(iv) Voltage drop across the series combination $= 24 \times (4 + 8) = 288\,\text{V}$
(v) Power delivered to the 8 Ω resistor $= V \times I = 192 \times 24 = 4608$ W

Power delivered to the 8 Ω resistor in series combination $= V \times I = 192 \times 24 = 4608$ W

Power delivered to the 4 Ω resistor in series combination $= V \times I = 96 \times 24 = 2304$ W

Total power delivered by the source $= V \times I = 288 \times 24 = 6912$ W

From the foregoing it can be concluded that there is no limit on the voltage and power delivered by a current source.

1.6.6 Dependent Energy Sources

An energy source, whose output voltage or current is either dependent on the voltage or current in another part of the circuit, is classified as a dependent (or controlled) source. All dependent sources are unidirectional and linear. Thus, both dependent voltage and current sources are obtainable. Figure 1.22 shows the symbolic representation of the four types of dependent sources.

It may be observed that there are four variants of controlled energy sources and in order to fully specify a source, four parameters are needed, that is, source voltage or source current (v_S or i_S), controlling voltage or current (v_i or i_s) in another part of the circuit, multiplying constants (such as $\alpha, \beta, \mu,$ or ρ) and reference polarity. It would also be useful to note that constants μ and β are dimensionless constants whereas α has the unit of A/V and ρ has the unit of V/A. The relationship for each type of dependent source along with the reference polarity is shown in Fig. 1.22.

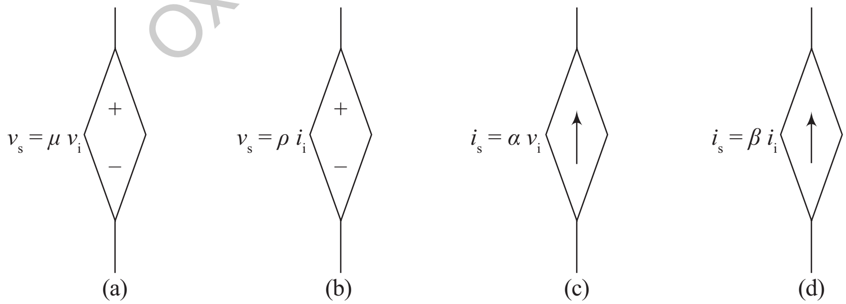


Fig. 1.22 Symbolic representation of dependent energy sources (a) Voltage-controlled voltage source (VCVS), (b) current-controlled voltage source (CCVS), (c) voltage-controlled current source (VCCS), (d) current-controlled current source (CCCS)

Example 1.27 Compute the load voltage across the resistor R_L for the given circuit in Fig. 1.23.

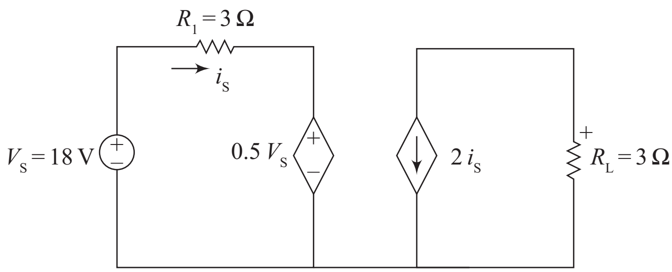


Fig. 1.23

Solution The circuit in Fig. 1.23, in addition to an independent voltage source $V_S = 18$ V, is also made up of a dependent VCVS = $0.5V_S$ and a dependent CCCS = $2i_S$.

Application of Ohm's law to the closed circuit containing the VCVS gives

$$18 + 3 \times i_S - 0.5 \times 18 = 0$$

or, $$i_S = (18 - 9)/3 = 3\text{A}$$

From the circuit containing CCCS, it is seen that the voltage across the load is

$$R_L = 2 \times (2i_S) = 2 \times (2 \times 3) = 12\text{V}$$

1.6.7 Practical Voltage and Current Sources

An ideal voltage source does not exist in practice. Figure 1.24 shows the simulation of a practical voltage source.

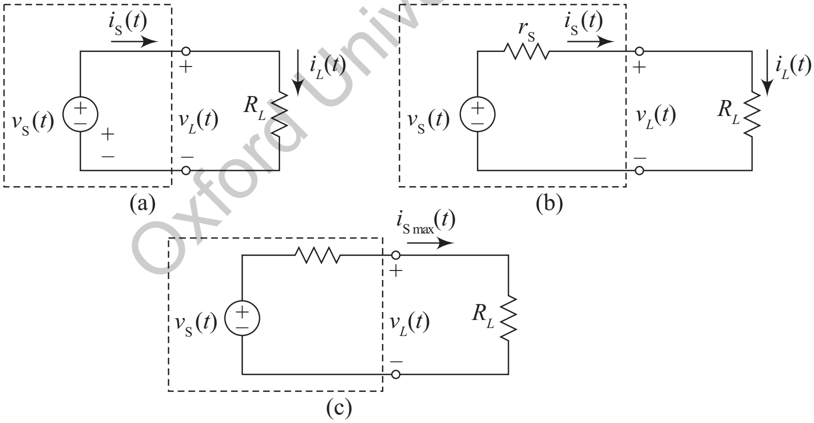


Fig. 1.24 Simulation of a practical voltage source

From Fig. 1.24(a), the load current i_L through the variable load resistor R_L Ω is given by

$$i_L(t) = i_S(t) = \frac{v_S(t)}{R_L}\text{A} \tag{1.19}$$

In addition, current, through R_L when a resistance r_S is connected in series with the ideal voltage source [see Fig. 1.24(b)] is written as

$$i_L(t) = i_S(t) = \frac{v_S(t)}{r_S + R_L}\text{V} \tag{1.20}$$

And the load voltage is

$$v_L(t) = i_L(t) \times R_L = \frac{v_S(t)}{r_S + R_L} R_L = \left(v_S(t) - i_L(t) \times r_S\right) \tag{1.21}$$

The following conclusions may be drawn from the above equations:

- From Eq. (1.19), it is seen that as $R_L \to 0$, current i_S supplied by the ideal source tends to ∞, a condition impossible to achieve.
- Equation (1.20) represents the load current supplied by a practical source. It may be noted that under a SC condition $(R_L = 0)$, the current supplied by the source is limited by its internal resistance r_S, that is, $i_{S_{max}}(t) = v_S(t)/r_S$.
- Equation (1.21) is employed to plot the v–i characteristics of a practical voltage source and is shown in Fig. 1.25.
- The resistance r_S, which has a typically low magnitude, is internal to the voltage source. Its presence affects the load voltage, and in the limiting condition as $r_S \to 0$, the magnitude of the load voltage v_L approaches the magnitude of the source voltage v_S.

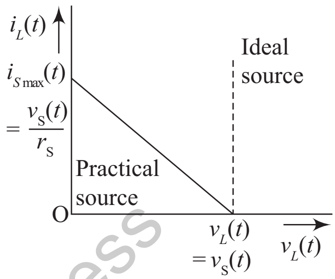


Fig. 1.25 *v–i* Characteristics of a practical voltage source

In a similar manner, a practical current source can be simulated. Modelling of a practical current source is shown in Fig. 1.26 and Fig. 1.27 depicts its *v–i* characteristics.

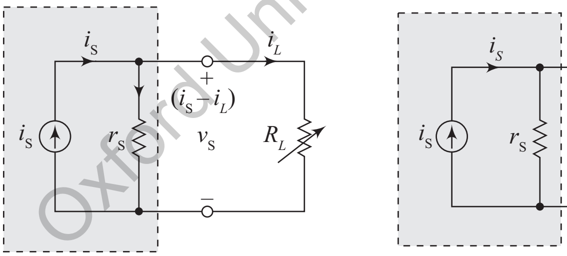


Fig. 1.26 Modelling of a practical current source

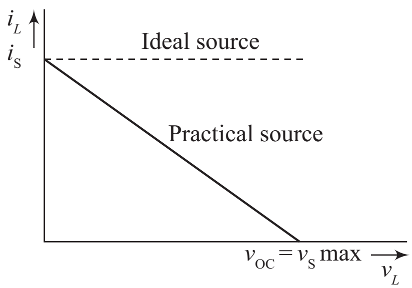


Fig. 1.27 *v–i* Characteristics of a practical current source

It is left as a tutorial exercise for the reader to verify the drooping nature of the characteristic and prove that the internal resistance possesses a high magnitude in order to simulate the behaviour of an ideal current source.

Example 1.28 A practical 48 V independent source can supply a maximum current of 120 A. Determine (a) the internal resistance of the source and (b) plot the load characteristics. Compute the load (c) current and (d) voltage when the source is a load of 50 Ω.

Solution

(a) The internal resistance of the source is determined by making use of $i_{S_{max}}(t) = v_S(t)/r_S$ as under

$$r_S = \frac{48}{120} = 0.4\,\Omega$$

(b) When the source is supplying maximum current, it is a SC condition: $v_L(t) = 0$ V and $i_L = 120$ A. Under OC condition, $v_L(t) = 48$ V and $i_L = 0$ A. The load characteristic is shown in Fig. 1.28.

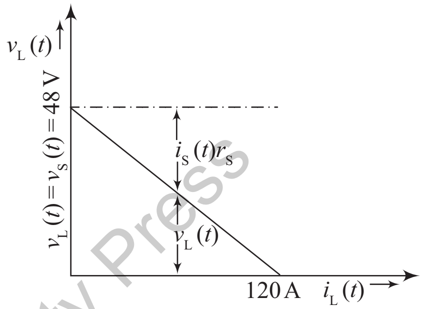


Fig. 1.28

(c) Load current is obtained by using Eq. (1.20) as under

$$i_L(t) = \frac{48}{0.4 + 50} = 0.952\,\text{A}$$

(d) Equation (1.21) is used to compute load voltage as follows:

$$v_L(t) = 0.952 \times 50 = 47.62\,\text{V}$$

Example 1.29 In the circuit shown in Fig. 1.26, the practical current source has a capacity of 10 A and an internal resistance of 100 Ω. Plot the *v–i* characteristics of the current source when load resistance is equal to (a) 10 Ω, (b) 50 Ω, and (c) 100 Ω.

Solution From Fig. 1.26, the following data is available: $i_S(t) = 10$ A and $r_S = 100\,\Omega$.

On OC $(R_L = \infty)$ $i_L(t) = 0$, and $v_L(t) = 10 \times 100 = 1000\,\text{V}$

(a) For $R_L = 10\,\Omega$, $i_L(t) = 10 \times \dfrac{100}{(100+10)} = 9.091\,\text{A}$,

and $v_L(t) = 9.091 \times 10 = 90.91\,\text{V}$

(b) For $R_L = 50\,\Omega$, $i_L(t) = 10 \times \dfrac{100}{(100+50)} = 6.667\,\text{A}$,

and $v_L(t) = 6.667 \times 50 = 333.333$ V

(c) For $R_L = 100\,\Omega$, $i_L(t) = 10 \times \dfrac{100}{(100+100)} = 5.0\,\text{A}$,

and $v_L(t) = 5.0 \times 100 = 500\,\text{V}$

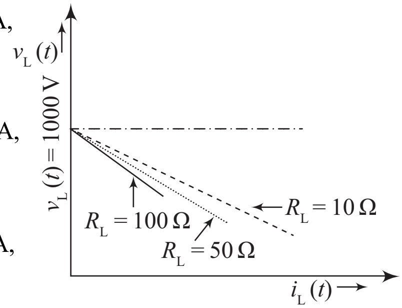


Fig. 1.29

The *v–i* characteristics are plotted in Fig. 1.29.

1.6.8 Source Transformation

Practical voltage sources can be transformed into current sources and vice versa since their terminal characteristics are linear. Figure 1.30 shows the transformation of a practical voltage source into a current source.

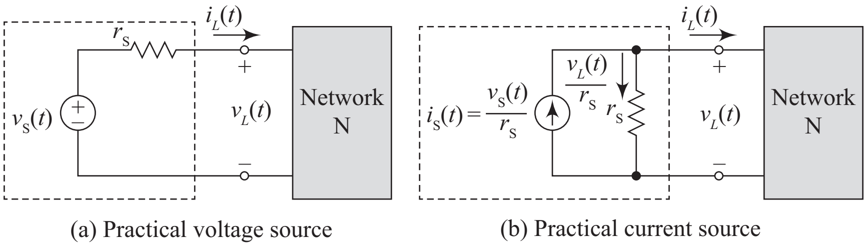


Fig. 1.30 Source transformation

From Fig. 1.30(a), the voltage at the terminals of the network N is expressed as

$$v_L(t) = v_S(t) - i_L(t) \times r_S \tag{1.22}$$

where $i_L(t)$ is the current flowing into the network.

Dividing Eq. (1.22) by r_s yields

$$\frac{v_L(t)}{r_S} = \frac{v_S(t)}{r_S} - i_L(t) \tag{1.23}$$

Equation (1.23) can be interpreted as consisting of a current source of magnitude $v_L(t)/r_S$ A and an internal resistance of magnitude r_S Ω connected across its terminals. Figure 1.30(b) schematically translates Eq. (1.23) and represents the transformation of a practical voltage source into an equivalent current source. Similarly, a practical current source can be transformed into a voltage source.

Example 1.30 Calculate the voltages across the 10 Ω and 5 Ω resistors in the circuit shown in Fig. 1.31, using source conversion technique.

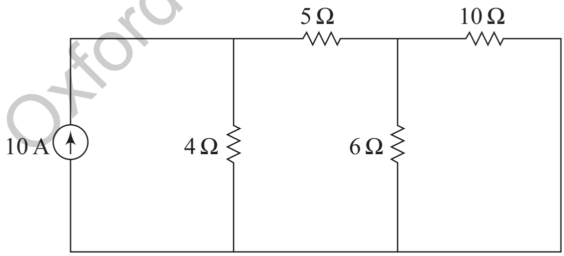


Fig. 1.31

Solution

Step 1: Transform the current source into a voltage and redraw the circuit as shown in Fig. 1.32.

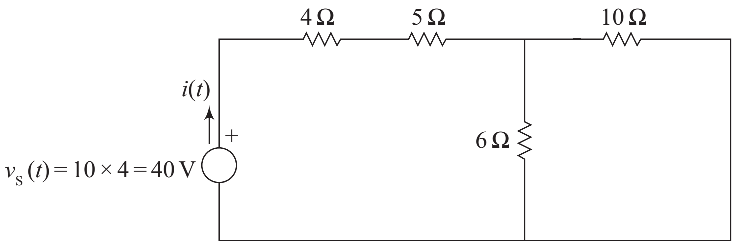


Fig. 1.32

Step 2: Calculate current $i(t)$ supplied by the voltage source.

$$i(t) = \frac{40}{\left(4+5+\dfrac{10\times 6}{10+6}\right)} = \frac{40}{12.75} = 3.137\,\text{A}$$

Step 3: Calculate the voltage across the 10 Ω resistor as under

$$v_{10\,\Omega}(t) = 40 - 3.137\times(4+5) = 11.765\,\text{V}$$

Step 4: Calculate the current through the 10 Ω resistor as follows:

$$i_{10\Omega}(t) = \frac{11.765}{10} = 1.177\,\text{A}$$

The computations are easily verified by computing the current through the 6 Ω resistor which is $i_{6\Omega} = \dfrac{11.765}{6} = 1.961$ A, and adding $i_{10\,\Omega} + i_{6\,\Omega}$ to get $i(t)$.

Example 1.31 Show that the circuits in Fig. 1.33(a) and (b) and Fig.1.34 (a) and (b) are equivalent.

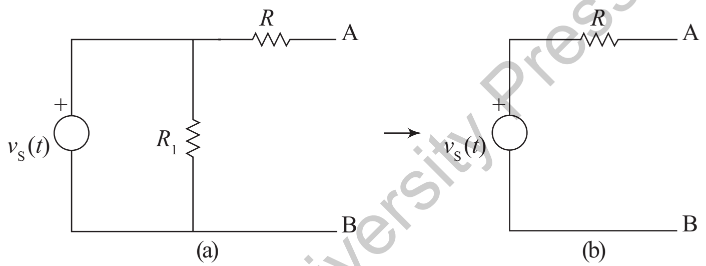


Fig. 1.33

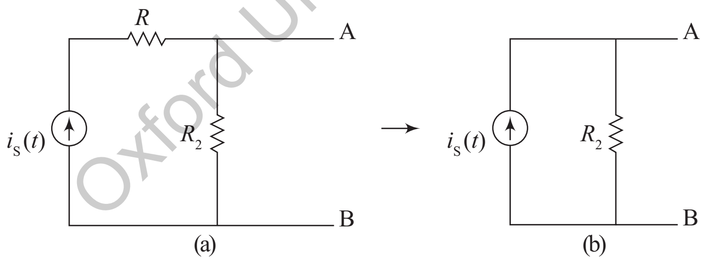


Fig. 1.34

Solution To show the equivalence between respective circuits, it is required to prove that the conditions across terminals A–B are the same.

It is seen that in both the circuits in Figs 1.33(a) and 1.33(b), the voltage across and current through the terminals A–B are $v_S(t)$ and zero, respectively. Hence, the two circuits are equivalent.

Similarly, in the circuits in Figs 1.34(a) and 1.34(b), the voltage across and current through the terminals A–B are $i_S(t)R_2$ and zero, respectively. Hence, the two circuits are equivalent.

From the foregoing, it can be concluded that a resistance connected in parallel across a voltage source and a resistance connected in series with a current source can be removed without affecting the terminal conditions in the circuit. Application of the equivalence principle is demonstrated in the next example.

Example 1.32 For the circuit shown in Fig. 1.35, use source transformation to calculate the voltage v_L across the 120 Ω resistor.

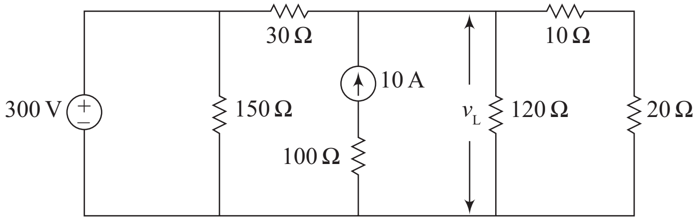


Fig. 1.35

Solution The given circuit is simplified by removing the 150 Ω resistor and shorting the 100 Ω resistors connected across the voltage and current sources, respectively. The simplified circuit is redrawn in Fig. 1.36 by combining the 10 Ω and 20 Ω resistors in series.

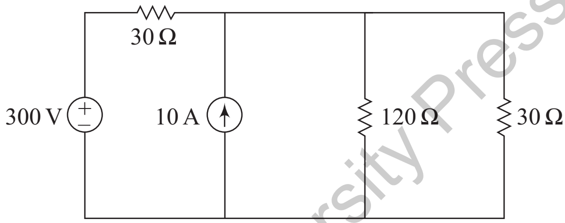


Fig. 1.36

Next, the voltage source of 300 V is transformed into a current source as shown in Fig. 1.37.

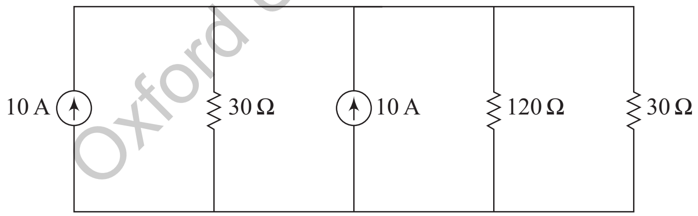


Fig. 1.37

In the next step, the two parallel current sources and the three parallel resistors are combined to arrive at the circuit shown in Fig. 1.38.

Thus, voltage $v_L = 20 \times 13.33 = 266.67\,\text{V}$.

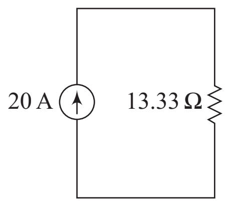


Fig. 1.38

1.7 Kirchhoff's Laws

The foundation of circuit analysis is based on the two laws of (i) current distribution and (ii) voltage division in a network and is named after the German physicist, Gustav Robert Kirchhoff.

The property of an electric charge, that it can neither be created nor be destroyed but must be conserved, forms the basis of *Kirchhoff's current law* (KCL). It states that the sum of the currents at a junction (also called a node) in a circuit is zero, at any instant of time. Mathematically KCL is written as

$$\sum_{m=1}^{m=n} i_m(t) = 0 \tag{1.24}$$

where $i_m(t)$ is the current in the m^{th} element at a node k and the total number of elements connected to the node is n. Figure 1.39 shows the application of KCL.

Since a direction is associated with the flow of current, it is necessary to define directions. Assume that the currents flowing into the junction are positive and currents flowing out are negative. Based on this assumption, KCL may be applied to the junction in Fig. 1.39 as under

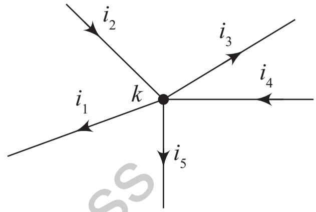


Fig. 1.39 Application of KCL

$$-i_1 + i_2 - i_3 + i_4 - i_5 = 0 \tag{1.25a}$$

or, $$i_2 + i_4 = i_1 + i_3 + i_5 \tag{1.25b}$$

Example 1.33 In Fig. 1.40, calculate v_1 at the node and i_1.

The data is as follows: $i_2 = 6$ A, $v_3 = 10e^{-2t}$, $v_4 = e^{-2t}$.

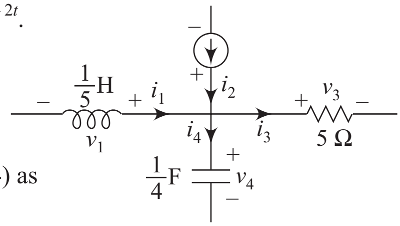


Fig. 1.40

Solution Application of Ohm's law gives

$$i_3 = \frac{10e^{-2t}}{5} = 2e^{-2t} \text{ A}$$

The current i_4 is computed by employing Eq. (1.14) as

$$i_4 = \frac{1}{4} \times \frac{d}{dt}\left(e^{-2t}\right) = -0.5e^{-2t} \text{ A}$$

Application of KCL to the node yields

$$i_1 + i_2 - i_3 - i_4 = 0$$

or $$\mathrm{i}_1 = -i_2 + i_3 + i_4 = -6 - 2e^{-2t} + (-0.5e^{-2t}) = -(6 + 2.5e^{-2t})\text{A}$$

The negative sign shows that the direction of flow of current is opposite to the assumed direction. In order to calculate v_1, Eq. (1.10) is used as under

$$v_1 = \frac{1}{5} \times \frac{d - \left(6 + 2.5e^{-2t}\right)}{dt} = e^{-2t} \text{ V}$$

Likewise that energy can neither be created nor be destroyed but must be conserved, forms the basis of *Kirchhoff's voltage law* (KVL). Thus, KVL states that in a closed circuit, the sum of voltages at any instant of time is zero. Mathematically, KVL is expressed in the following manner.

$$\sum_{k=1}^{k=n} v_k(t) = 0 \tag{1.26}$$

where $v_k(t)$ represents the voltage across the k^{th} element in a closed loop containing n elements.

In order to demonstrate the application of KVL, consider the circuit shown in Fig. 1.41.

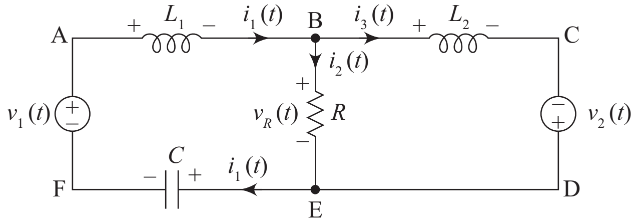


Fig. 1.41 Application of KVL

As in the case of current flow, a direction is associated with potential difference. Therefore, assume that an increase in potential difference (i.e., a voltage rise from −ve polarity to +ve polarity) is positive and conversely a decrease in potential difference (i.e., a voltage drop from +ve polarity to −ve polarity) is negative. Starting at the bottom node F and applying KVL to the closed loop FABEF yields,

$$v_1(t) - L_1\frac{di_1}{dt} - i_2R - \frac{1}{C}\int i_1 dt = 0 \tag{1.27a}$$

In order to firm up the concept of the application of KVL, consider the closed circuit FABCDEF and again start at node F.

Hence, $$v_1(t) - L_1\frac{di_1}{dt} - L_2\frac{di_3}{dt} + v_2(t) - \frac{1}{C}\int i_1 dt = 0 \tag{1.27b}$$

It is left to the reader, as a tutorial exercise, to apply KVL to the closed circuit EBCDE. *Hint*: Start at the bottom node.

Example 1.34 Apply Kirchhoff's laws to determine the source current I_S and the power consumed in the 6 Ω resistor for the circuit shown in Fig. 1.42.

Solution Current through the 6 Ω resistor is $I_1 = 4$ A

Voltage of node C, $v_c = 6 \times 4 = 24$V

Application of KCL to node C yields $I_2 + I_3 = 6 - 4 = 2$A

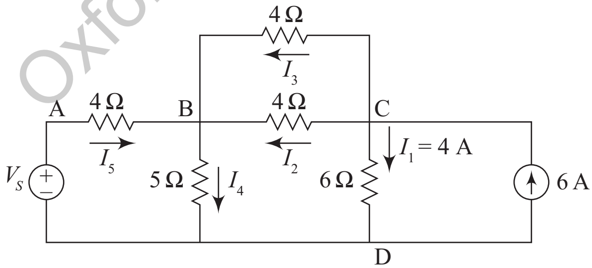


Fig. 1.42

Since the two 4 Ω resistors are in parallel, $I_2 = I_3 = 1.0$ A

Voltage of node B is $24 - I_2 \times 4 = 24 - 1.0 \times 4 = 20$ V

Current through the 5 Ω resistor is $I_4 = \frac{20}{5} = 4$ A

Applying KCL to node B leads to the source current. $I_S = I_4 + I_2 + I_3 = 4 - 1.0 - 1.0 = 2$ A

Energy consumed by the 6 Ω resistor $= I_4^2 \times 5 = 2^2 \times 5 = 20$ W .

$V_S = V_B + 4 \times I_S = 20 + 4 \times 2 = 28$ V

Example 1.35 Given that $L_1 = 0.25$ H, $L_2 = 0.5$ H, $C = 0.25$ F, $R = 2$ Ω, $v_R(t) = 4 \cos t$ V, and $i_3 = 4 \sin t$ A, determine $v_1(t)$ and $v_2(t)$ for the circuit shown in Fig. 1.41.

Solution The current through R is given by $i_2(t) = \dfrac{v_R(t)}{R} = \dfrac{4\cos t}{2} = 2\cos t\,\text{A}$

Application of KCL to node B leads to

$$i_1(t) = i_2(t) + i_3(t) = (2\cos t + 4\sin t)\ \text{A}$$

Commencing with the node F, apply KVL to the closed circuit FABEF as follows:

$$-v_1(t) + 0.25 \times \frac{d}{dt}(2\cos t + 4\sin t) + 2 \times 2\cos t + \frac{1}{0.25}\int (2\cos t + 4\sin t)\,dt = 0$$

or, $\quad v_1(t) = -0.5\sin t + \cos t + 4\cos t + 8\sin t - 16\cos t = (7.5\sin t - 11\cos t)\text{V}$

Put $\quad K\sin\varphi = -11 \text{ and } K\cos\varphi = 7.5.$

Therefore, $K = \sqrt{121 + 56.25} = 13.3135$ and $\varphi = \tan^{-1}\left(-\dfrac{11}{7.5}\right) = -55.71°$

Thus, $v_1(t) = 13.3135 \sin(t - 55.71°)$

Now apply KVL to EBCDE and start with node E.

$$-2 \times 2\cos t + 0.5 \times \frac{d}{dt}(4\sin t) - v_2(t) = 0$$

or, $\quad v_2(t) = -4\cos t + 2\cos t = -2\cos t\ \text{V}$

Example 1.36 Determine the voltage at (a) node A and (b) power supplied by the voltage source in the circuit shown in Fig. 1.43. All the relevant data is indicated in the circuit diagram.

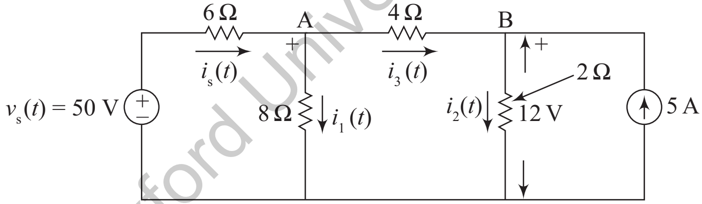


Fig. 1.43

Solution It would be useful to start from the independent current source side. Since the voltage of node B is 12 V, current through the 2 Ω resistor $i_2(t) = 12/2 = 6\,\text{A}$.

Application of KCL at node B gives $i_3(t) = 6 - 5 = 1\text{A}$

Voltage at node A $= 12 + 1 \times 4 = 16$ V

Current through the 8 Ω resistor $i_i(t) = 16/8 = 2\,\text{A}$

Current supplied by the voltage source $i_S(t) = i_1(t) + i_3(t) = 2 + 1 = 3$ A

Power supplied by the voltage source $v_S(t)i_S(t) = 50 \times 3 = 150\,\text{W}$

Example 1.37 For the circuit shown in Fig. 1.44, compute (a) current i, (b) voltage v across the dependent current source. Prove that the power generated is equal to the power absorbed.

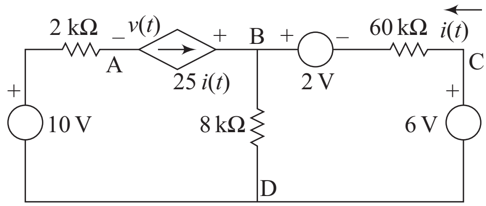


Fig. 1.44

Solution Current through the 8 kΩ resistor is = $25i(t) + i(t) = 26i(t)$

Application of Ohm's law around the closed circuit DCBD yields

$$-6+60\times10^3\times i(t)-2+8\times10^3\times26i(t)=0$$

or $$i(t)=\frac{8}{(60+208)\times10^3}=29.85\,\mu\text{A}$$

Voltage at point A is determined as under

$$v_\text{A}(t)=-10+2\times10^3\times25\times29.85\times10^{-6}=-8.5075\,\text{V}$$

Voltage at point B is given by

$$v_\text{B}(t)=8\times10^3\times26\times29.85\times10^{-6}=6.2090\text{ V}$$

$$v(t)=v_\text{A}(t)+v_\text{B}(t)=-8.5075+6.2090=2.2985$$

Power generated by the voltage sources $P_\text{g}=(6+2)\times29.85+10\times25\times29.85=7701.3\mu\text{W}$

Power is absorbed by the three resistors and the current dependent source and is obtained as under

$$P_\text{a}=6\times10^3\times i(t)^2+8\times10^3\times\left[26i(t)\right]^2+2\times10^3\times\left[25i(t)\right]^2-v(t)\times25i(t)$$

$$=6\times10^3\times\left(29.85\times10^{-6}\right)^2+8\times10^3\times\left(26\times29.85\times10^{-6}\right)^2+2\times10^3\times\left(25\times29.85\times10^{-6}\right)^2$$

$$-(-2.2985)\times\left(25\times29.85\times10^{-6}\right)=7701.3\,\mu\text{W}$$

Thus, the power generated is equal to the power absorbed.

1.8 Connection of Circuit Elements

Circuits can be connected in several ways to obtain desired outputs. This section discusses the different methods of connecting the circuit elements.

1.8.1 Series Connections

When circuit elements are connected end to end, the elements are said to be connected in series. A distinct property of the series connection is that the same current flows through all the elements connected in the circuit.

Resistors in series and the voltage divider circuit:

Figure 1.45 (a) shows resistors R_1 Ω, R_2 Ω, R_3 Ω,......, R_n Ω connected in series across a voltage source $v_S(t)$ V.

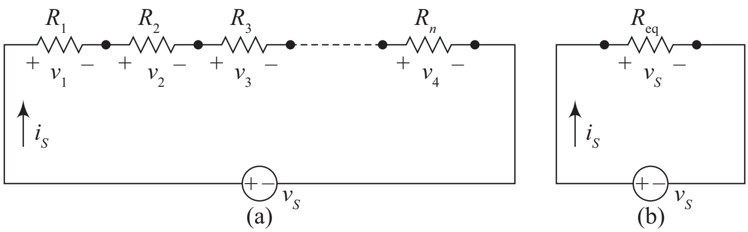


Fig. 1.45 (a) Resistors in series, (b) equivalent resistance

If the current flowing through the circuit is $i_S(t)$ A and KVL is applied around the closed circuit, then

$$i_S(t)R_1 + i_S(t)R_2 + i_S(t)R_3 + \ldots + i_S(t)R_n - v_S(t) = 0$$

$$i_S(t) = \frac{v_S(t)}{(R_1 + R_2 + R_3 + \ldots + R_n)} \tag{1.28a}$$

Figure 1.45(b) shows a circuit in which a resistor R_{eq} is connected across a voltage source $v_S(t)$ such that the same current $i_S(t)$, as in the circuit of Fig. 1.45(a), flows through it. Thus,

$$i_S(t) = \frac{v_S(t)}{R_{eq}} \tag{1.28b}$$

Comparison of Eqs (1.28a) and (1.28b) gives,

$$R_{eq} = R_1 + R_2 + R_3 + \ldots + R_n \tag{1.29}$$

Equation (1.29) shows that resistors connected in series can be directly added to obtain the equivalent resistor.

Referring to Fig. 1.45(a), it is seen that by Ohm's law,

$$v_1(t) = i_S(t) \times R_1$$

Substitution off $i_S(t)$ from Eq. (1.28b) yields

$$v_1(t) = \frac{R_1}{R_{eq}} v_S(t) \tag{1.30}$$

Similarly, $v_2(t) = \frac{R_2}{R_{eq}} v_S(t), v_3(t) = \frac{R_3}{R_{eq}} v_S(t), \ldots v_n(t) = \frac{R_n}{R_{eq}} v_S(t)$

Equation (1.30) shows that it is possible to obtain any desired voltage output by dividing it across a resistor. Such a circuit is called a *voltage divider* circuit and is shown in Fig. 1.46. It may be noted that R_{eq}, is called the input resistance R_{in}, R_1 is the output resistance, R_o, $v_s(t)$ is the input voltage, and $v_1(t)$ is the output voltage $v_o(t)$.

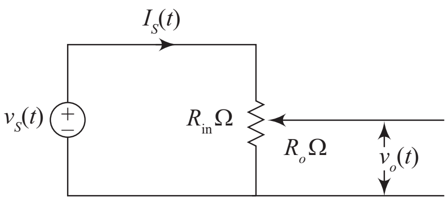


Fig. 1.46 Voltage divider circuit

Example 1.38 Design a voltage divider to obtain a variable voltage for a DC source of 220 V and a current supply of 2.0 A. (a) Determine the output resistance for an output voltage of 60 V. (b) If the output resistance is 75 Ω, calculate the percentage voltage output.

Solution Referring to Fig. 1.46, $R_{in} = \frac{v_S(t)}{i_S(t)} = \frac{220}{2.0} = 110\,\Omega$

(a) Equation (1.30) is used to determine the output resistance as follows:

$$R_o = \frac{v_o(t) \times R_{in}}{v_S(t)} = \frac{60 \times 110}{220} = 30\,\Omega$$

(b) Again from Eq. (1.30), percentage output voltage is given by

$$\frac{v_o(t)}{v_S(t)} = \frac{R_o}{R_{in}} \times 100 = \frac{75}{220} \times 100 = 34.09\%$$

Inductors in series:

Figure 1.47 shows inductors L_1 H, L_2 H, L_3 H,........, L_n H connected in series across a time-varying voltage source $v_S(t)$ V.

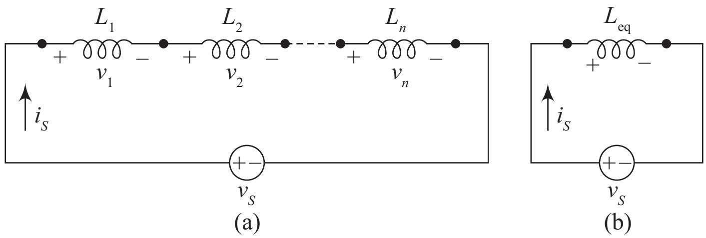


Fig. 1.47 (a) Inductors in series, (b) equivalent inductor

Assume that the circuit current is $i_S(t)$ A. Application of KVL to the circuit leads to

$$L_1\frac{d}{dt}[i_S(t)]+L_2\frac{d}{dt}[i_S(t)]+L_3\frac{d}{dt}[i_S(t)]+\ldots+L_n\frac{d}{dt}[i_S(t)]-v_S(t)=0$$

or
$$v_S(t)=L_1\frac{d}{dt}[i_S(t)]+L_2\frac{d}{dt}[i_S(t)]+L_3\frac{d}{dt}[i_S(t)]+\ldots+L_n\frac{d}{dt}[i_S(t)] \tag{1.31a}$$

Application of Ohm's law to Fig. 1.47(b) gives

$$v_S(t)=L_{eq}\frac{d}{dt}[i_S(t)] \tag{1.31b}$$

Comparison of Eqs (1.31a) and (1.31b) gives the equivalent inductance as

$$L_{eq}=L_1+L_2+L_3+\ldots+L_n \tag{1.32}$$

Capacitors in series: Similarly an equivalent circuit for capacitors connected in series can be developed. Figure 1.48 shows capacitors connected in series across a time-varying voltage source $v_S(t)$.

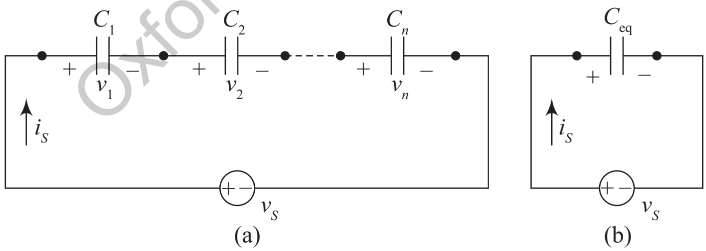


Fig. 1.48 (a) Capacitors in series, (b) equivalent capacitor

Equation (1.33) gives the expression for the equivalent capacitance.

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots+\frac{1}{C_n} \tag{1.33}$$

Derivation of the expression is left as an exercise for the reader.

1.8.2 Parallel Connections

When the two ends of all the circuit elements are joined together at two nodes, the elements are said to be connected in parallel. A characteristic of elements connected in parallel is that same voltage appears across the terminals.

Resistors in parallel and the current divider rule: Resistors $R_1\ \Omega, R_2\ \Omega, R_3\ \Omega, \ldots, R_n\ \Omega$ are connected in parallel across a voltage source $v_S(t)$ V and are shown in Fig. 1.49(a).

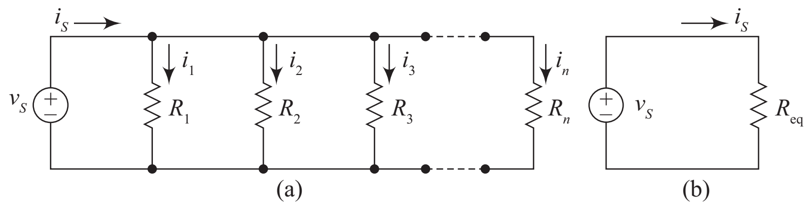


Fig. 1.49 (a) Resistors in parallel, (b) equivalent resistors

If $i_{S_1}(t), i_{S_2}(t), i_{S_3}(t), \ldots i_{S_n}(t)$ represent currents flowing through the respective resistors $R_1, R_2, R_3, \ldots, R_n$, application of KCL to the node gives

$$i_S(t) = i_{S_1}(t) + i_{S_2}(t) + i_{S_3}(t) + \ldots + i_{S_n}(t) \tag{1.34}$$

where $i_S(t)$ is the total current supplied by the voltage source.

Current in the i^{th} element in Fig. 1.49(a) is determined by Ohm's law as

$$i_{S_i} = \frac{v_S(t)}{R_i} \tag{1.35a}$$

Substitution of Eq. (1.35a) for various currents in Eq. (1.34) yields

$$i_S(t) = v_S(t)\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}\right] \tag{1.35b}$$

From the equivalent circuit in Fig. 1.49(b), it is seen that

$$i_S(t) = \frac{v_S(t)}{R_{eq}} \tag{1.35c}$$

Comparison of Eqs (1.35b) and (1.35c) leads to

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \frac{1}{R_n}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}}\ \Omega \tag{1.36a}$$

Thus, equivalent resistance of n parallel-connected resistances is the reciprocal of the sum of the reciprocals of individual resistances. In the remaining text in this book, parallel combination of resistors will be mathematically represented in the following manner.

$$R_1 \| R_2 \| R_3 \| \ldots \| R_n$$

In terms of the conductance of a resistance, Eq. (1.36a) may be expressed as

$$G_{eq} = (G_1 + G_2 + G_3 + \ldots + G_n)\,\mathrm{S} \tag{1.36b}$$

In Eq. (1.36b), $G_{eq}, G_1, G_2, G_3, \ldots, G_n$ are conductances corresponding to resistances $R_{eq}, R_1, R_2, R_3, \ldots, R_n$, respectively.

Complementing the voltage divider circuit is the *current divider* circuit which is associated with resistors in parallel.

Writing the current components $i_{S_1}(t), i_{S_2}(t), i_{S_3}(t), \ldots i_{S_n}(t)$ in Eq. (1.34), in terms of $i_S(t)$ and the corresponding resistor values gives

$$
\begin{aligned}
i_1(t) &= \frac{R_{eq}}{R_1} i_S(t) = \frac{1/R_1}{1/R_{eq}} i_S(t) = \frac{G_1}{G_{eq}} i_S(t)\,\text{A} \\
i_2(t) &= \frac{R_{eq}}{R_2} i_S(t) = \frac{1/R_2}{1/R_{eq}} i_S(t) = \frac{G_2}{G_{eq}} i_S(t)\,\text{A} \\
i_3(t) &= \frac{R_{eq}}{R_3} i_S(t) = \frac{1/R_3}{1/R_{eq}} i_S(t) = \frac{G_3}{G_{eq}} i_S(t)\,\text{A} \qquad (1.37) \\
&\cdots\cdots\cdots\cdots \\
i_n(t) &= \frac{R_{eq}}{R_n} i_S(t) = \frac{1/R_n}{1/R_{eq}} i_S(t) = \frac{G_n}{G_{eq}} i_S(t)\,\text{A}
\end{aligned}
$$

Application of the current divider principle is shown in the following example.

Example 1.39 For the current divider circuit shown in Fig. 1.50, determine the value of R_L if the current I_S supplied by the voltage source is 50 mA. What is the voltage across R_L? Compute the power consumed by R_L. All values are shown in the figure.

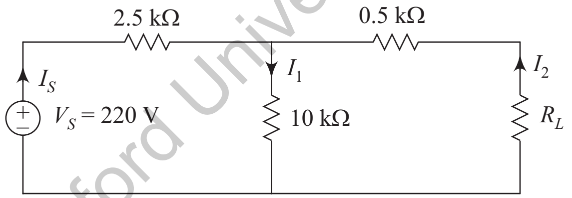


Fig. 1.50

Solution

Equivalent resistance of the circuit $R_{eq} = 2.5 + \dfrac{10 \times (0.5 + R_L)}{(10 + 0.5 + R_L)}$ (1.39.1)

By Ohm's law $R_{eq} = \dfrac{V_S}{I_S} = \dfrac{220}{50} = 4.4\ \text{k}\Omega$ (1.39.2)

Equating Eqs (1.39.1) and (1.39.2) gives

$$2.5 + \frac{10 \times (0.5 + R_L)}{(10 + 0.5 + R_L)} = 4.4 \qquad (1.39.3)$$

Simplification of Eq. (1.39.3) yields

$$R_L = 1.85\ \text{k}\Omega$$

Current through $R_L = 50 \times \dfrac{10}{(10 + 0.5 + 1.85)} = 40.50$ mA

Voltage across $R_L = 40.50 \times 1.85 = 74.80\,\text{V}$

Power consumed by $R_L = 74.80 \times \left(40.50 \times 10^{-3}\right) = 3.03\,\text{W}$

Inductors in parallel: Inductors $L_1\,\text{H}, L_2\,\text{H}, L_3\,\text{H}, \ldots L_n\,\text{H}$ are shown connected in parallel, across a time-varying voltage source $v_S(t)$, in Fig. 1.51 (a) along with its equivalent circuit in Fig. 1 .51(b).

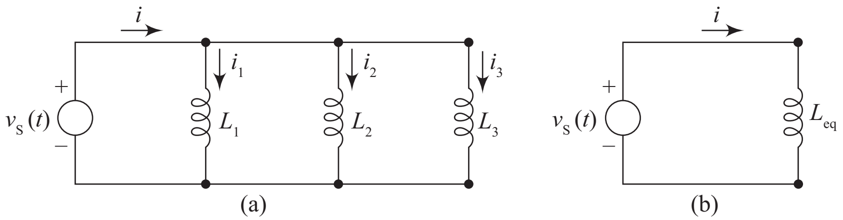


Fig 1.51 (a) Inductors in parallel, (b) equivalent inductor circuit

Assuming that the initial currents at $t = 0$ at are zero and equating the source currents in both the circuits gives

$$\frac{1}{L_1}\int v_S(t)\,dt+\frac{1}{L_2}\int v_S(t)\,dt+\frac{1}{L_3}\int v_S(t)\,dt+\ldots\frac{1}{L_n}\int v_S(t)\,dt=\frac{1}{L_{eq}}\int v_S(t)\,dt$$

or,
$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}+\ldots\frac{1}{L_n} \qquad (1.38)$$

Equation (1.38) expresses the equivalent inductance of $L_1, L_2, L_3, \ldots L_n$ inductors connected in parallel.

Example 1.40 Compute (a) the equivalent inductance, (b)$i_1(t)$, (c) $i_2(t)$, and (d)$i(t)$ for the inductive circuit shown in Fig. 1.52. Assume the voltage at the terminals is $v_S(t) = -40e^{-8t}$ mV and that the energy in both the inductors for $t \le 0$s is zero.

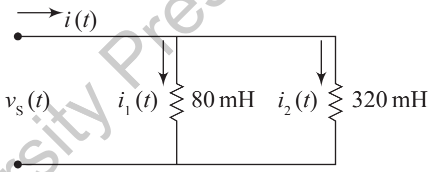


Fig. 1.52

Solution

(a) Assuming $L_1 = 80$ mH and $L_2 = 320$ mH, the parallel combination of inductors is computed by using Eq. (1.38) as follows:

$$\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2} \text{ or } L=\frac{L_1L_2}{L_1+L_2}=\frac{80\times 320}{400}=64\,\text{mH}$$

(b) Noting that for $t \le 0$ s, the stored energy is zero; Eq. (1.11) is suitably modified and the currents calculated as under

$$i_1(t)=\frac{1}{80}\int_0^t\left(-40e^{-8t}\right)=\frac{1}{80}\left[-40\times\frac{e^{-8t}}{(-8)}\right]=0.0625e^{-8t}\text{ A}$$

(c) $$i_2(t)=\frac{1}{320}\int_0^t\left(-40e^{-8t}\right)=\frac{1}{320}\left[-40\times\frac{e^{-8t}}{(-8)}\right]=0.0156e^{-8t}\text{A}$$

(d) The circuit current $i(t)=i_1(t)+i_2(t)=0.0781e^{-8t}$ A

Capacitors in parallel: Figure 1.53 (a) shows capacitors C_1 F, C_2 F, C_3 F...C_n F connected in parallel across a voltage source $v_S(t)$ V and Fig. 1.53(b) is the equivalent circuit in which C_{eq} is the equivalent capacitor such that the charge acquired, in both the cases, is the same.

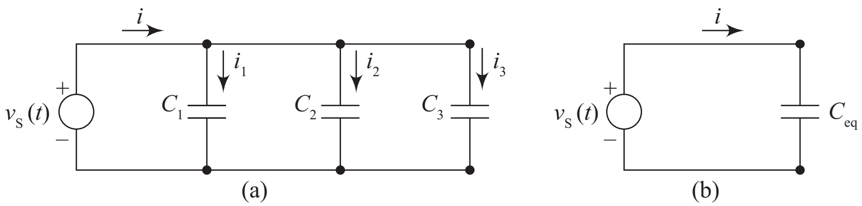


Fig. 1.53 (a) Capacitors in parallel, (b) equivalent capacitor circuit

By equating the source currents in the two circuits, it is seen that

$$\left(C_1 + C_2 + C_3 + \ldots C_n\right)\frac{d}{dt}\left[v_S(t)\right] = C_{eq}\frac{d}{dt}\left[v_S(t)\right]$$

or
$$C_{eq} = \left(C_1 + C_2 + C_3 + \ldots C_n\right)\text{ F} \tag{1.39}$$

Equation (1.39) shows that when capacitors C_1 F, C_2 F, C_3 F...C_n F are connected in parallel, the equivalent capacitance is obtained by a direct sum of all the capacitors.

Example 1.41 For the circuit shown in Fig. 1.54, calculate the capacitance of the equivalent capacitor between terminals A and B.

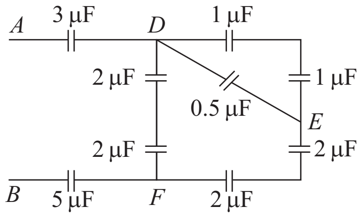


Fig. 1.54

Solution Two capacitors, 1 μF each, in series between points D and E can be combined, using Eq. (1.33), to make one capacitor of 0.5 μF. Similarly, two sets of series capacitances of 2 μF each between points D and F and E and F combine to make 1 μF each. Figure 1.54 is redrawn replacing series-connected capacitors by their equivalents as shown in Fig. 1.55(a).

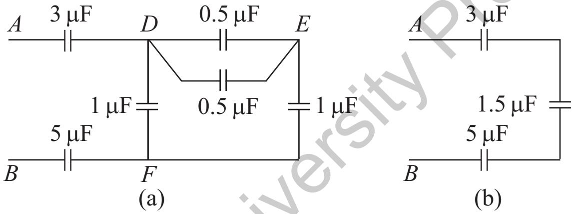


Fig. 1.55

In Fig. 1.55(a), the two parallel capacitors of 0.5 μF and 0.5 μF between points D and E can be added [using Eq. (1.39)] to an equivalent capacitor of 1 μF, which in series with 1 μF capacitor between points E and F combine to make equivalent capacitor of 0.5 μF between points D and F. Again, two parallel capacitors of 1 μF and 0.5 μF between points D and F add to form equivalent capacitor of 1.5 μF. The new equivalent circuit is shown in Fig. 1.55(b). Equation (1.33) yields the equivalent capacitor of Fig. 1.55(b) as

$$C_{eq} = \frac{1}{(1/3)+(1/1.5)+(1/5)} = \frac{5}{6} = 0.83\ \mu\text{F}$$

1.8.3 Series-Parallel Connections

In order to obtain desired outputs, circuits in practice, more often than not, are a series–parallel combination of resistors, inductors, and capacitors. The techniques for analysing series–parallel connections are best explained by taking up a few examples.

Example 1.42 A DC voltage source of 12 V is connected across the terminals A–B of the circuit shown in Fig. 1.56. Determine the source current and power supplied by the source. What is the power consumed by the 2 Ω resistor? Values of all resistors are shown in the figure.

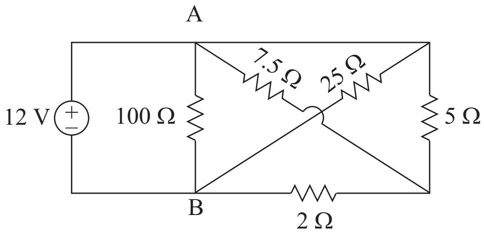


Fig. 1.56

Solution In order to visualize the series–parallel connections in the circuit, it is redrawn in Fig. 1.57.

Equivalent resistance of branch consisting of 7.5, 5.0 and 2 Ω resistors

$$= \frac{7.5 \times 5.0}{7.5 + 5.0} + 2.0 = 5.0\,\Omega$$

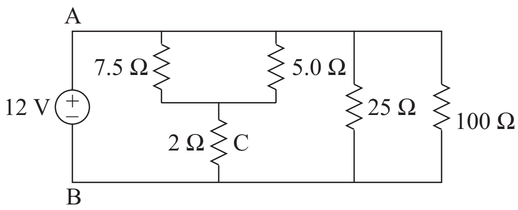


Fig. 1.57

The equivalent resistance R_{eq} at the terminals A–B is given by $\frac{1}{R_{eq}} = \frac{1}{5} + \frac{1}{25} + \frac{1}{100} = 0.25$

or $\quad R_{eq} = 4\,\Omega$

Source current $I_S = \frac{V_S}{R_{eq}} = \frac{12}{4} = 3.0\,\text{A}$

Power supplied by the source $= V_S \times I_S = 12 \times 3 = 36\,\text{W}$

The equivalent resistance of the 25 Ω and 100 Ω parallel branches $= \frac{25 \times 100}{(25+100)} = 20\,\Omega$

The current through the 2 Ω resistor $= I_S \times \frac{20}{(20+5)} = 3.0 \times \frac{20}{25} = 2.4\,\text{A}$

Therefore, power consumed by the 2 Ω resistor $= (2.4)^2 \times 2 = 11.52\,\text{W}$

Example 1.43 In the circuit shown in Fig. 1.58 calculate (a) currents I, I_1, and I_2; (b) the power consumed by each resistor; (c) the voltage drop V_2 across the 2 Ω resistor.

Solution

(a) $$I = \frac{12}{2 + (12 \quad 24)} = \frac{12}{2 + \frac{12 \times 24}{12 + 24}} = \frac{12}{10} = 1.2\,\text{A}$$

$$I_1 = I \times \frac{24}{24+12} = 1.2 \times \frac{2}{3} = 0.8\text{A}$$

$$I_2 = I \times \frac{12}{24+12} = 1.2 \times \frac{1}{3} = 0.4\,\text{A}$$

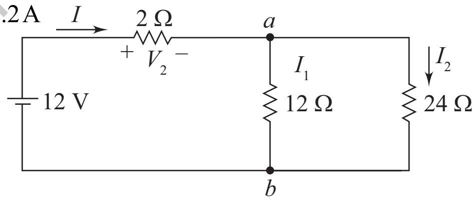


Fig. 1.58

(b) Power consumed in the 2 Ω resistor = $I^2 \times 2 = 1.2^2 \times 2 = 2.88$ W

Power consumed in the 12 Ω resistor = $I_1^2 \times 12 = 0.8^2 \times 12 = 7.68$ W

Power consumed in the 24 Ω resistor = $I_2^2 \times 24 = 0.4^2 \times 24 = 3.84$ W

(c) Voltage drop $V_2 = I \times 2 = 1.2 \times 2 = 2.4$ V

Example 1.44 The RLC circuit in Fig. 1.59 is operating in the steady-state condition. Compute the energy stored in the capacitor and inductor.

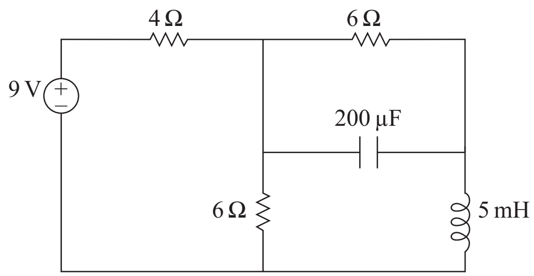


Fig. 1.59

Solution Under steady-state condition, the capacitor is on OC and the inductor is on SC. Figure 1.60 shows the circuit in the steady-state condition.

In order to determine the stored energies, it is necessary to calculate the capacitor OC voltage V_C and the inductor SC current I_L.

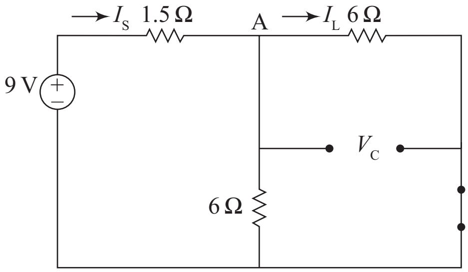


Fig. 1.60

The source current $I_S = \dfrac{9}{1.5 + 6 \times 6/(6+6)} = 2\,\text{A}$

The voltage at point A, $V_C = 9 - 2 \times 1.5 = 6\,\text{V}$

The current through the inductor $I_L = 1\,\text{A}$

Energy stored in the capacitor $= \dfrac{1}{2} \times 200 \times 10^{-6} \times (6)^2 = 0.0036\,\text{J}$

Energy stored in the inductor $= \dfrac{1}{2} \times 5 \times 10^{-3} \times (1)^2 = 0.0025\,\text{J}$

Example 1.45 In Fig.1.61, the energy stored in the capacitor is equal to the energy stored in the inductor when the circuit is operating under steady-state condition. What is the magnitude of R? All data is shown in the figure.

Solution Figure 1.62 shows the circuit when it is operating under steady-state condition.

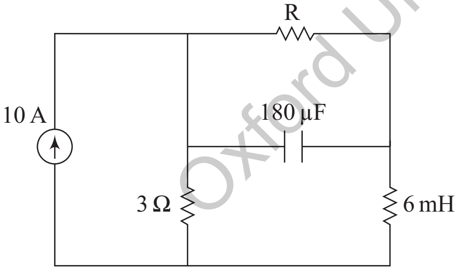


Fig. 1.61

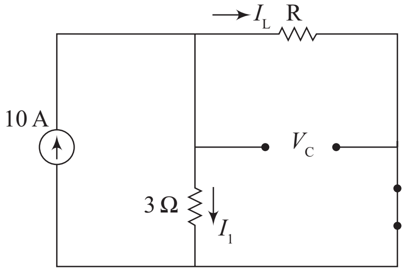


Fig. 1.62

Assume that the steady-state voltage across the capacitor is V_C volts.

Current through the inductor $I_L = \dfrac{V_C}{R}\,\text{A}$

$$\text{Energy stored in the capacitor} = \frac{1}{2} \times 180 \times 10^{-6} \times V_C^2\ \text{J} \tag{1.45.1}$$

$$\text{Energy stored in the inductor} = \frac{1}{2} \times 6 \times 10^{-3} \times \left(\frac{V_C}{R}\right)^2\ \text{J} \tag{1.45.2}$$

Equating Eqs (1.47.1) and (1.47.2) results in

$$R = \sqrt{\frac{6 \times 10^{-3}}{180 \times 10^{-6}}} = 5.774\,\Omega$$

Example 1.46 Show that the output in the circuit in Fig. 1.63 is $-RC\dfrac{dv_i(t)}{dt}$ when the input voltage is $v_i(t)$.

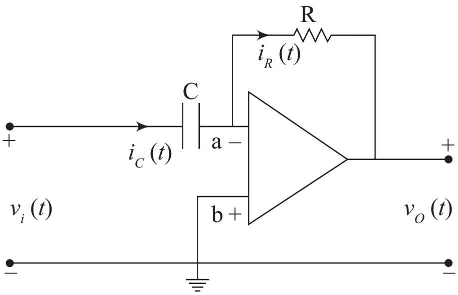


Fig. 1.63

Solution The given circuit is easily identified as an amplifier circuit. Since, points 'a' and 'b' are at the same potential, application of KCL at point 'a' yields

$$i_R(t) = i_C(t) \tag{1.46.1}$$

In addition, $i_R(t) = -\dfrac{v_0(t)}{R}$ and $i_C = C\dfrac{d}{dt}\left[v_i(t)\right]$

Substituting in Eq. (1.46.1) and rearranging results in

$$v_0(t) = -RC\frac{d}{dt}\left[v_i(t)\right] \tag{1.46.2}$$

Equation (1.46.2) shows that the output voltage is a differential of the input voltage.

Example 1.47 The applied input voltage to the circuit in Example 1.46, is

$$v_i(t) = 1500t \text{ when } 0 < t < 3 \text{ ms}$$

$$v_i(t) = 9 - 1500t \text{ when } 3 < t < 6 \text{ ms}$$

(a) Determine the form of output voltage.
(b) Use MATLAB facility to sketch the input and output voltages. Assume $R = 6$ kΩ, and $C = 0.25$ μF and that the capacitor carries no initial charge.

Solution

(a) Equation (1.46.2) is used to compute the output voltage as under

For $0 < t < 3$, output voltage $v_0(t) = 6\times10^3 \times 0.25\times10^{-6}\dfrac{d}{dt}[1500t] = 2.25\text{V}$

For $0 < t < 3$, output voltage $v_0(t) = 6\times10^3 \times 0.25\times10^{-6}\dfrac{d}{dt}[9-1500t] = -2.25\text{V}$

(b) The time t and input voltage $v_i(t)$ coordinates for the input voltage are first expressed as vectors as shown below.

$$t = [0, 3, 6, 9, 12]$$

$$v_i(t) = [0, 4.5, 0, 4.5, 0]$$

Each $t - v_i(t)$ pair along with the line command is used to plot the input voltage.

```
>> line([0, 3], [0, 4.5])           % Plots co-ordinates (0, 0) and
                                                        (3, 4.5)
>> hold on
>> line([3, 6], [4.5, 0])          % Plots co-ordinates (3, 4.5) and
                                                          (6, 0)
```

```
>> line([6, 9], [0, 4.5])          % Plots co-ordinates (6, 0) and
                                                        (9, 4.5)
>> line([9, 12], [4.5, 0])        % Plots co-ordinates (9, 4.5) and
                                                        (12, 0)
>> grid on
>> xlabel('Time t in milli sec')
>> ylabel('Input voltage in volts')
```

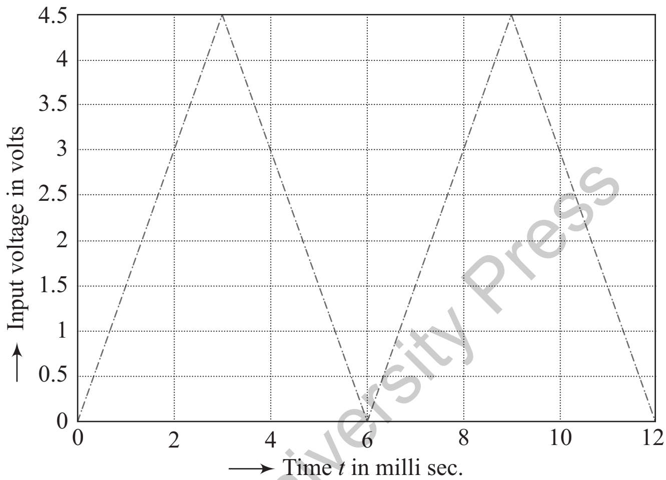


Fig. 1.64

Figure 1.64 shows the plot of the input voltage $v_i(t)$ versus t.

By following a similar procedure, line command is used to plot the output voltage $v_o(t)$ versus t.

```
>> line([0, 3], [2.25, 2.25])     % Plots co-ordinates (0, 2.25) and
                                                       (3, 2.25)
>> hold on
>> line([3, 3], [2.25, -2.25])    % Plots co-ordinates (3, 2.25) and
                                                      (3, -2.25)
>> line([3, 6], [-2.25, -2.25])      % Plots co-ordinates (3, -2.25)
                                                  and (6, -2.25)
>> line([6, 6], [-2.25, 0])          % Plots co-ordinates (6, -2.25)
                                                      and (6, 0)
>> grid on
>> xlabel('Time t in millisec')
>> ylabel('Output voltage v in
   volts')
>> line([6, 6], [-2.25, 2.25])       % Plots co-ordinates (6, -2.25)
                                                   and (6, 2.25)
>> line([6, 9], [2.25, 2.25])     % Plots co-ordinates (6, 2.25) and
                                                       (9, 2.25)
```

```
>>line ([9, 9], [2.25, -2.25])   % Plots co-ordinates (9, 2.25) and
                                                            (9, -2.25)
>>line ([9, 12], [-2.25, -2.25])   % Plots co-ordinates (9, -2.25)
                                                        and (12, -2.25)
>>line ([12, 12], [-2.25, 2.25])   % Plots co-ordinates (12, -2.25)
                                                        and (12, 2.25)
>>line ([12, 13], [2.25, 2.25])    % Plots co-ordinates (12, 2.25)
                                                        and (13, 2.25)
```

Figure 1.65 shows the plot of output voltage $v_o(t)$ versus t.

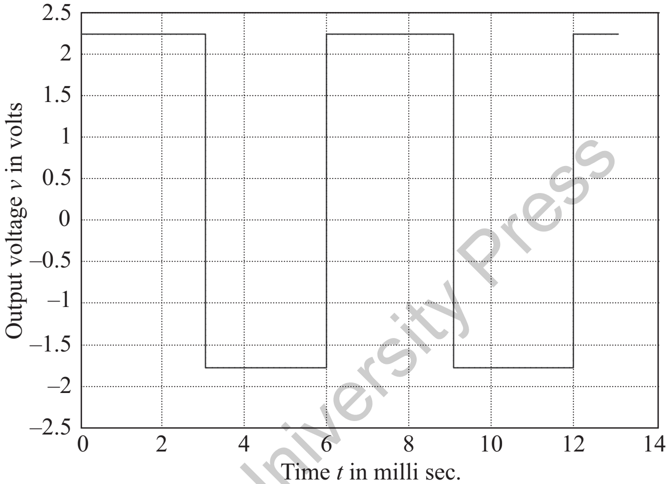


Fig. 1.65

1.9 STAR (Y)–DELTA (Δ), (Δ)–(Y) TRANSFORMATIONS

Combinations of elements to form series, parallel, and series–parallel circuits become so complex that their simplification often becomes very complicated. Simplification of such circuits is facilitated by identifying star connections and transforming them into delta connections and vice versa.

Figure 1.66 (a) shows resistors R_{1n}, R_{2n}, and R_{3n} connected in star between nodes 1, 2, and 3, respectively, whereas Fig. 1.66(b) represents resistors R_{1-2}, R_{2-3}, and R_{3-1} connected in delta between the corresponding nodes.

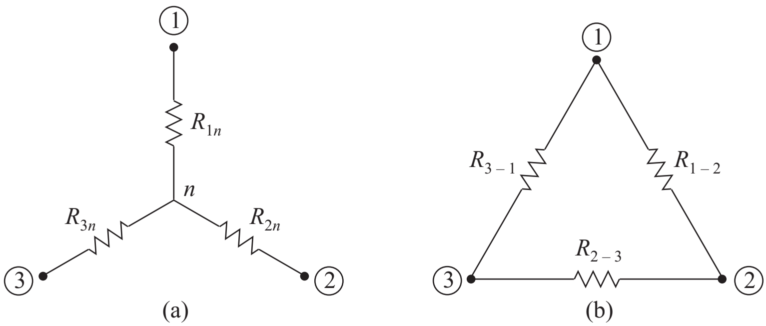


Fig. 1.66 (a) Star connection, (b) delta connection

In order to effect transformation of one type of connection into the other type, it is necessary that the resistance between any two pair of nodes of a network being transformed is equal to the resistance between the same pair of nodes of the other network. Equating the resistance between nodes 1 and 2 in Figs 1.66(a) and (b), it is seen that

$$R_{1n} + R_{2n} = R_{1-2} \parallel \left(R_{2-3} + R_{3-1}\right) = \frac{R_{1-2} \times R_{2-3} + R_{1-2} \times R_{3-1}}{R_{1-2} + R_{2-3} + R_{3-1}} \quad (1.40a)$$

Similarly, resistance between nodes 2 and 3 gives

$$R_{2n} + R_{3n} = R_{2-3} \parallel \left(R_{3-1} + R_{1-2}\right) = \frac{R_{2-3} \times R_{3-1} + R_{2-3}R_{1-2}}{R_{1-2} + R_{2-3} + R_{3-1}} \quad (1.40b)$$

And between nodes 3 and 1 yields

$$R_{3n} + R_{1n} = R_{3-1} \parallel \left(R_{1-2} + R_{2-3}\right) = \frac{R_{3-1} \times R_{1-2} + R_{3-1} \times R_{2-3}}{R_{1-2} + R_{2-3} + R_{3-1}} \quad (1.40c)$$

For a transformation of the delta network of Fig. 1.66(b) into an equivalent star network of Fig. 1.66(a), Eqs (140 a–c) are solved simultaneously.

Subtraction of Eq. (1.40b) from Eq. (1.40a) gives

$$R_{1n} - R_{3n} = \frac{R_{1-2} \times R_{3-1} - R_{2-3} \times R_{3-1}}{R_{1-2} + R_{2-3} + R_{3-1}} \quad (1.40d)$$

Addition of Eqs (140c) and (140d) and dividing the sum by 2 results in

$$R_{1n} = \frac{R_{3-1} \times R_{1-2}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega \quad (1.41a)$$

Similarly expressions for R_{2n} and R_{3n} can be derived and are given below:

$$R_{2n} = \frac{R_{1-2} \times R_{2-3}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega \quad (1.41b)$$

$$R_{3n} = \frac{R_{2-3} \times R_{3-1}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega \quad (1.41c)$$

From Eqs (1.41), it is seen that transformation equations can be developed by inspection by following the thumb rule given below:

'The equivalent star resistance connected to a node is equal to the product of the two delta resistances connected to the same node divided by the sum of the delta resistances.'

Simultaneous solution of Eqs (1.40) or (1.41) results in expressions for transforming a star connected network into a delta network. The equations are

$$R_{1-2} = R_{1n} + R_{2n} + \frac{R_{1n}R_{2n}}{R_{3n}}\,\Omega \quad (1.42a)$$

$$R_{2-3} = R_{2n} + R_{3n} + \frac{R_{2n}R_{3n}}{R_{1n}}\,\Omega \quad (1.42b)$$

$$R_{3-1} = R_{3n} + R_{1n} + \frac{R_{3n}R_{1n}}{R_{2n}}\,\Omega \quad (1.42c)$$

The derivation of Eqs (1.42) is left as a tutorial exercise for the reader.

The thumb rule for transforming a star network into equivalent delta network is stated below.

'The equivalent delta resistance between two nodes is the sum of two star resistances connected to those nodes plus the product of the same two star resistances divided by the third star resistance.'

Example 1.48 The circuit shown in Fig. 1.67 consists of eight resistors, each of 3 Ω, and is connected as shown below. Determine the source current when a DC voltage source of 6 V is connected between (a) A and B and (b) A and C.

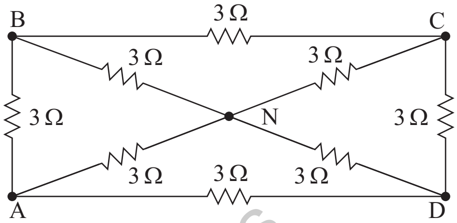


Fig. 1.67

Solution

(a) In order to calculate the source current, it is required to determine the equivalent resistance between nodes A–B. As a first step, transform delta connections between nodes NAD and NBC into equivalent star connections. The resistance of the equivalent star connection is $\frac{3\times 3}{(3+3+3)} = 1\,\Omega$. The equivalent circuit is shown in Fig. 1.68(a)

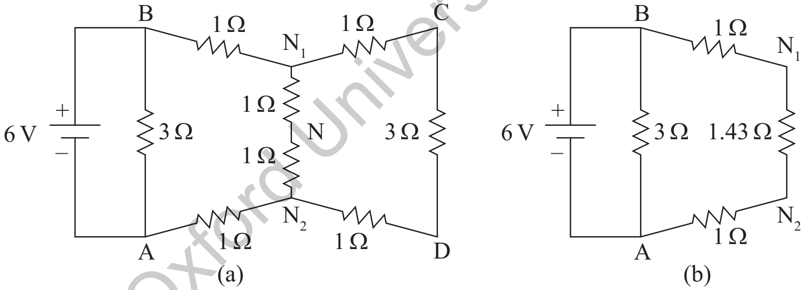


Fig. 1.68

From the circuit in Fig. 1.68(a), it is seen that resistances between nodes N_1 N_2 || with series resistors between nodes N_1C, CD, and DN_2. Thus, equivalent resistance of the parallel combination between nodes N_1 N_2 is $\frac{2\times 5}{(2+1+3+1)} = 1.43\,\Omega$

The equivalent resistance between nodes AB is obtained by the parallel combination of the 3 Ω resistances with the series combination of resistors between nodes AN_2, N_2N_1, and N_1B as shown in Fig. 1.68(b). Hence,

$$R_{AB} = \frac{3\times(1+1.43+1)}{(3+3.43)} = 1.60\,\Omega$$

Therefore, source current $= \frac{6}{1.6} = 3.75\,\text{A}$

(b) When the supply source is connected between nodes A and C, the two delta networks ANB and DNC are transformed into two equivalent star networks and the transformed circuit is shown in Fig. 1.69(a).

The circuit in Fig. 1.69(b) is obtained by combining the series resistances between nodes N_1B and BC and N_2D and DA. The delta AN_1N_2 is transformed into an equivalent

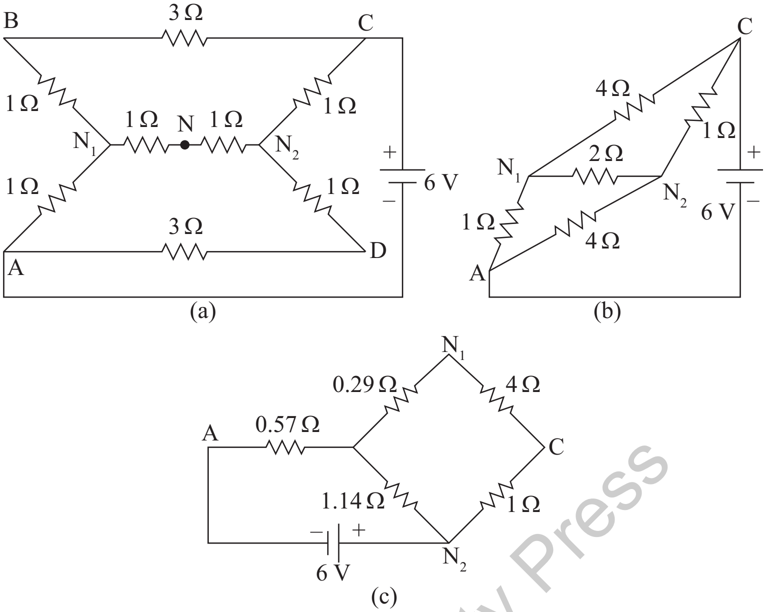


Fig. 1.69

star and the circuit is shown in Fig. 1.69(c). The resistance between nodes A and C is computed as follows:

$$R_{AC} = 0.57 + \frac{(0.29 + 4.0)\times(1.14 + 1.0)}{(4.29 + 2.14)} = 2.0\,\Omega$$

The source current $= \frac{6}{2} = 3.0\,\text{A}$.

Recapitulation

- Electrical materials are classified into conductors, semiconductors, and insulators.
- Voltage is work done per unit charge: $v = \frac{dw}{dq}$ J/C or V.
- Current, $i = \frac{dq}{dt}$ C/sec or A
- Electric power, $p = v \times i = \frac{dw}{dq} \times \frac{dq}{dt} = \frac{v^2}{R} = Gv^2 = \frac{i^2}{G}$ J/sec or W
- Energy, $E = p \times t = v \times i \times t = vit$ J, or, watt-sec
- Resistance of a conductor, $R = \rho \frac{l}{a} \Omega$
- Resistivity, $\rho = \frac{R \times a}{l} \Omega$-m
- Conductance, $G = \sigma \times \frac{a}{l}$ S
- Resistance of a conductor, $R_2 = R_1\left[1 + \alpha_1\left(t_2 - t_1\right)\right] \Omega$
- Power dissipated in a resistor, $p = i^2 R = \frac{v^2}{R}$ W

- Energy dissipated in a resistor, $W = \int_0^t p\,dt = p \times t = \left(i^2 R\right) \times t = \left(\dfrac{v^2}{R}\right) \times t$ W-sec
- Inductance of an inductor, $L = \dfrac{v}{\dfrac{di}{dt}}$ V/A or H
- Current in an inductor, $i = \dfrac{1}{L}\int_0^t v\,dt + i(0)\,\mathrm{A}$, where $i(0)$ is the current in the circuit at $t = 0$.
- Instantaneous power in the inductor at time t, $p = vi = Li\dfrac{di}{dt}$ W
- Energy stored in an inductor, $W_\mathrm{L} = \int_0^i (L\dfrac{di}{dt})i\,dt = L\int_0^i i\,di = \dfrac{1}{2}Li^2$ J
- Current through a capacitor, $i = \dfrac{dq}{dt} = C\dfrac{dv}{dt}$ A
- Voltage across a capacitor at time t, $v = \dfrac{1}{C}\int_0^t i\,dt + v(0)$, where $v(0)$ is voltage across the capacitor at $t = 0$.
- Instantaneous power in the capacitor, $p = vi = Cv\dfrac{dv}{dt}$ W
- Energy stored in the capacitor, $W_\mathrm{C} = \int_0^v (Cv\dfrac{dv}{dt})\,dt = C\int_0^v v\,dv = \dfrac{1}{2}Cv^2$ J
- v–i characteristics of a practical voltage source,

  $$v_\mathrm{L}(t) = i_\mathrm{L}(t) \times R_\mathrm{L} = \frac{v_\mathrm{S}(t)}{r_\mathrm{S} + R_\mathrm{L}} R_\mathrm{L} = \left(v_\mathrm{S}(t) - i_\mathrm{L}(t) \times r_\mathrm{S}\right)$$
- v–i characteristics of a practical current source, $v_L(t) = i_L(t) \times R_L = r_\mathrm{S}\left(i_\mathrm{S}(t) - i_L(t)\right)$
- Kirchhoff's current law, $\sum_{k=1}^{k=n} i_k = 0$
- Kirchhoff's voltage law, $\sum_{k=1}^{k=n} v_k = 0$
- Resistors in series, $R_\mathrm{eq} = R_1 + R_2 + R_3 + \ldots + R_n$
- Voltage divider rule, $v_1(t) = \dfrac{R_1}{R_\mathrm{eq}} v_\mathrm{S}(t)$
- Inductors in series, $L_\mathrm{eq} = L_1 + L_2 + L_3 + \ldots + L_n$
- Capacitors in series, $\dfrac{1}{C_\mathrm{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \ldots + \dfrac{1}{C_n}$
- Resistors in parallel, $\dfrac{1}{R_\mathrm{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots \dfrac{1}{R_n}$ or $R_\mathrm{eq} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots + \dfrac{1}{R_n}}\,\Omega$
- In terms of conductances, $G_\mathrm{eq} = (G_1 + G_2 + G_3 + \ldots + G_n)$S
- Current divider rule, $i_1(t) = \dfrac{R_\mathrm{eq}}{R_1} i_\mathrm{S}(t) = \dfrac{1/R_1}{1/R_\mathrm{eq}} i_\mathrm{S}(t) = \dfrac{G_1}{G_\mathrm{eq}} i_\mathrm{S}(t)$ A
- Inductors in parallel, $\dfrac{1}{L_\mathrm{eq}} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} + \ldots \dfrac{1}{L_n}$
- Capacitors in parallel, $C_\mathrm{eq} = \left(C_1 + C_2 + C_3 + \ldots C_n\right)$F

- Delta–star transformation, $R_{1n} = \dfrac{R_{3-1} \times R_{1-2}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega$, $R_{2n} = \dfrac{R_{1-2} \times R_{2-3}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega$,

  $R_{3n} = \dfrac{R_{2-3} \times R_{3-1}}{R_{1-2} + R_{2-3} + R_{3-1}}\,\Omega$

- Star–delta transformation, $R_{1-2} = R_{1n} + R_{2n} + \dfrac{R_{1n}R_{2n}}{R_{3n}}\,\Omega$, $R_{2-3} = R_{2n} + R_{3n} + \dfrac{R_{2n}R_{3n}}{R_{1n}}\,\Omega$,

  $R_{3-1} = R_{3n} + R_{1n} + \dfrac{R_{3n}R_{1n}}{R_{2n}}\,\Omega$

Exercises

Review Questions

1. Name different types of electrical materials and discuss their classification.
2. Describe the structure of an atom.
3. State Coulomb's law and explain (a) voltage and (b) current.
4. (a) How are voltage and current correlated to power and energy?
   (b) Explain passive sign convention and discuss its significance.
5. Specify the basic circuit elements and state how they are categorized?
6. Write clear and concise notes on the characteristics of (a) resistors, (b) inductors, and (c) capacitors.
7. Discuss the significance of temperature coefficient of resistance of a material.
8. Classify different types of energy sources. Draw their symbolic representation circuits and discuss the properties of each.
9. Explain with the help of load characteristics, the difference between ideal and practical voltage and current sources.
10. Define and explain Kirchhoff's laws. State the basis of these laws.
11. Explain series connection of resistors and the voltage divider circuit.
12. Explain parallel connection of resistors and the current divider circuit.
13. Derive expressions for (a) inductors and (b) capacitors connected in series.
14. Derive expressions for (a) inductors and (b) capacitors connected in parallel.
15. State the rules for (a) star–delta and (b) delta–star transformation by inspection.
16. Derive expressions for delta–star conversion of networks.

Multiple Choice Questions

1. The width of the forbidden zone in a conductor is
   (a) overlapping
   (b) more than that of an insulating material
   (c) equal to that of a semiconductor
   (d) none of these

2. Which of the following is a dielectric?
   (a) Carbon (b) silicon (c) iron (d) Mica

3. What is the rate of electron drift if a current of 3.2 A is flowing through a conductor?
   (a) 3.2 electrons/s (b) 1.0×10^{19} electrons/s
   (c) 2.0×10^{19} electrons/s (d) 3.2×10^{19} electrons/s

4. The magnitude of the static force between two charged bodies, separated by r metre, is F N. What is the magnitude of the charge when the distance is doubled?

(a) $2F$ N (b) $0.5F$ N (c) $0.25F$ N (d) none of these

5. Power p J/sec is generated when an object is moved inside a magnetic field. What is the power generated when the magnetic field is halved and velocity of the object is increased three times?

(a) p J/s (b) $0.67\,p$ J/s (c) $6.0\,p$ J/s (d) $1.5\,p$ J/s

6. Which of the following is representative of power?

(a) $\dfrac{[v_S(t)]^2}{R}$ (b) $v_S(t)i_S(t)$ (c) $[i_S(t)]^2 R$ (d) all of these

7. A 5 kΩ resistor is connected across a 50 V supply. The power consumed by the resistor is

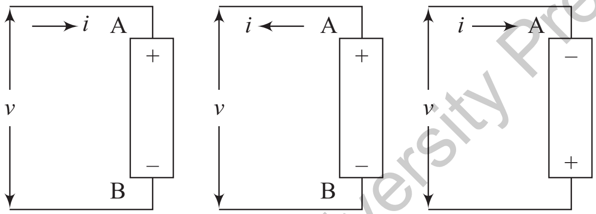
(a) 1000 mW (b) 500 mW (c) 250 mW (d) 10 mW

8. Which of the following is a unit of energy?

(a) Joule (b) Joule/sec (c) Joule-sec (d) none of these

9. Which of the following is not generating power as per the passive sign convention?

(a) (b) (c) (d) none of these



10. Which of the following is not a bilateral element?

(a) transistor (b) resistor (c) inductor (d) capacitor

11. A conductor of diameter d and length l has a resistance of R Ω. What is the value of the resistance if the conductor diameter and length are both halved?

(a) $8R$ (b) $4R$ (c) $2R$ (d) R

12. The power consumed by a conductor of diameter d and length l is W when a current of I A flows through it. What will be the power consumed if the conductor diameter, length, and current are halved?

(a) 8 W (b) 4 W (c) 2 W (d) 0.5 W

13. A 16 W resistor has a maximum current rating of 400 mA. What is the maximum current rating if the power rating of the resistor is limited to 1 W?

(a) 400 mA (b) 300 mA (c) 200 mA (d) 100 mA

14. Resistivity has the unit of

(a) Ω-metre^2 (b) Ω-metre (c) Ω/metre (d) Ω/metre^2

15. Two resistors connected in series draw a current of 5 A from a voltage source of 100 V. When one of the resistors is connected across the same voltage source, the current in the resistor is 20 A. The resistance of the disconnected resistor is

(a) 15 Ω (b) 10 Ω (c) 5 Ω (d) 1.5 Ω

16. The current flowing in a 2 H inductor increases from 0 to 10 A in 1 s and then decreases to zero in the next 2.5 s. The voltage at the end of 3.5 s in the inductor is equal to

(a) 20 V (b) 15 V (c) 8 V (d) −8 V

17. A current of 100 mA flows through a capacitor of 100 μF for 1 s. If the capacitor is initially uncharged, the charge on the capacitor is

(a) 0.001 C (b) 0.01 C (c) 0.1 C (d) 1.0 C

18. The power supplied by the source in Q. 17 is

(a) 10 mW (b) 100 mW (c) 10 W (d) 100 W

19. The two laws which form the basis of circuit analysis were stated by

(a) Bohr (b) Ohm (c) Kirchhoff (d) Faraday

20. When two capacitors are connected in series across a voltage source $v_S(t) = 2e^{-2t}$, the current supplied by the source is $i_S(t) = -12e^{-2t}$. When one of the capacitors is removed from the circuit, the current supplied by the source is $i_S(t) = -16e^{-2t}$. The capacitance of the disconnected capacitor is

(a) 2 (b) 4 (c) 6 (d) 12

21. A branch in a circuit is said to be active when it contains

(a) an energy source (b) resistor (c) inductor (d) capacitor

22. A circuit is said to be linear when the current–voltage relationship can be expressed by linear

(a) algebraic equations (b) differential equations
(c) integral equations (d) all of these

23. Which of the following is not a characteristic of an independent voltage source?

(a) voltage independent of magnitude of current drawn
(b) voltage dependent of magnitude of current drawn
(c) independent of the direction of current flow
(d) can supply or receive uninterrupted energy at constant voltage

24. Which of the following pair is dimensionless?

(a) α–β (b) β–μ (c) μ–ρ (d) ρ–α

25. A practical voltage source can be represented by

(a) an ideal voltage source with it internal resistance connected in series
(b) an ideal voltage source with it internal resistance connected across its terminals
(c) by neglecting the internal resistance
(d) none of these

26. A practical current source $i_S(t)$ A has an internal resistance r_S Ω. It can be transformed into a voltage source by putting

(a) $v_S = \dfrac{i_S(t)}{r_S}$ and neglecting the internal resistance

(b) $v_S = \dfrac{i_S(t)}{r_S}$ and connecting the internal resistance in series

(c) $v_S = i_S(t) \times r_S$ and connecting the internal resistance in series

(d) $v_S = i_S(t) \times r_S$ and connecting the internal resistance in parallel

27. An ideal voltage source of 10 V has internal resistance of 0.2 Ω and it supplies a load current of 10 A. The power supplied by the practical voltage source is

(a) 100 W (b) 80 W (c) 10 W (d) 20 W

28. Two resistors, each of 10 kΩ and 15 kΩ, are connected in series across a DC voltage source to form a voltage divider. What should be the magnitude of the source voltage in order to obtain an output of 60 V across the 15 kΩ resistor?

(a) 75 V (b) 100 V (c) 125 V (d) 150 V

29. Two resistances of 5 Ω and 20 Ω are connected in parallel. The parallel combination is connected in series with a 1 Ω resistance and this series–parallel combination is connected across a DC source of 50 V. The current in the 20 Ω resistor is

(a) 10 A (b) 8 A (c) 2 A (d) none of these

30. In Q. **29**, the power dissipated in the 5 Ω resistor is

(a) 500 W (b) 320 W (c) 100 W (d) 80 W

31. The current flowing through two series inductors of 3 H and 6 H is $i_S(t) = 10\sin 2t$ A. The source voltage across the combined inductors is given by

(a) $10\sin 2t$ (b) $20\cos 2t$ (c) $120\cos 2t$ (d) $180\cos 2t$

32. In Q. **31**, the voltage output, as a percentage of the source voltage, is

(a) 100% (b) 66.67% (c) 33.33% (d) none of these

33. Star-to-delta and vice versa transformations are employed to simplify circuit elements connected in

(a) series–parallel (b) series (c) parallel (d) none of these

34. Inductors, each of 9 H, are connected to form a rectangle ABCD. Another inductor of 9 H is connected between the diagonal nodes A and C. A voltage source is connected between nodes A and D which supplies a current of $i_S = 4e^{-4t}$ A. The voltage of the supply source is equal to

(a) $16e^{-4t}$ V (b) $-16e^{-4t}$ V (c) $-90e^{-4t}$ V (d) $90e^{-4t}$ V

Unsolved Problems

1.1 A voltage of 220 V is applied across a 1000 W heater. Determine the following: (i) resistance of the heater, (ii) current supplied, and (iii) the charge transferred in 10 s.

1.2 A voltage source $v_S(t) = 311\sin\omega t$ V is applied across a resistor of 500 Ω. Write expressions for (i) resistor current and (ii) power. If ω = 314 rad/s, draw the voltage, current, and power waveforms. Discuss the results.

1.3 In problem **1.2**, determine the charge transferred in (i) 0.01 s and (ii) 0.02 s.

1.4 Repeat problem **1.2** with (a) an inductor of 500 mH and (b) a capacitor of 500 Ω connected across the voltage source.

1.5 The current flowing through a conductor is given by

$$i = 6 \text{ A for } 0 < t < 1 \text{ s}$$
$$i = 6t^2 \text{ A for } t > 1$$

Compute the total charge entering the conductor from $t = 0$ to $t = 2$ s.

1.6 The current entering an electrical conductor is

$$i = 20\cos 5000t \text{ A}.$$

Assume the charge is zero at the instant the current is passing through its maximum value. Find the expression for $q(t)$.

1.7 A fully discharged 6-V battery is slowly (trickle) charged by a battery charger for 6 hours. If the rate of charging is set at $i(t) = 6e^{-t/3600}$ A for $0 \le t \le 6$h and $i(t) = 0$ for $t > 6$ h, compute for the charging period (i) total charge transferred to the battery, (ii) maximum power absorbed by the battery, (iii) total energy in kJ supplied by the charger, and (iv) average power in watts absorbed by the battery.

1.8 A 220 V DC motor is supplying a load of 30 kW at an efficiency of 80%. Calculate (i) input power and (ii) motor current. If energy is priced at Rs 3.50/kWh, what is the cost of running the motor for 6 h.

1.9 A voltage source, $v_S(t) = 5e^{-5t}$ V is applied across a capacitor of 2 F. What is the current flowing through the capacitor? Calculate the current in the capacitor at $t = \infty$.

1.10 A rectangle having sides of 50 cm and 25 cm is made up of copper wire of diameter 4.0 cm. The rectangle is opened out and stretched in a straight wire. What is the resistance of the wire? Take $\rho = 1.72 \times 10^{-8}$ Ωm

1.11 A current of 7.5 A flows through the parallel combination of two wires, one of which is an aluminium wire 10.0 m long and the other wire is of an unknown metal and is 7.0 m long. The current through the aluminium wire is 5.5 A. The diameters of the aluminium and unknown metal wires are 1.5 and 0.6 mm, respectively. Compute the resistivity of the unknown metal wire. Assume resistivity of aluminium equal to 2.8×10^{-8} Ωm.

1.12 A heating coil is made by winding a bare copper wire of diameter 0.75 mm on to a porcelain cylinder 25 cm long and having a diameter of 5 cm. The distance between the consecutive turns is equal to the diameter of the wire. If the heating coil is connected across a 200 V DC supply, calculate (i) the current supplied and (ii) the heat produced. Determine the heat dissipated per square cm. Neglect the end areas for heat dissipation and assume resistivity of copper at 1.74×10^{-8}Ωm.

1.13 Calculate the equivalent capacitance between terminals (a) A–B and (b) A–G for the network shown in Fig. P 1.13. Assume each capacitance to be of C farad. Determine the source current when a voltage source of $v_S(t) = 3e^{-3t}$ V is connected, in turn between each pair of terminals.

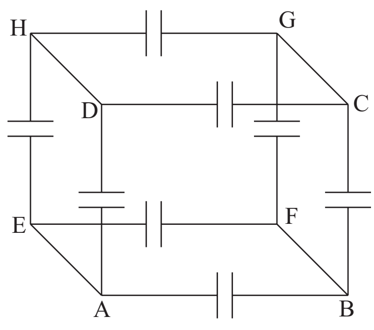


Fig. P 1.13

1.14 For the circuit shown in Fig. P1.14, compute the value of R. Calculate (a) the voltage across terminals A–B, (b) power dissipated in R, and (c) total power consumed in the circuit.

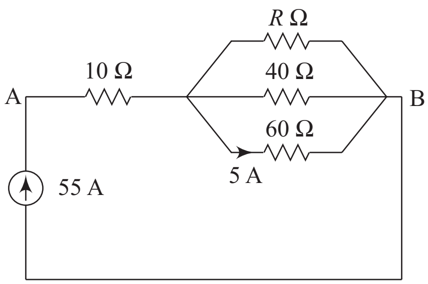


Fig. P 1.14

1.15 An ideal voltage source of 230 V is connected across a 200 W bulb. Calculate the supply current and resistance of the bulb. Three bulbs of 100 W each are now connected, along with the 200 W bulb, in series across the voltage source. Determine (a) source current and (b) voltage drop across each bulb.

1.16 Repeat problem **1.15** with the ideal voltage source replaced by an ideal current source of 100 A. Determine (a) the supply current, (b) the voltage drop across each bulb, and (c) the power dissipated in each of the bulbs.

1.17 A short-circuit test on a practical voltage source gave a current of 1.2 A. If the open-circuit voltage of the source is 36 V, compute the internal resistance of the source. The source delivers a load current of 0.3 A, when it is connected across a load. Calculate (i) load resistance, (ii) voltage drop across the load, and (iii) power dissipated.

1.18 Calculate (i) internal resistance, (ii) open-circuit voltage, and (iii) voltage regulation of a voltage source from the following loading conditions: (a) V_L = 105 V, I_L = 500 mA, and (b) V_L = 90 V, I_L = 1.0 A.

1.19 Use source transformation technique to determine *I* in the given circuit.

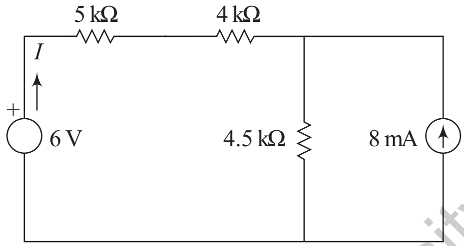


Fig. P 1.19

1.20 Compute the current and power dissipated in the 3 Ω resistance in the given circuit by the source transformation method.

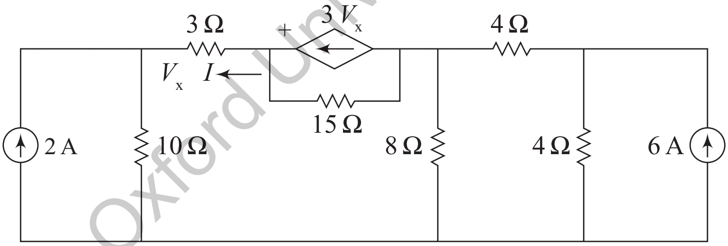


Fig. P 1.20

1.21 An inductor L_1 = 0.05 H is connected in series with a parallel combination of two inductors L_2 = 0.02 H and L_3 = 0.04 H. (a) Find the equivalent inductance of the combination. (b) Determine the value of the emf across L_2 when the current in L_1 is changing at the rate of 1500 A/s.

1.22 Two capacitors, each of 4 μF and 10 μF, are connected in series as shown in P1.22. If the capacitors are charged to initial voltages of $v_1 = -3\,\text{V}$ and $v_2 = -6\,\text{V}$, determine the total energy in the capacitors at $t \to \infty$ when a current $i(t) = 250e^{-12.5t}\,\mu\text{A}$ for $t \geq 0$ is applied across the terminals of the circuit. [**Hint**: *Determine the energy in each capacitor separately*.]

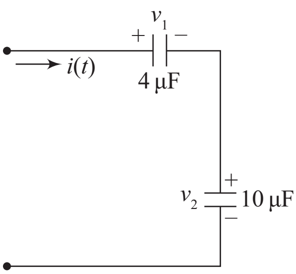


Fig. P 1.22

1.23 Figure P 1.23 below shows a voltage divider circuit. Calculate the output voltage V_o. What is the output voltage if the internal resistance of the voltage source is neglected? Calculate the percentage error introduced by neglecting internal resistance. Assume an internal resistance of 1.5 Ω for the voltage source.

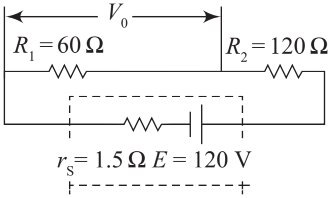


Fig. P 1.23

1.24 A load resistor R_L is connected across the output voltage $v_0(t)$ in Fig. 1.46. Develop an expression for the output voltage and derive the condition for the output voltage to remain constant.

1.25 In the voltage divider circuit in Fig. 1.46, $R_{in} = 150$ kΩ and $R_o = 125$ kΩ. If the commercial resistors have a tolerance of ±10%, calculate the maximum and minimum output voltage $v_o(t)$.

1.26 Figure P 1.26 shows a current divider circuit. Compute (a) the value of R which will cause a current of 3 A to flow through the 50 Ω resistor, (b) power dissipated in R, and (c) magnitude of power required to be generated by the current source to meet the requirement of power dissipation in R.

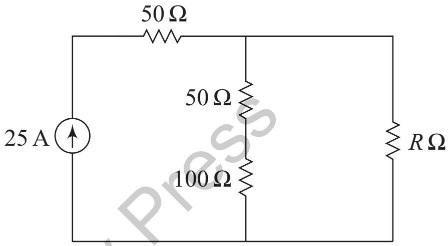


Fig. P 1.26

1.27 Show that the output of the circuit in Fig. P 1.27 is an integral of the input. Assume that the capacitor is not charged.

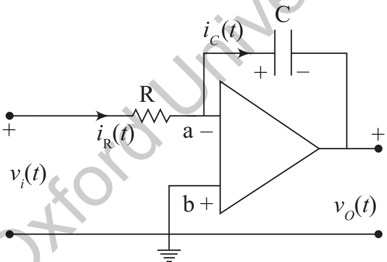


Fig. P 1.27

1.28 Using the method of series–parallel combination, determine $i_S(t), i_1(t), v_o(t)$, and $v_1(t)$ for the circuit shown in Fig. P 1.28, when a source voltage of $v_S(t) = 4 \sin 3t$ V is applied across terminals A–B. All values of the inductors are shown in the following figure.

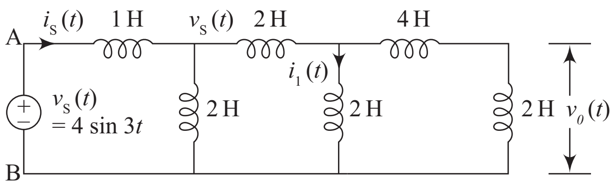


Fig. P 1.28

1.29 The currents flowing through three capacitors, connected in parallel, are 25, 50, and 75 A, when a voltage varying at the rate of 100 V/s is applied across the terminals. Determine the equivalent capacitance.

1.30 The three capacitors of Problem 1.29 are now connected in series across a voltage source of 230 V. Compute (a) the equivalent capacitance, (b) the charge on each capacitor, (c) the voltage drop across each capacitor, and (d) the total energy stored.

1.31 Determine the resistance between the points A and B of the networks shown in Figs P 1.32.

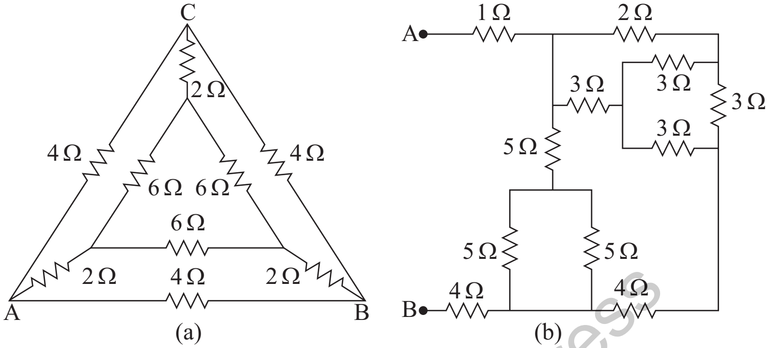


Fig. P 1.32

Answers to Multiple Choice Questions

1. (a)	**2**. (d)	**3**. (c)	**4**. (c)	**5**. (d)	**6**. (d)	**7**. (b)
8. (a)	**9**. (a)	**10**. (a)	**11**. (c)	**12**. (d)	**13**. (d)	**14**. (b)
15. (a)	**16**. (d)	**17**. (c)	**18**. (d)	**19**. (c)	**20**. (d)	**21**. (a)
22. (d)	**23**. (b)	**24**. (b)	**25**. (a)	**26**. (c)	**27**. (b)	**28**. (c)
29. (c)	**30**. (b)	**31**. (d)	**32**. (b)	**33**. (a)	**34**. (c)	