Principles of Electromagnetics

6th edition

Asian Adaptation

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Prairie View A&M University

Adapted by S.V. Kulkarni

Indian Institute of Technology Bombay

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To my wife, Kikelomo

–Matthew N.O. Sadiku

To God who gave me wisdom and strength

–S.V. Kulkarni

Preface

Electromagnetics is a branch of Electrical and Electronics Engineering which entails the study of the principles, synthesis, and physical interpretation of electric and magnetic fields. The subject requires thorough knowledge of vector calculus and an ability to imagine field distribution in space. The various applications of electromagnetics include power transformers, rotating machines, and actuators (low-frequency devices) and microwave devices, waveguides, antennas, and radars (high-frequency devices). The principles of electromagnetics help us understand the design and operation of these low- and high-frequency devices. The main objective of this book is to present the fundamental laws and principles of electromagnetics and its applications in a clearer and more interesting manner than other books do.

ABOUT THE BOOK

The Asian adaptation of *Principles of Electromagnetics*, sixth edition, is a comprehensive textbook designed for undergraduate students of Electrical and Electronics Engineering. Using a vectors-first approach, the book explains electrostatics, magnetostatics, fields, waves, and applications such as transmission lines, waveguides, and antennas. The book also provides a balanced presentation of static and time-varying fields, preparing students for employment in today's industrial and manufacturing sectors.

KEY FEATURES

- Treats mathematical theorems separately from physical concepts, making it easier for students to grasp the theorems
- Presents real-world applications of the concepts covered at the end of each chapter
- Provides MATLAB codes developed for the computer implementation of the concepts presented in each chapter
- Devotes an entire part to the different numerical techniques with practical applications and computer programs
- Comprises numerous examples, each worked step-by-step, and a set of multiple-choice questions at the end of each chapter
- Contains more than 450 figures to help students visualize the different electromagnetic phenomena

Each revision of this book has involved many changes that have made the contents of the book even better. The fully revised and updated sixth edition now features the following:

- An appendix called *Summary of Important Concepts in Electromagnetics* explains the fundamentals of electromagnetics succinctly. This appendix will help students consolidate their understanding of the subject. Numerous comments and explanations have been added at various places so that theories and concepts are understood better.
- The text contains new material/sections on constant coordinate surfaces, classification of vector fields, torque on a dipole, homogeneous and heterogeneous dielectric systems, classification

of magnetic materials, permanent magnets, wave polarization, transients on transmission lines, transmission lines as circuit elements, and current and mode excitation in waveguides.
- Coverage of numerical methods has been enhanced, with separate chapters dedicated to the different types of methods. These have been exemplified by solving real-life problems using all the techniques through additional MATLAB codes. The finite difference time domain method has been newly added.
- Sixteen new application notes have been added, which explain the connections between the concepts discussed in the text and the real world.
- There are additional solved examples in all the chapters.
- New practice exercises and chapter-end problems have been added.

Although this book is intended to be self-explanatory and useful for self-instruction, the personal contact that is always needed in teaching has not been forgotten. The actual choice of course topics, as well as their emphasis, depends on the preference of the individual instructor. For example, an instructor who feels too much importance has been devoted to vector analysis or static fields may skip some of the material; however, students may use them as reference. In addition, it is pertinent to note, having covered Chapters 1–3, it is possible to explore Chapters 9–15. Instructors who disagree with the vector calculus-first approach may proceed with Chapters 1 and 2, skip to Chapter 4, and then refer to Chapter 3 as needed. Enough material has been covered for the two-semester courses. If the text is to be covered in one semester, covering Chapters 1–9 is recommended; some sections may be skipped, explained briefly, or assigned as homework. Sections marked with the dagger sign (†) may be in this category.

ONLINE RESOURCES

The following resources are available at http://oupinheonline.com to support the faculty and students using this book.

For Faculty
- Solutions Manual
- Figures-only PPTs
- Math Assessment with Solutions

For Students
- Multiple-choice Questions

ACKNOWLEDGMENTS

I thank Dr Sudarshan Nelatury of Penn State University for providing the new application notes, as well as solving the problems present in the book and working with me on the solutions manual. I appreciate the help of Dr Josh Nickel of Santa Clara University for developing the MATLAB code at the end of each chapter. Special thanks are due to Nancy Blaine and Patrick Lynch at Oxford University Press for their efforts. I also thank the reviewers who provided helpful feedback for this edition:

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Matthew N.O. Sadiku
Prairie View, Texas, USA

I thank Dr Sadiku and Oxford University Press India for giving me the opportunity to add value to this already established and popular textbook on electromagnetics.

I also thank my Institute, IIT Bombay, for providing an excellent ambience for completing the work. I am grateful to Prof. R.K. Shevgaonkar, my colleague and Professor, IIT Bombay, Prof. G.B. Kumbhar, IIT Roorkee, and Dr K.P. Ray, Programme Director & Head, RF & Microwave System Division, SAMEER, for their useful comments.

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Writing a book is a difficult task and requires support and encouragement from family members. The overwhelming support and understanding of my wife, Sushama, is admirable. My son, Anandchaitanya, was quite considerate in allowing me to work even on weekends; thanks are due to the little one for his sacrifice and patience.

S.V. Kulkarni
IIT Bombay, Mumbai, India

A Note to the Student

Electromagnetic theory is generally regarded by students as one of the most difficult courses in physics or the electrical engineering curriculum. But this misconception may be proved wrong if you take some precautions. From experience, the following ideas are provided to help you perform to the best of your ability with the aid of this textbook:

1. Pay particular attention to Part 1 on vector analysis, the mathematical tool for this course. Without a clear understanding of this section, you may have problems with the rest of the book.
2. Do not attempt to memorize too many formulas. Memorize only the basic ones, which are usually boxed, and try to derive others from these. Try to understand how formulas are related. Obviously, there is nothing like a general formula for solving all problems. Each formula has some limitations owing to the assumptions made in obtaining it. Be aware of those assumptions and use the formula accordingly.
3. Try to identify the key words or terms in a given definition or law. Knowing the meaning of these key words is essential for proper application of the definition or law.
4. Attempt to solve as many problems as you can. Practice is the best way to gain skill. The best way to understand the formulas and assimilate the material is by solving problems. It is recommended that you solve at least the problems in the Practice Exercise immediately following each illustrative example. Sketch a diagram illustrating the problem before attempting to solve it mathematically. Sketching the diagram not only makes the problem easier to solve, it also helps you understand the problem by simplifying and organizing your thinking process. Note that unless otherwise stated, all distances are in meters. For example (2, −1, 5) actually means (2 m, −1 m, 5 m).

You may use MATLAB to do number crunching and plotting. A brief introduction to MATLAB is provided in Appendix D.
Important formulas in calculus, vectors, and complex analysis are provided in Appendix B. Answers to problems are given in Appendix F.

FEATURES OF

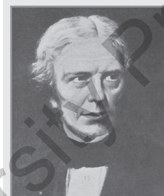
PART 1
VECTOR ANALYSIS

Coverage of Vector Analysis

Vector analysis is covered in the beginning of the book and the concepts gradually applied, thus helping students separate mathematical theorems from physical concepts. This makes it easier for them to grasp the generality of those theorems.

Historical Profile of Scientists

Select chapters open with the profile of a pioneer in the field of electromagnetics, describing the contribution of the scientist in this area of study.



Michael Faraday (1791–1867), an English physicist, is known for his pioneering experi tricity and magnetism. Many consider him experimentalist who ever lived.

Born at Newington, near London, to a po received little more than an elementary educa seven-year apprenticeship as a bookbinder, F oped his interest in science and in particular a result, Faraday started a second apprentice

†11.8 APPLICATION NOTE—MICROSTRIP LINES AND C

†A. Microstrip Transmission Lines

Microstrip lines belong to a group of lines widely used in present-day electronics. Apar mission lines for microwave integrated circu

†12.10 APPLICATION NOTE—CLOAKING AND INVISIB

The practice of using metamaterials to Metamaterials are ideal for cloaking becau index. All materials have an index of refrac

Application Notes

The last section in each chapter is devoted to the applications of the concepts covered therein. This helps students understand how the concepts apply to real-life situations.

Boxes

Important formulas are boxed to help students identify the essential ones. Key terms are defined and highlighted to ensure students clearly understand the subject matter.

$$\boxed{\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}} \quad (2.2$$

Stokes's theorem states that the circulation of a vec to the surface integral of the curl of **A** over the oper provided **A** and $\nabla \times$ **A** are continuous on *S*.

THE BOOK

MATLAB Programs

Each chapter concludes with a MATLAB code developed for computer implementation of the concepts studied in that chapter. A short tutorial on MATLAB is provided in Appendix D.

MATLAB 10.1

```
% This script assists with the solution and graphing of E
% We use symbolic variables in the creation of the wavefo
% that describes the expression for the electric field

clear
syms E omega Beta t x                 % symbolic variables
                                      % time, and frequency

% Enter the frequency (in rad/s)
```

EXAMPLE 2.1 Given point $P(-2, 6, 3)$ and vector $\mathbf{A} = y\mathbf{a}_x + (x + z)\mathbf{a}_y$ drical and spherical coordinates. Evaluate $\mathbf{A}$ at P in the Cartesian, cylindri

***Solution*:** At point P, $x = -2$, $y = 6$, $z = 3$. Hence,

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.32$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{6}{-2} = 108.43°$$

$$z = 3$$

Examples

Each chapter includes worked-out examples which give students the confidence to solve problems themselves. Each illustrative example is followed by a problem in the form of a Practice Exercise with its answer.

Review Questions

Each chapter ends with review questions in the form of multiple-choice questions with answers immediately following them. This encourages students to check the answers and gain immediate feedback.

REVIEW QUESTIONS

11.1 Which of the following statements are not true o

(a) R and L are series elements.

(b) G and C are shunt elements.

(c) $G = \dfrac{1}{R}$.

Answers 11.1c,d,e, 11.2b,c, 11.3c, 11.4a,c, 11.5c, 11
(viii) A, 11.7a, 11.8 (a) T, (b) F, (c) F, (d) T, (e) F, (f)

***3.36** Given that $\mathbf{F} = x^2y\mathbf{a}_x - y\mathbf{a}_y$, find

(a) $\oint_L \mathbf{F} \cdot d\mathbf{l}$ where L is shown in Figure 3.31.

(b) $\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$, where S is the area bounded b

(c) Is Stokes's theorem satisfied?

****3.41** A vector field is given by

$$Q = \frac{\sqrt{x^2 + y^2 + z^2}}{\sqrt{x^2 + y^2}}\left[(x - y)\mathbf{a}_x + (x + y)\mathbf{a}_y\right.$$

Evaluate the following integrals:

End-chapter Problems

A large number of problems are provided and presented in the same order as the material in the main text. Problems of intermediate difficulty are identified by a single asterisk (*); the most difficult problems are marked with a double asterisk (**).

Companion Online Resources for Instructors and Students

Visit www.oupinheonline.com to access both teaching and learning solutions online.

The following resources are available to support the faculty and students using this book:

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- Solutions Manual
- Figures-only PPTs
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For Students

- Multiple-choice Questions

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  - ▷ Recent titles

Step 3: Select Title

Select title for which you are looking for resources.

Step 4: Search Results with Resources available

Operations Research, 1/e
S.R. Yadav & A.K. Malik

Select resources by chapter

Student Resources

Instructor Resources

Click here

Step 5: To download Results

Download All

Step 6: Login to download

The page you wish to access is password-protected.
Please login to access the resources

User Name :
Password :

Login

Forgot Password? | New Student? Register | New Instructor? Register

New Instructor? Register

Step 7: Registration Form

Please fill correct details and *marked fields are mandatory

Instructor Registration

Personal Details

User Name * : (Enter a valid email address as your User Name e.g. support@gmail.com)
Password * :
Confirm Password * :
Security Question * : --Select--
Security Answer * :
Salutation * : --Select--
Name * :
Designation * :
Department * :
Mobile * :
Alternative Email Id :

Username should be email ID

Please fill correct Mobile no. to get SMS after verification

Institute Details

Institute Name * : (Please enter full Institute Name.)
University * : --Select--
Address * : (Please enter complete address of the institute.)
Country * : --Select--
State :
City * :

Course Taught

SNo.	Course	Sem/Year	Enrolment
1		--Select--	
2		--Select--	
3		--Select--	
4		--Select--	
5		--Select--	

Submit

SUBMIT

Step 8: Message after completing the registration form

Thank you for registering with us. We shall revert to you within 48 hours after verifying the details provided by you. Once validated please login using your username and the password and access the resources.

Step 9: Verification

You will receive a confirmation on your mobile & email ID.

For any further query please write to us at HEMarketing.in@oup.com with your mobile number

Step 10: Visit us again after validation

- Go to www.oupinheonline.com
- Login from Member Login

Member Login

User Name :

Password :

Login

Forgot Password?

Step 11: My Subscriptions

My Subscriptions

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Brief Contents

Detailed Contents

PART 1
VECTOR ANALYSIS

CODES OF ETHICS

Engineering is a profession that makes significant contributions to the economic and social well-being of people all over the world. As members of this important profession, engineers are expected to exhibit the highest standards of honesty and integrity. Unfortunately, the engineering curriculum is so crowded that there is no room for a course on ethics in most schools. Although there are over 850 codes of ethics for different professions all over the world, the code of ethics of the Institute of Electrical and Electronics Engineers (IEEE) is presented here to give students a flavor of the importance of ethics in engineering professions.

We, the members of the IEEE, in recognition of the importance of our technologies in affecting the quality of life throughout the world, and in accepting a personal obligation to our profession, its members and the communities we serve, do hereby commit ourselves to the highest ethical and professional conduct and agree:

1. to accept responsibility in making engineering decisions consistent with the safety, health, and welfare of the public, and to disclose promptly factors that might endanger the public or the environment;
2. to avoid real or perceived conflicts of interest whenever possible, and to disclose them to affected parties when they do exist;
3. to be honest and realistic in stating claims or estimates based on available data;
4. to reject bribery in all its forms;
5. to improve the understanding of technology, its appropriate application, and potential consequences;
6. to maintain and improve our technical competence and to undertake technological tasks for others only if qualified by training or experience, or after full disclosure of pertinent limitations;
7. to seek, accept, and offer honest criticism of technical work, to acknowledge and correct errors, and to credit properly the contributions of others;
8. to treat fairly all persons regardless of such factors as race, religion, gender, disability, age, or national origin;
9. to avoid injuring others, their property, reputation, or employment by false or malicious action;
10. to assist colleagues and co-workers in their professional development and to support them in following this code of ethics.

—Courtesy of IEEE

CHAPTER 1

Vector Algebra

One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man.

—ELBERT HUBBARD

1.1 INTRODUCTION

Electromagnetics (EM) may be regarded as the study of the interactions between electric charges at rest and in motion. It entails the analysis, synthesis, physical interpretation, and application of electric and magnetic fields.

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

EM principles find applications in various allied disciplines such as microwaves, antennas, electric machines, satellite communications, bioelectromagnetics, plasmas, nuclear research, fiber optics, electromagnetic interference and compatibility, electromechanical energy conversion, radar meteorology, and remote sensing.[1,2] In physical medicine, for example, EM power, in the form of either shortwaves or microwaves, is used to heat deep tissues and to stimulate certain physiological responses in order to relieve certain pathological conditions. EM fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operations. Dielectric heating equipment uses shortwaves to join or seal thin sheets of plastic materials. EM energy offers many new and exciting possibilities in agriculture. It is used, for example, to change vegetable taste by reducing acidity.

EM devices include transformers, electric relays, radio/TV, telephones, electric motors, transmission lines, waveguides, antennas, optical fibers, radars, and lasers. The design of these devices requires thorough knowledge of the laws and principles of EM.

[1]For numerous applications of electrostatics, see J. M. Crowley, *Fundamentals of Applied Electrostatics*. New York: John Wiley & Sons, 1986.

[2]For other areas of applications of EM, see, for example, D. Teplitz, ed., *Electromagnetism: Paths to Research*. New York: Plenum Press, 1982.

†1.2 A PREVIEW OF THE BOOK

The subject of electromagnetic phenomena in this book can be summarized in Maxwell's equations:

$$\nabla \cdot \mathbf{D} = \rho_v \tag{1.1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{1.2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.3}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{1.4}$$

where ∇ = the vector differential operator
$\mathbf{D}$ = the electric flux density
$\mathbf{B}$ = the magnetic flux density
$\mathbf{E}$ = the electric field intensity
$\mathbf{H}$ = the magnetic field intensity
ρ_v = the volume charge density
$\mathbf{J}$ = the current density

Maxwell based these equations on previously known results, both experimental and theoretical. A quick look at these equations shows that we shall be dealing with vector quantities. It is consequently logical that we spend some time in Part 1 examining the mathematical tools required for this course. The derivation of eqs. (1.1) to (1.4) for time-invariant conditions and the physical significance of the quantities $\mathbf{D}$, $\mathbf{B}$, $\mathbf{E}$, $\mathbf{H}$, $\mathbf{J}$, and ρ_v will be our aim in Parts 2 and 3. In Part 4, we shall reexamine the equations for time-varying situations and apply them in our study of practical EM devices.

1.3 SCALARS AND VECTORS

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended. We must learn its rules and techniques before we can confidently apply it. Since most students taking this course have little exposure to vector analysis, considerable attention is given to it in this and the next two chapters.[3] This chapter introduces the basic concepts of vector algebra in Cartesian coordinates only. The next chapter builds on this and extends to other coordinate systems.

A quantity can be either a scalar or a vector.

> A **scalar** is a quantity that has only magnitude.

Quantities such as time, mass, distance, temperature, entropy, electric potential, and population are scalars.

† Indicates sections that may be skipped, explained briefly, or assigned as homework if the text is covered in one semester.

[3] The reader who feels no need for review of vector algebra can skip to the next chapter.

A **vector** is a quantity that has both magnitude and direction.

Vector quantities include velocity, force, displacement, and electric field intensity. Another class of physical quantities is called *tensors*, of which scalars and vectors are special cases. For most of the time, we shall be concerned with scalars and vectors.[4]

To distinguish between a scalar and a vector it is customary to represent a vector by a letter with an arrow on top of it, such as $\vec{A}$ and $\vec{B}$, or by a letter in boldface type such as **A** and **B**. A scalar is represented simply by a letter—for example, A, B, U, and V.

EM theory is essentially a study of some particular fields.

A **field** is a function that specifies a particular quantity everywhere in a region.

If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. Examples of scalar fields are temperature distribution in a building, sound intensity in a theater, electric potential in a region, and refractive index of a stratified medium. The gravitational force on a body in space and the velocity of raindrops in the atmosphere are examples of vector fields.

1.4 UNIT VECTOR

A vector **A** has both magnitude and direction. The *magnitude* of **A** is a scalar written as A or $|\mathbf{A}|$. A *unit vector* $\mathbf{a}_A$ along **A** is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along **A**, that is,

$$\boxed{\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}} \tag{1.5}$$

Note that $|\mathbf{a}_A| = 1$. Thus we may write **A** as

$$\mathbf{A} = A\mathbf{a}_A \tag{1.6}$$

which completely specifies **A** in terms of its magnitude A and its direction $\mathbf{a}_A$.

A vector **A** in Cartesian (or rectangular) coordinates may be represented as

$$(A_x, A_y, A_z) \quad \text{or} \quad A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \tag{1.7}$$

where A_x, A_y, and A_z are called the *components of* **A** in the x-, y-, and z-directions, respectively; $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors in the x-, y-, and z-directions, respectively. For example, $\mathbf{a}_x$ is a dimensionless vector of magnitude one in the direction of the increase of the x-axis. The unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are illustrated in Figure 1.1(a), and the components of **A** along the coordinate axes are shown in Figure 1.1(b). It should be noted that the projection of **A** on the xy-plane ($z = 0$) is a vector which is the addition of its vector components in the x and y directions; this is a vector addition (see Section 1.5). The magnitude of vector **A** is given by

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.8}$$

and the unit vector along **A** is given by

[4] For an elementary treatment of tensors, see, for example, A. I. Borisenko and I. E. Tarapor, *Vector and Tensor Analysis with Applications.* New York: Dover, 1979.

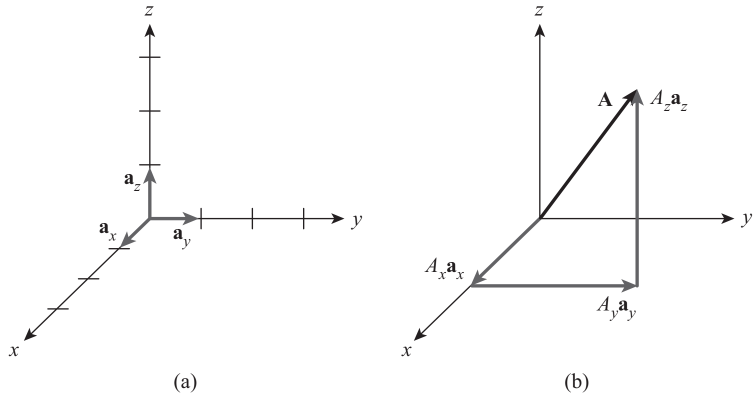

Figure 1.1 (**a**) Unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$, (**b**) components of **A** along $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$.

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \tag{1.9}$$

1.5 VECTOR ADDITION AND SUBTRACTION

Two vectors **A** and **B** can be added together to give another vector **C**; that is,

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \tag{1.10}$$

The vector addition is carried out component by component. Thus, if $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$.

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z \tag{1.11}$$

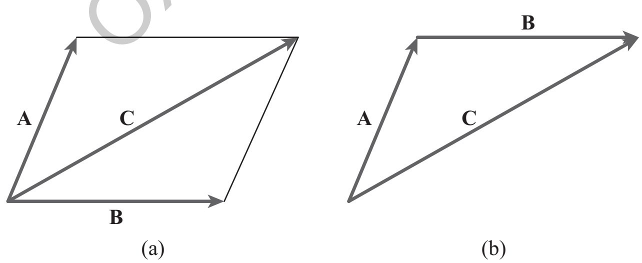

Figure 1.2 Vector addition **C** = **A** + **B**: (**a**) parallelogram rule, (**b**) head-to-tail rule.

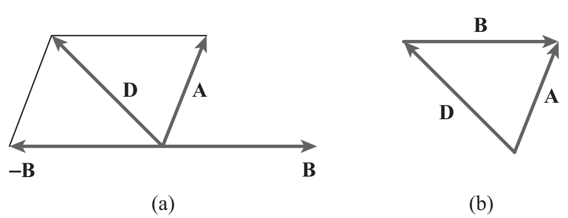

Figure 1.3 Vector subtraction **D** = **A** – **B**: (**a**) parallelogram rule, (**b**) head-to-tail rule.

Vector subtraction is similarly carried out as

$$\mathbf{D} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z \tag{1.12}$$

Graphically, vector addition and subtraction are obtained by either the parallelogram rule or the head-to-tail rule as portrayed in Figures 1.2 and 1.3, respectively.

The three basic laws of algebra obeyed by any given vectors **A**, **B**, and **C**, are summarized as follows:

Law	Addition	Multiplication
Commutative	$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	$k\mathbf{A} + \mathbf{A}k$
Associative	$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	$k(l\mathbf{A}) = (kl)\mathbf{A}$
Distributive	$k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$	

where k and ℓ are scalars. Multiplication of a vector with another vector will be discussed in Section 1.7.

1.6 POSITION AND DISTANCE VECTORS

A point P in Cartesian coordinates may be represented by (x, y, z).

The **position vector $\mathbf{r}_P$** (or **radius vector**) of point P is defined as the directed distance from the origin O to P, that is,

$$\mathbf{r}_p = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z \tag{1.13}$$

The position vector of point P is useful in defining its position in space. Point (3, 4, 5), for example, and its position vector $3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$ are shown in Figure 1.4. Its distance from the origin is $\sqrt{3^2 + 4^2 + 5^2} = 7.071$. This distance can also be calculated as follows. The projection of the position vector in the xy-plane ($z = 0$) is:

$$\mathbf{r}_{P'} = 3\mathbf{a}_x + 4\mathbf{a}_y \rightarrow |\mathbf{r}_{P'}| = OP' = \sqrt{3^2 + 4^2} = 5$$

The vector addition of $\mathbf{r}_{P'}$ and $\mathbf{r}_{P'P}$ results in the position vector of the point P. The angle between the two vectors, $\mathbf{r}_{P'}$ and $\mathbf{r}_{P'P} (= 5\mathbf{a}_z)$, is 90°.

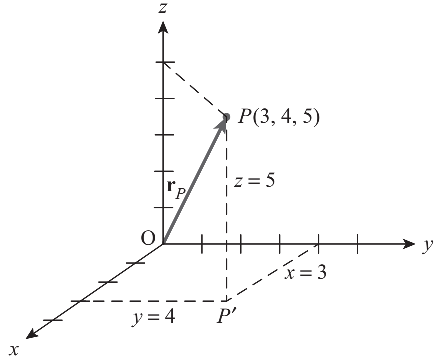

Figure 1.4 Illustration of position vector $\mathbf{r}_P = 3\mathbf{a}_x + 4\mathbf{a}_y + 5\mathbf{a}_z$.

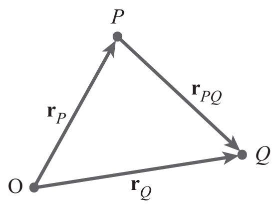

Figure 1.5 Distance vector $\mathbf{r}_{PQ}$.

$$\mathbf{r}_p = \mathbf{r}_{p'} + \mathbf{r}_{P'P} \text{ and } \left|\mathbf{r}_p\right| = OP = \sqrt{5^2 + 5^2} = 7.071$$

It may be noted that $\mathbf{r}_{PP'}$ is at 90° to all possible vectors, in the *xy*-plane, originating from point *P′*.

The **distance vector** is the displacement from one point to another.

If two points *P* and *Q* are given by (x_P, y_P, z_P) and (x_Q, y_Q, z_Q), the *distance vector* (or *separation vector*) is the displacement from *P* to *Q* as shown in Figure 1.5; that is,

$$\mathbf{r}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (x_Q - x_P)\mathbf{a}_x + (y_Q - y_P)\mathbf{a}_y + (z_Q - z_P)\mathbf{a}_z \tag{1.14}$$

The difference between a point *P* and a vector **A** should be noted. Though both *P* and **A** may be represented in the same manner as (*x*, *y*, *z*) and (A_x, A_y, A_z), respectively, the point *P* is not a vector; only its position vector $\mathbf{r}_P$ is a vector. Vector **A** may depend on point *P*, however. For example, if $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ and *P* is (2, −1, 4), then **A** at *P* would be $-4\mathbf{a}_x + \mathbf{a}_y - 32\mathbf{a}_z$. A vector field is said to be *constant* or *uniform* if it does not depend on space variables *x*, *y*, and *z*. For example, vector $\mathbf{B} = 3\mathbf{a}_x - 2\mathbf{a}_y + 10\mathbf{a}_z$ is a uniform vector while vector $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ is not uniform because **B** is the same everywhere, whereas **A** varies from point to point.

EXAMPLE 1.1 If $\mathbf{A} = 10\mathbf{a}_x - 4\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + \mathbf{a}_y$, find (a) the component of **A** along $\mathbf{a}_y$, (b) the magnitude of 3**A** − **B**, (c) a unit vector along **A** + 2**B**.

***Solution*:**

(a) The component of **A** along $\mathbf{a}_y$ is $A_y = -4$.

(b) $3\mathbf{A} - \mathbf{B} = 3(10, -4, 6) - (2, 1, 0)$

$$= (30, -12, 18) - (2, 1, 0)$$
$$= (28, -13, 18)$$

Hence,

$$|3\mathbf{A} - \mathbf{B}| = \sqrt{28^2 + (-13)^2 + (18)^2} = \sqrt{1277} = 35.74$$

(c) Let $\mathbf{C} = \mathbf{A} + 2\mathbf{B} = (10, -4, 6) + (4, 2, 0) = (14, -2, 6)$.

A unit vector along **C** is

$$\mathbf{a}_c = \frac{\mathbf{C}}{|\mathbf{C}|} = \frac{(14, -2, 6)}{\sqrt{14^2 + (-2)^2 + 6^2}}$$

or

$$\mathbf{a}_c = 0.9113\mathbf{a}_x - 0.1302\mathbf{a}_y + 0.3906\mathbf{a}_z$$

Note that $|\mathbf{a}_c| = 1$ as expected.

PRACTICE EXERCISE 1.1

Given vectors $\mathbf{A} = \mathbf{a}_x + 3\mathbf{a}_z$ and $\mathbf{B} = 5\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z$, determine

(a) $|\mathbf{A} + \mathbf{B}|$

(b) 5**A** − **B**

(c) The component of **A** along $\mathbf{a}_y$

(d) A unit vector parallel to 3**A** + **B**

Answer: (a) 7, (b) (0, −2, 21), (c) 0, (d) ± (0.9117, 0.2279, 0.3419).

EXAMPLE 1.2 Points P and Q are located at (0, 2, 4) and (−3, 1, 5). Calculate

(a) The position of vector $\mathbf{r}_P$
(b) The distance vector from P to Q
(c) The distance between P and Q
(d) A vector parallel to PQ with magnitude of 10

***Solution*:**

(a) $\mathbf{r}_P = 0\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z = 2\mathbf{a}_y + 4\mathbf{a}_z$

(b) $\mathbf{r}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (-3, 1, 5) - (0, 2, 4) = (-3, -1, 1)$

or $\mathbf{r}_{PQ} = -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$

(c) Since $\mathbf{r}_{PQ}$ is the distance vector from P to Q, the distance between P and Q is the magnitude of this vector; that is,

$$d = |\mathbf{r}_{PQ}| = \sqrt{9+1+1} = 3.317$$

Alternatively:

$$d = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2}$$
$$= \sqrt{9+1+1} = 3.317$$

(d) Let the required vector be **A**, then

$$\mathbf{A} = A\mathbf{a}_A$$

where $A = 10$ is the magnitude of **A**. Since **A** is parallel to PQ, it must have the same unit vector as $\mathbf{r}_{PQ}$ or $\mathbf{r}_{QP}$. Hence,

$$\mathbf{a}_A = \pm\frac{\mathbf{r}_{PQ}}{|\mathbf{r}_{PQ}|} = \pm\frac{(-3, -1, 1)}{3.317}$$

and

$$\mathbf{A} = \pm\frac{10(-3, -1, 1)}{3.317} = \pm(-9.045\mathbf{a}_x - 3.015\mathbf{a}_y + 3.015\mathbf{a}_z)$$

PRACTICE EXERCISE 1.2

Given points $P(1, -3, 5)$, $Q(2, 4, 6)$, and $R(0, 3, 8)$, find (a) the position vectors of P and R, (b) the distance vector $\mathbf{r}_{QR}$, (c) the distance between Q and R.

Answer: (a) $\mathbf{a}_x - 3\mathbf{a}_y + 5\mathbf{a}_z$, $3\mathbf{a}_y + 8\mathbf{a}_z$, (b) $-2\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$, (c) 3.

EXAMPLE 1.3 A river flows southeast at 10 km/hr and a boat floats upon it with its bow pointed in the direction of travel. A man walks upon the deck at 2 km/hr in a direction to the right and perpendicular to the direction of the boat's movement. Find the velocity of the man with respect to the earth.

***Solution*:** Consider Figure 1.6 as illustrating the problem. The velocity of the boat is

$$\mathbf{u}_b = 10(\cos 45°\,\mathbf{a}_x - \sin 45°\,\mathbf{a}_y)$$
$$= 7.071\mathbf{a}_x - 7.071\mathbf{a}_y \text{ km/hr}$$

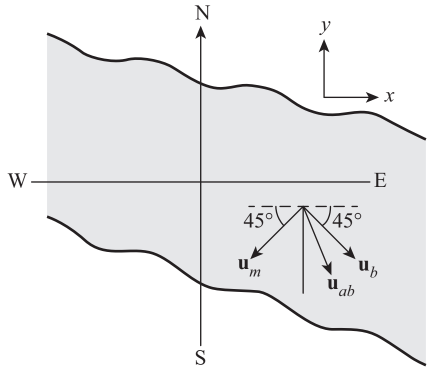

Figure 1.6 For Example 1.3.

The velocity of the man with respect to the boat (relative velocity) is

$$\mathbf{u}_m = 2(-\cos 45°\,\mathbf{a}_x - \sin 45°\,\mathbf{a}_y)$$
$$= -1.414\mathbf{a}_x - 1.414\mathbf{a}_y \text{ km/hr}$$

Thus the absolute velocity of the man is

$$\mathbf{u}_{ab} = \mathbf{u}_m + \mathbf{u}_b = 5.657\mathbf{a}_x - 8.485\mathbf{a}_y$$
$$|\mathbf{u}_{ab}| = 10.2 \angle{-56.3°}$$

that is, 10.2 km/hr at 56.3° south of east.

PRACTICE EXERCISE 1.3

An airplane has a ground speed of 350 km/hr in the direction due west. If there is a wind blowing northwest at 40 km/hr, calculate the true air speed and heading of the airplane.

Answer: 379.3 km/hr, 4.275° north of west.

1.7 VECTOR MULTIPLICATION

When two vectors **A** and **B** are multiplied, the result is either a scalar or a vector depending on how they are multiplied. Thus there are two types of vector multiplication:

1. Scalar (or dot) product: $\mathbf{A} \cdot \mathbf{B}$
2. Vector (or cross) product: $\mathbf{A} \times \mathbf{B}$

Multiplication of three vectors **A**, **B**, and **C** can result in either:

3. Scalar triple product: $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$

or

4. Vector triple product: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$

A. Dot Product

The **dot product** of two vectors **A** and **B**, written as **A · B**, is defined geometrically as the product of the magnitudes of **A** and **B** and the cosine of the smaller angle between them, when they are drawn tail to tail.

Thus,

$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta_{AB}} \tag{1.15}$$

where θ_{AB} is the *smaller* angle between **A** and **B**. The result of $\mathbf{A}\cdot\mathbf{B}$ is called either the *scalar product* because it results into a scalar quantity, or the *dot product* due to the dot sign. If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$, then

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z} \tag{1.16}$$

which is obtained by multiplying **A** and **B** component by component. Two vectors **A** and **B** are said to be *orthogonal* (or perpendicular) with each other if $\mathbf{A}\cdot\mathbf{B} = 0$.

Note that dot product obeys the following:

(i) *Commutative law:*

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A} \tag{1.17}$$

(ii) *Distributive law:*

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C} \tag{1.18}$$

(iii)

$$\mathbf{A}\cdot\mathbf{A} = |\mathbf{A}|^2 = A^2 \tag{1.19}$$

Also note that

$$\mathbf{a}_x\cdot\mathbf{a}_y = \mathbf{a}_y\cdot\mathbf{a}_z = \mathbf{a}_z\cdot\mathbf{a}_x = 0 \tag{1.20a}$$

$$\mathbf{a}_x\cdot\mathbf{a}_x = \mathbf{a}_y\cdot\mathbf{a}_y = \mathbf{a}_z\cdot\mathbf{a}_z = 1 \tag{1.20b}$$

It is easy to prove the identities in eqs. (1.17) to (1.20) by applying eq. (1.15) or (1.16).

B. Cross Product

The **cross product** of two vectors **A** and **B**, written as $\mathbf{A}\times\mathbf{B}$, is a vector quantity whose magnitude is the area of the parallelogram formed by **A** and **B** (see Figure 1.7) and is in the direction of advance of a right-handed screw as A is turned into B.

Thus,

$$\boxed{\mathbf{A}\times\mathbf{B} = AB\sin\theta_{AB}\,\mathbf{a}_n} \tag{1.21}$$

where $\mathbf{a}_n$ is a unit vector normal to the plane containing **A** and **B**. The direction of $\mathbf{a}_n$ is taken as the direction of the right thumb when the fingers of the right hand rotate from **A** to **B** as shown in Figure 1.8(a). Alternatively, the direction of $\mathbf{a}_n$ is taken as that of the advance of a right-handed

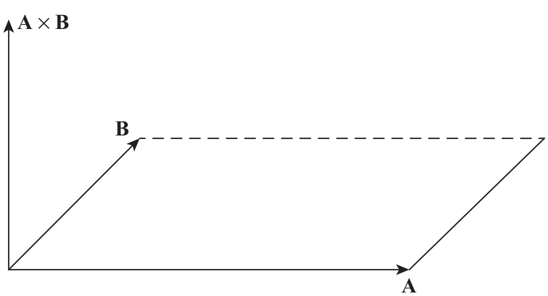

Figure 1.7 The cross product of **A** and **B** is a vector with magnitude equal to the area of the parallelogram and direction as indicated.

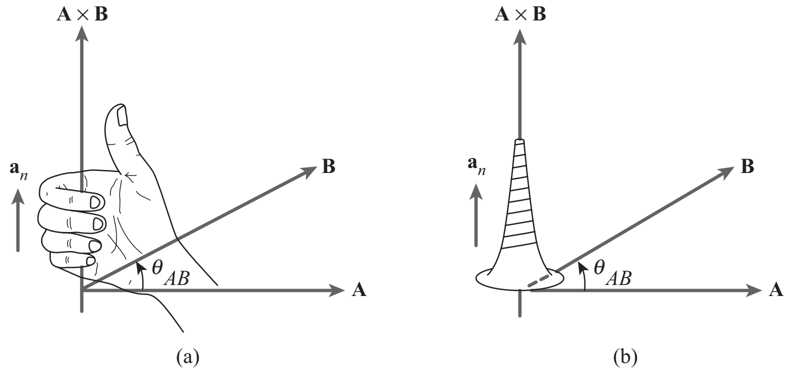

Figure 1.8 Direction of $\mathbf{A} \times \mathbf{B}$ and $\mathbf{a}_n$ using (a) the right-hand rule and (b) the right-handed-screw rule.

screw as **A** is turned into **B** as shown in Figure 1.8(b). Here **A, B** and $\mathbf{A} \times \mathbf{B}$ form a right-handed triplet in which when **A** is rotated towards **B** through an angle θ_{AB}, which is less than π as in Figure 1.7, $\mathbf{A} \times \mathbf{B}$ points in the direction of a right-handed screw turned anticlockwise. However, if the screw is turned clockwise from **A** to **B** through an angle greater than π in Figure 1.7, this leads to a left handed triplet with $\mathbf{A} \times \mathbf{B}$ directed downward (in the direction opposite to that shown in the figure).

The vector multiplication of eq. (1.21) is called *cross product* owing to the cross sign; it is also called *vector product* because the result is a vector. If $\mathbf{A} = (A_x, A_y, A_z)$ and $\mathbf{B} = (B_x, B_y, B_z)$, then

$$\boxed{\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}} \tag{1.22a}$$

$$= (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z \tag{1.22b}$$

which is obtained by "crossing" terms in cyclic permutation, hence the name "cross product."

Note that the cross product has the following basic properties:

(i) It is not commutative:

$$\mathbf{A}\times\mathbf{B} \neq \mathbf{B}\times\mathbf{A} \tag{1.23a}$$

It is anticommutative:

$$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A} \tag{1.23b}$$

(ii) It is not associative:

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) \neq (\mathbf{A}\times\mathbf{B})\times\mathbf{C} \tag{1.24}$$

(iii) It is distributive:

$$\mathbf{A}\times(\mathbf{B}+\mathbf{C}) = (\mathbf{A}\times\mathbf{B}) + \mathbf{A}\times\mathbf{C} \tag{1.25}$$

(iv)

$$\mathbf{A}\times\mathbf{A} = \mathbf{0} \tag{1.26}$$

Also note that

$$\begin{aligned} \mathbf{a}_x \times \mathbf{a}_y &= \mathbf{a}_z \\ \mathbf{a}_y \times \mathbf{a}_z &= \mathbf{a}_x \\ \mathbf{a}_z \times \mathbf{a}_x &= \mathbf{a}_y \end{aligned} \tag{1.27}$$

which are obtained in cyclic permutation and illustrated in Figure 1.9. The identities in eqs. (1.23) to (1.27) are easily verified by using eq. (1.21) or (1.22). It should be noted that in obtaining $\mathbf{a}_n$, we have used the right-hand or right-handed-screw rule because we want to be consistent with our coordinate system illustrated in Figure 1.1, which is right-handed. A right-handed coordinate system is one in which the right-hand rule is satisfied: that is, $\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$ is obeyed. In a left-handed system, we follow the left-hand or left-handed screw rule and $\mathbf{a}_x \times \mathbf{a}_y = -\mathbf{a}_z$ is satisfied. Throughout this book, we shall stick to right-handed coordinate systems.

Just as multiplication of two vectors gives a scalar or vector result, multiplication of three vectors **A**, **B**, and **C** gives a scalar or vector result, depending on how the vectors are multiplied. Thus, we have a scalar or vector triple product.

C. Scalar Triple Product

Given three vectors **A**, **B**, and **C**, we define the scalar triple product as

$$\boxed{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})} \tag{1.28}$$

obtained in cyclic permutation. If $\mathbf{A} = (A_x, A_y, A_z)$, $\mathbf{B} = (B_x, B_y, B_z)$, and $\mathbf{C} = (C_x, C_y, C_z)$, then $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ is the volume of a parallelepiped having **A**, **B**, and **C** as edges and is easily obtained by finding the determinant of the 3×3 matrix formed by **A**, **B**, and **C**; that is,

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \tag{1.29}$$

Since the result of this vector multiplication is scalar, eq. (1.28) or (1.29) is called the *scalar triple product.*

D. Vector Triple Product

For vectors **A**, **B**, and **C**, we define the vector triple product as

$$\boxed{\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})} \tag{1.30}$$

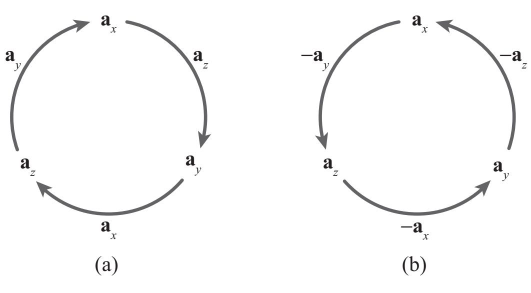

Figure 1.9 Cross product using cyclic permutation. (**a**) Moving clockwise leads to positive results. (**b**) Moving counterclockwise leads to negative results.

which may be remembered as the "bac-cab" rule. It should be noted that

$$(\mathbf{A}\cdot\mathbf{B})\mathbf{C} \neq \mathbf{A}(\mathbf{B}\cdot\mathbf{C}) \tag{1.31}$$

but

$$(\mathbf{A}\cdot\mathbf{B})\mathbf{C} = \mathbf{C}(\mathbf{A}\cdot\mathbf{B}) \tag{1.32}$$

1.8 COMPONENTS OF A VECTOR

A direct application of scalar product is its use in determining the projection (or component) of a vector in a given direction. The projection can be scalar or vector. Given a vector **A**, we define the *scalar component* A_B of **A** along vector **B** as [see Figure 1.10(a)]

$$\boxed{A_B = A\cos\theta_{AB} = |\mathbf{A}||\mathbf{a}_B|\cos\theta_{AB}}$$

or

$$\boxed{A_B = \mathbf{A}\cdot\mathbf{a}_B} \tag{1.33}$$

The *vector component* $\mathbf{A}_B$ of **A** along **B** is simply the scalar component in eq. (1.33) multiplied by a unit vector along **B**; that is,

$$\boxed{\mathbf{A}_B = A_B\mathbf{a}_B = (\mathbf{A}\cdot\mathbf{a}_B)\mathbf{a}_B} \tag{1.34}$$

Both the scalar and vector components of **A** are illustrated in Figure 1.10. Notice from Figure 1.10(b) that the vector can be resolved into two orthogonal components: one component $\mathbf{A}_B$ parallel to **B**, another $(\mathbf{A} - \mathbf{A}_B)$ perpendicular to **B**. In fact, our Cartesian representation of a vector is essentially resolving the vector into three mutually orthogonal components as in Figure 1.1(b).

We have considered addition, subtraction, and multiplication of vectors. However, division of vectors **A/B** has not been considered because it is undefined except when **A** and **B** are parallel so that ($\mathbf{A} = k\mathbf{B}$), where k is a constant. Differentiation and integration of vectors will be considered in Chapter 3.

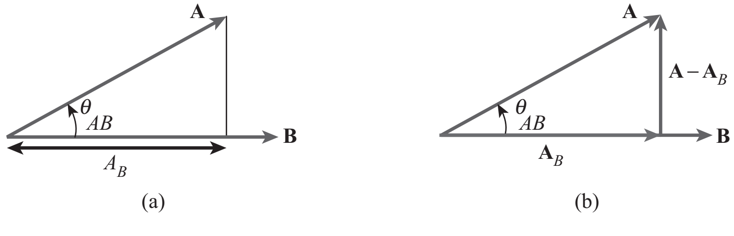

Figure 1.10 Components of **A** along **B**: (**a**) scalar component A_B, (**b**) vector component $\mathbf{A}_B$.

EXAMPLE 1.4 Given vectors $\mathbf{A} = 3\mathbf{a}_x + 4\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_y - 5\mathbf{a}_z$, find the angle between **A** and **B**.

***Solution*:** The angle θ_{AB} can be found by using either dot product or cross product.

$$\mathbf{A}\cdot\mathbf{B} = (3, 4, 1)\cdot(0, 2, -5)$$
$$= 0 + 8 - 5 = 3$$
$$|\mathbf{A}| = \sqrt{3^2 + 4^2 + 1} = \sqrt{26}$$
$$|\mathbf{B}| = \sqrt{0^2 + 2^2 + (-5)^2} = \sqrt{29}$$
$$\cos\theta_{AB} = \frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|} = \frac{3}{\sqrt{(26)(29)}} = 0.1092$$
$$\theta_{AB} = \cos^{-1} 0.1092 = 83.73°$$

Alternatively:

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 3 & 4 & 1 \\ 0 & 2 & -5 \end{vmatrix}$$
$$= (-20 - 2)\,\mathbf{a}_x + (0 + 15)\,\mathbf{a}_y + (6 - 0)\,\mathbf{a}_z$$
$$= (-22, 15, 6)$$
$$|\mathbf{A}\times\mathbf{B}| = \sqrt{(-22)^2 + 15^2 + 6^2} = \sqrt{745}$$
$$\sin\theta_{AB} = \frac{|\mathbf{A}\times\mathbf{B}|}{|\mathbf{A}||\mathbf{B}|} = \frac{\sqrt{745}}{\sqrt{(26)(29)}} = 0.994$$
$$\theta_{AB} = \sin^{-1} 0.994 = 83.73°$$

PRACTICE EXERCISE 1.4

If $\mathbf{A} = \mathbf{a}_x + 3\mathbf{a}_z$ and $\mathbf{B} = 5\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z$, find θ_{AB}.

Answer: 120.6°.

EXAMPLE 1.5 Three field quantities are given by

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_z$$
$$\mathbf{Q} = 2\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{R} = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$$

Determine

(a) $(\mathbf{P}+\mathbf{Q})\times(\mathbf{P}-\mathbf{Q})$

(b) $\mathbf{Q}\cdot\mathbf{R}\times\mathbf{P}$

(c) $\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R}$

(d) $\sin\theta_{QR}$

(e) $\mathbf{P}\times(\mathbf{Q}\times\mathbf{R})$

(f) A unit vector perpendicular to both **Q** and **R**

(g) The component of **P** along **Q**

***Solution*:**

(a) $$\begin{aligned}(\mathbf{P}+\mathbf{Q})\times(\mathbf{P}-\mathbf{Q}) &= \mathbf{P}\times(\mathbf{P}-\mathbf{Q})+\mathbf{Q}\times(\mathbf{P}-\mathbf{Q})\\ &= \mathbf{P}\times\mathbf{P}-\mathbf{P}\times\mathbf{Q}+\mathbf{Q}\times\mathbf{P}-\mathbf{Q}\times\mathbf{Q}\\ &= \mathbf{0}+\mathbf{Q}\times\mathbf{P}+\mathbf{Q}\times\mathbf{P}-\mathbf{0}\\ &= 2\mathbf{Q}\times\mathbf{P}\\ &= 2\begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ 2 & -1 & 2\\ 2 & 0 & -1\end{vmatrix}\\ &= 2(1-0)\,\mathbf{a}_x+2(4+2)\,\mathbf{a}_y+2(0+2)\,\mathbf{a}_z\\ &= 2\mathbf{a}_x+12\mathbf{a}_y+4\mathbf{a}_z\end{aligned}$$

(b) The only way $\mathbf{Q}\cdot\mathbf{R}\times\mathbf{P}$ makes sense is

$$\begin{aligned}\mathbf{Q}\cdot(\mathbf{R}\times\mathbf{P}) &= (2,-1,2)\cdot\begin{vmatrix}\mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z\\ 2 & -3 & 1\\ 2 & 0 & -1\end{vmatrix}\\ &= (2,-1,2)\cdot(3,4,6)\\ &= 6-4+12=14\end{aligned}$$

Alternatively:

$$\mathbf{Q}\cdot(\mathbf{R}\times\mathbf{P}) = \begin{vmatrix}2 & -1 & 2\\ 2 & -3 & 1\\ 2 & 0 & -1\end{vmatrix}$$

To find the determinant of a 3×3 matrix, we repeat the first two rows and cross multiply; when the cross multiplication is from right to left, the result should be negated as shown diagrammatically here. This technique of finding a determinant applies only to a 3×3 matrix. Hence,

$$\mathbf{Q}\cdot(\mathbf{R}\times\mathbf{P}) = \begin{array}{r|rrr|l} & 2 & -1 & 2 & \\ & 2 & -3 & 1 & \\ - & 2 & 0 & -1 & + \\ - & 2 & -1 & 2 & + \\ - & 2 & -3 & 1 & + \end{array}$$

$$\begin{aligned}&= +6+0-2+12-0-2\\ &= 14\end{aligned}$$

as obtained before.

(c) From eq. (1.28)

$$\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{R}) = \mathbf{Q}\cdot(\mathbf{R}\times\mathbf{P}) = 14$$

or

$$\begin{aligned}\mathbf{P}\cdot(\mathbf{Q}\times\mathbf{R}) &= (2,0,-1)\cdot(5,2,-4)\\ &= 10+0+4\\ &= 14\end{aligned}$$

(d) $\sin\theta_{QR} = \dfrac{|\mathbf{Q}\times\mathbf{R}|}{|\mathbf{Q}||\mathbf{R}|} = \dfrac{(5, 2, -4)|}{|(2, -1, 2)||2, -3, 1|}$

$$= \frac{\sqrt{45}}{3\sqrt{14}} = \frac{\sqrt{5}}{\sqrt{14}} = 0.5976$$

(e) $\mathbf{P}\times(\mathbf{Q}\times\mathbf{R}) = (2, 0, -1)\times(5, 2, -4)$
$= (2, 3, 4)$

Alternatively, using the bac-cab rule,

$$\begin{aligned}\mathbf{P}\times(\mathbf{Q}\times\mathbf{R}) &= \mathbf{Q}(\mathbf{P}\cdot\mathbf{R}) - \mathbf{R}(\mathbf{P}\cdot\mathbf{Q})\\ &= (2, -1, 2)\,(4+0-1) - (2, -3, 1)\,(4+0-2)\\ &= (2, 3, 4)\end{aligned}$$

(f) A unit vector perpendicular to both **Q** and **R** is given by

$$\begin{aligned}\mathbf{a} &= \frac{\pm\mathbf{Q}\times\mathbf{R}}{|\mathbf{Q}\times\mathbf{R}|} = \frac{\pm(5, 2, -4)}{\sqrt{45}}\\ &= \pm(0.745, 0.298, -0.596)\end{aligned}$$

Note that $|\mathbf{a}| = 1$, $\mathbf{a}\cdot\mathbf{Q} = 0 = \mathbf{a}\cdot\mathbf{R}$. Any of these can be used to check **a**.

(g) The component of **P** along **Q** is

$$\begin{aligned}\mathbf{P}_Q &= |\mathbf{P}|\cos\theta_{PQ}\mathbf{a}_Q\\ &= (\mathbf{P}\cdot\mathbf{a}_Q)\mathbf{a}_Q = \left(\mathbf{P}\cdot\frac{\mathbf{Q}}{|\mathbf{Q}|}\right)\left(\frac{\mathbf{Q}}{|\mathbf{Q}|}\right) = \frac{(\mathbf{P}\cdot\mathbf{Q})\mathbf{Q}}{|\mathbf{Q}|^2}\\ &= \frac{(4+0-2)(2, -1, 2)}{(4+1+4)} = \frac{2}{9}(2, -1, 2)\\ &= 0.4444\mathbf{a}_x - 0.2222\mathbf{a}_y + 0.4444\mathbf{a}_z\end{aligned}$$

PRACTICE EXERCISE 1.5

Let $\mathbf{E} = 3\mathbf{a}_y + 4\mathbf{a}_z$ and $\mathbf{F} = 4\mathbf{a}_x - 10\mathbf{a}_y + 5\mathbf{a}_z$.

(a) Find the component of **E** along **F**.
(b) Determine a unit vector perpendicular to both **E** and **F**.

Answer: (a) (−0.2837, 0.7092, −0.3546), (b) ±(0.9398, 0.2734, −0.205).

EXAMPLE 1.6 Derive the cosine formula

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and the sine formula

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

using dot product and cross product, respectively.

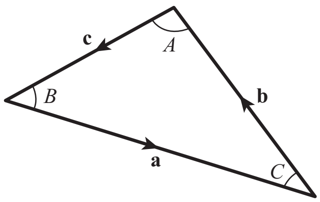

Figure 1.11 For Example 1.6.

***Solution*:** Consider a triangle as shown in Figure 1.11. From the figure, we notice that

$$\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$$

that is,

$$\mathbf{b}+\mathbf{c}=-\mathbf{a}$$

Hence,

$$\begin{aligned} a^2 &= \mathbf{a}\cdot\mathbf{a}=(\mathbf{b}+\mathbf{c})\cdot(\mathbf{b}+\mathbf{c}) \\ &= \mathbf{b}\cdot\mathbf{b}+\mathbf{c}\cdot\mathbf{c}+2\mathbf{b}\cdot\mathbf{c} \\ a^2 &= b^2+c^2-2bc\cos A \end{aligned}$$

where $(\pi - A)$ is the angle between **b** and **c**.
The area of a triangle is half of the product of its height and base. Hence,

$$\left|\frac{1}{2}\mathbf{a}\times\mathbf{b}\right|=\left|\frac{1}{2}\mathbf{b}\times\mathbf{c}\right|=\left|\frac{1}{2}\mathbf{c}\times\mathbf{a}\right|$$

$$ab\sin C = bc\sin A = ca\sin B$$

Dividing through by *abc* gives

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

PRACTICE EXERCISE 1.6

Show that vectors $\mathbf{a} = (4, 0, -1)$, $\mathbf{b} = (1, 3, 4)$, and $\mathbf{c} = (-5, -3, -3)$ form the sides of a triangle. Is this a right angle triangle? Calculate the area of the triangle.

Answer: Yes, 10.5.

EXAMPLE 1.7 Show that points $P_1(5, 2, -4)$, $P_2(1, 1, 2)$, and $P_3(-3, 0, 8)$ all lie on a straight line. Determine the shortest distance between the line and point $P_4(3, -1, 0)$.

***Solution*:** The distance vector $\mathbf{r}_{P_1P_2}$ is given by

$$\begin{aligned} \mathbf{r}_{P_1P_2} &= \mathbf{r}_{P_2}-\mathbf{r}_{P_1}=(1,1,2)-(5,2,-4) \\ &= (-4,-1,6) \end{aligned}$$

Similarly,

$$\begin{aligned} \mathbf{r}_{P_1P_3} = \mathbf{r}_{P_3}-\mathbf{r}_{P_1} &= (-3,0,8)-(5,2,-4) \\ &= (-8,-2,12) \\ \mathbf{r}_{P_1P_4} = \mathbf{r}_{P_4}-\mathbf{r}_{P_1} &= (3,-1,0)-(5,2,-4) \\ &= (-2,-3,4) \end{aligned}$$

$$\begin{aligned} \mathbf{r}_{P_1P_2}\times\mathbf{r}_{P_1P_3} &= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ -4 & -1 & 6 \\ -8 & -2 & 12 \end{vmatrix} \\ &= (0,0,0) \end{aligned}$$

showing that the angle between $\mathbf{r}_{P_1P_2}$ and $\mathbf{r}_{P_1P_3}$ is zero ($\sin\theta = 0$). This implies that P_1, P_2, and P_3 lie on a straight line.

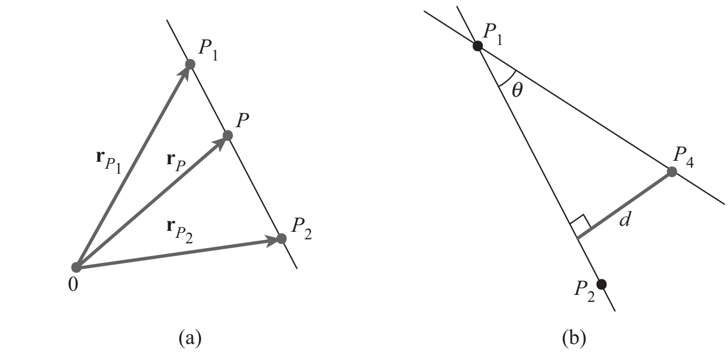

Figure 1.12 For Example 1.7.

Alternatively, the vector equation of the straight line is easily determined from Figure 1.12(a). For any point P on the line joining P_1 and P_2

$$\mathbf{r}_{P_1P} = \lambda \mathbf{r}_{P_1P_2}$$

where λ is a constant. Hence the position vector $\mathbf{r}_P$ of the point P must satisfy

$$\mathbf{r}_p - \mathbf{r}_{P_1} = \lambda(\mathbf{r}_{P_2} - \mathbf{r}_{P_1})$$

that is,

$$\begin{aligned} \mathbf{r}_p &= \mathbf{r}_{P_1} + \lambda(\mathbf{r}_{P_2} - \mathbf{r}_{P_1}) \\ &= (5, 2, -4) - \lambda(4, 1, -6) \\ \mathbf{r}_p &= (5 - 4\lambda, 2 - \lambda, -4 + 6\lambda) \end{aligned}$$

This is the vector equation of the straight line joining P_1 and P_2. If P_3 is on this line, the position vector of P_3 must satisfy the equation; $\mathbf{r}_3$ does satisfy the equation when $\lambda = 2$.

The shortest distance between the line and point $P_4(3, -1, 0)$ is the perpendicular distance from the point to the line. From Figure 1.12(b), it is clear that

$$\begin{aligned} d = r_{P_1P_4} \sin\theta &= \left|\mathbf{r}_{P_1P_4} \times \mathbf{a}_{P_1P_2}\right| \\ &= \frac{|(-2, -3, 4) \times (-4, -1, 6)|}{|(-4, -1, 6)|} \\ &= \frac{\sqrt{312}}{\sqrt{53}} = 2.426 \end{aligned}$$

Any point on the line may be used as a reference point. Thus, instead of using P_1 as a reference point, we could use P_3. If $\angle P_4P_3P_2 = \theta'$, then

$$d = \left|\mathbf{r}_{P_3P_4}\right| \sin\theta' = \left|\mathbf{r}_{P_3P_4} \times \mathbf{a}_{P_3P_2}\right|$$

PRACTICE EXERCISE 1.7

If P_1 is (1, 2, −3) and P_2 is (−4, 0, 5), find

(a) The distance P_1P_2
(b) The vector equation of the line P_1P_2
(c) The shortest distance between the line P_1P_2 and point P_3 (7, −1, 2)

Answer: (a) 9.644, (b) $(1-5\lambda)\mathbf{a}_x + 2(1-\lambda)\mathbf{a}_y + (8\lambda - 3)\mathbf{a}_z$, (c) 8.2.

MATLAB 1.1

```
% This script allows the user to input two vectors and
% then compute their dot product, cross product, sum,
% and difference
clear
vA = input('Enter vector A in the format [x y z]... \n > ');
if isempty(vA); vA = [0 0 0]; end % if the input is
                % entered incorrectly set the vector to 0
vB = input('Enter vector B in the format [x y z]... \n > ');
if isempty(vB); vB = [0 0 0]; end
disp('Magnitude of A:')
disp(norm(vA))            % norm finds the magnitude of a
                          % multi-dimensional vector
disp('Magnitude of B:')
disp(norm(vB))
disp('Unit vector in direction of A:')
disp(vA/norm(vA))         % unit vector is the vector
                          % divided by its magnitude
disp('Unit vector in direction of B:')
disp(vB/norm(vB))
disp('Sum A+B:')
disp(vA+vB)
disp('Difference A-B:')
disp(vA-vB)
disp('Dot product (A · B):')
disp(dot(vA,vB))          % dot takes the dot product of vectors
disp('Cross product (A × B):')
disp(cross(vA,vB))        % cross takes cross product of vectors
```

ADDITIONAL EXAMPLES

EXAMPLE 1.8 Electric field intensity is produced by a charge distribution as explained in Appendix A and Chapter 4. It is a vector and denoted by **E**. The electric field components at a point *P* due to two different sources (charge distributions) are as follows:

$\mathbf{E}_1$ = 10 V/m at an angle of 30° with the horizontal in the anticlockwise direction

$\mathbf{E}_2$ = 12 V/m at an angle of 50° with the horizontal in the clockwise direction

(a) Determine the net electric field at the point *P*.

(b) Convert the given values into vector quantities and determine the net electric field using vector addition.

***Solution*:**

(a) By the parallelogram rule, the net electric field is given by

$$\begin{aligned}|\mathbf{E}| &= \sqrt{|\mathbf{E}_1|^2 + |\mathbf{E}_2|^2 - \left(2|\mathbf{E}_1||\mathbf{E}_2|\cos(\mathbf{E}_1, \mathbf{E}_2)\right)} \\ &= \sqrt{10^2 + 12^2 - \left(2 \times 10 \times 12 \cos(100^\circ)\right)} \\ &= 16.90 \text{ V/m}\end{aligned}$$

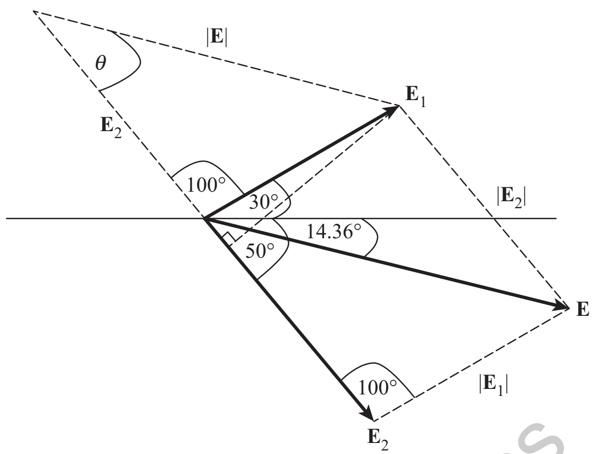

Figure 1.13 For Example 1.8.

Note: In the cosine rule we have to take the angle between the two vectors when the head of one vector is connected to the tail of the other vector as shown in Figure 1.13.

Let θ be the angle made by the resultant electric field with $\mathbf{E}_2$

$$\theta = \tan^{-1}\left(\frac{|\mathbf{E}_1|\sin 80^\circ}{|\mathbf{E}_2| + |\mathbf{E}_1|\cos 80^\circ}\right)$$

$$= \tan^{-1}\left(\frac{10\sin 80^\circ}{12 + 10\cos 80^\circ}\right)$$

$$= 35.64^\circ$$

The angle made by $\mathbf{E}$ with the horizontal is 14.36° in the clockwise direction.

$$\mathbf{E} = 16.90\cos(-14.36)^\circ\, \mathbf{a}_x + 16.90\sin(-14.36)^\circ\, \mathbf{a}_y = 16.373\mathbf{a}_x - 4.192\mathbf{a}_y \text{ V/m}$$

(b) In vector notation,

$$\mathbf{E}_1 = 10\cos 30\mathbf{a}_x + 10\sin 30\mathbf{a}_y = 8.660\mathbf{a}_x + 5\mathbf{a}_y \text{ V/m}$$

$$\mathbf{E}_2 = 12\cos(-50^\circ)\mathbf{a}_x + 12\sin(-50^\circ)\mathbf{a}_y = 7.713\mathbf{a}_x - 9.192\mathbf{a}_y \text{ V/m}$$

The net electric field at the point is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = 8.660\mathbf{a}_x + 5\mathbf{a}_y + 7.713\mathbf{a}_x - 9.192\mathbf{a}_y$$

$$\mathbf{E} = 16.373\mathbf{a}_x - 4.192\mathbf{a}_y \text{ V/m}$$

It can be observed that the procedure of vector addition is simple and straightforward.

PRACTICE EXERCISE 1.8

If $\mathbf{A} = 4\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$ and $\mathbf{B} = 12\mathbf{a}_x + 18\mathbf{a}_y - 8\mathbf{a}_z$, determine:

(a) $\mathbf{A} - 3\mathbf{B}$

(b) $(2\mathbf{A} + 5\mathbf{B})/|\mathbf{B}|$

(c) $\mathbf{a}_x \times \mathbf{A}$

(d) $(\mathbf{B} \times \mathbf{a}_x) \cdot \mathbf{a}_y$

Answers: (a) $-32\mathbf{a}_x - 56\mathbf{a}_y + 30\mathbf{a}_z$ (b) $2.94\mathbf{a}_x + 3.72\mathbf{a}_y - 1.214\mathbf{a}_z$ (c) $-6\mathbf{a}_y - 2\mathbf{a}_z$ (d) -8

EXAMPLE 1.9 Let us consider a two-dimensional plane having a uniform electric field of $3\mathbf{a}_x - 2\mathbf{a}_y$ V/m. Determine the dot product between the electric field and
(a) the vector joining points (0, 0) and (12, −8);
(b) the vector joining points (0, 0) and (8, 12); and
(c) the vector joining points (0, 0) and (8, −12).

***Solution*:** Electric field in the given plane $\mathbf{E} = 3\mathbf{a}_x - 2\mathbf{a}_y$ V/m

(a) The position vector joining the points (0, 0) and (12, −8) is $\mathbf{r} = 12\mathbf{a}_x - 8\mathbf{a}_y$ m
The unit vector along the **E** field is $0.83\mathbf{a}_x - 0.55\mathbf{a}_y$
The unit vector along the **r** vector is $0.83\mathbf{a}_x - 0.55\mathbf{a}_y$
Both vectors are in the same direction, and the dot product is:

$$\left(3\mathbf{a}_x - 2\mathbf{a}_y\right)\cdot\left(12\mathbf{a}_x - 8\mathbf{a}_y\right) = 36 + 16 = 52\text{ V}$$

(b) The position vector joining the points (0, 0) and (8, 12) is $\mathbf{r} = 8\mathbf{a}_x + 12\mathbf{a}_y$ m
The dot product is: $\left(3\mathbf{a}_x - 2\mathbf{a}_y\right)\cdot\left(8\mathbf{a}_x + 12\mathbf{a}_y\right) = 24 - 24 = 0$
The dot product has the minimum magnitude when **r** is in the direction orthogonal to **E**.

(c) The position vector joining the points (0, 0) and (8, −12) is $\mathbf{r} = 8\mathbf{a}_x - 12\mathbf{a}_y$ m
The dot product is: $\left(3\mathbf{a}_x - 2\mathbf{a}_y\right)\cdot\left(8\mathbf{a}_x - 12\mathbf{a}_y\right) = 24 + 24 = 48\text{ V}$

Incidentally, dot product gives the magnitude of the potential difference between two points under consideration, as explained in Appendix A and in Section 4.7 of Chapter 4. The rate of change of potential is maximum along **E** (case-a) and minimum along the orthogonal direction (case-b) which is an equipotential contour. It has intermediate values for other directions (case-c).

PRACTICE EXERCISE 1.9

Determine the dot product, cross product, and angle between $\mathbf{P} = 2\mathbf{a}_x - 6\mathbf{a}_y + 5\mathbf{a}_z$ and $\mathbf{Q} = 3\mathbf{a}_y + \mathbf{a}_z$

Answer: −13, $-21\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$, 120.66°

EXAMPLE 1.10 A wave propagation phenomenon can be explained in terms of two vectors: electric field intensity (**E**) and magnetic field intensity (**H**). A uniform plane wave propagating from a radiating source is characterized by constant amplitudes of **E** and **H** vectors in any plane transverse to the direction of propagation. Consider a uniform plane wave originating from an antenna and traveling through a homogenous unbounded medium. The electric field and magnetic field at an instant of time at a point in a plane near the receiver is $75.196\mathbf{a}_x + 43.415\mathbf{a}_y$ V/m and $-0.115\mathbf{a}_x + 0.199\mathbf{a}_y$ A/m respectively. Determine the instantaneous power transferred to that point by the antenna at the instant of time.

***Solution*:** The following data is given—

$$\text{Electric field vector } (\mathbf{E}) = 75.196\mathbf{a}_x + 43.415\mathbf{a}_y \text{ V/m}$$

$$\text{Magnetic field vector } (\mathbf{H}) = -0.115\mathbf{a}_x + 0.199\mathbf{a}_y \text{ A/m}$$

The instantaneous power density in the wave is given by Poynting Vector (**P**) which is the cross product of **E** and **H**:

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

$$= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 75.196 & 43.415 & 0 \\ -0.115 & 0.199 & 0 \end{vmatrix}$$

$$= (14.96 + 4.99)\mathbf{a}_z$$

$$= 19.95\,\mathbf{a}_z \text{ W/m}^2$$

It should be noted that **E** and **H** are orthogonal in space for a uniform plane wave (see in Appendix A and Chapter 10). This fact can be verified by taking the dot product of the two vectors in this example, which is zero. Needless to say that **E**, **H**, and **P** form a right-handed system. The direction of **P** is the direction of wave propagation.

PRACTICE EXERCISE 1.10

Find the area of the parallelogram formed by the vectors $\mathbf{D} = 4\mathbf{a}_x - \mathbf{a}_y + 5\mathbf{a}_z$ and $\mathbf{E} = -\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z$

Answer: 8.646

EXAMPLE 1.11 Consider a straight line in the *xy*-plane represented by $3x + 2y = 6$. Find the unit vector directed from the origin perperdicular to this line.

***Solution*:** The line $3x + 2y = 6$ intersects the *x*-axis and the *y*-axis in points $A(2, 0)$ and $B(0, 3)$ respectively. The equation of the line segment from (2, 0) to (0, 3) is

$$\mathbf{r}_{AB} = (2-0)\mathbf{a}_x + (0-3)\mathbf{a}_y = 2\mathbf{a}_x - 3\mathbf{a}_y$$

Let us consider a point $P(x, y)$ on the given line such that the vector joining the origin and(x, y) is perpendicular to the line. As per eq. (1.14), the vector directed from the origin to $P(x, y)$ is as follows:

$$\mathbf{r}_P = \mathbf{r}_{OP} = x\mathbf{a}_x + y\mathbf{a}_y$$

The unit vector along it is given by the following equation:

$$\mathbf{a}_{OP} = \frac{x\mathbf{a}_x + \mathrm{y}\mathbf{a}_y}{\sqrt{x^2 + \mathrm{y}^2}}$$

As the vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_P$ are orthogonal, their dot product will be zero:

$$\mathbf{r}_{AB} \cdot \mathbf{r}_P = 0$$

$$2x - 3y = 0$$

Solving the above equation with $3x + 2y = 6$ gives $x = 1.38$ and $y = 0.92$
Therefore,

$$\mathbf{r}_P = 1.38\mathbf{a}_x + 0.92\mathbf{a}_y$$

And the unit vector along it is given as follows:

$$\mathbf{a}_{OP} = \frac{1.38\mathbf{a}_x + 0.92\mathbf{a}_y}{\sqrt{(1.38)^2 + (0.92)^2}} = 0.83\mathbf{a}_x + 0.55\mathbf{a}_y$$

PRACTICE EXERCISE 1.11

If $\mathbf{A} = 4\mathbf{a}_x - 6\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = 2\mathbf{a}_x + 5\mathbf{a}_z$, find:

(a) $\mathbf{A} \cdot \mathbf{B} + 2|\mathbf{B}|^2$

(b) a unit vector perpendicular to both **A** and **B**

Answers: (a) 71, (b) $\pm(-0.8111\mathbf{a}_x - 0.4867\mathbf{a}_y + 0.3244\mathbf{a}_z)$

SUMMARY

1. A field is a function that specifies a quantity in space. For example, $\mathbf{A}(x, y, z)$ is a vector field, whereas $V(x, y, z)$ is a scalar field.
2. A vector **A** is uniquely specified by its magnitude and a unit vector along it, that is, $\mathbf{A} = A\mathbf{a}_A$.
3. Multiplying two vectors **A** and **B** results in either a scalar $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$ or a vector $\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB}\, \mathbf{a}_n$. Multiplying three vectors **A**, **B**, and **C** yields a scalar $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$ or a vector $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$.
4. The scalar projection (or component) of vector **A** onto **B** is $A_B = \mathbf{A} \cdot \mathbf{a}_B$, whereas vector projection of **A** onto **B** is $\mathbf{A}_B = A_B\mathbf{a}_B$.

REVIEW QUESTIONS

1.1 Tell which of the following quantities is not a vector: (a) force, (b) momentum, (c) acceleration, (d) work, (e) weight.

1.2 Which of the following is not a scalar field?
- (a) Displacement of a mosquito in space
- (b) Light intensity in a drawing room
- (c) Temperature distribution in your classroom
- (d) Atmospheric pressure in a given region
- (e) Humidity of a city

1.3 Of the rectangular coordinate systems shown in Figure 1.14, which are not right handed?

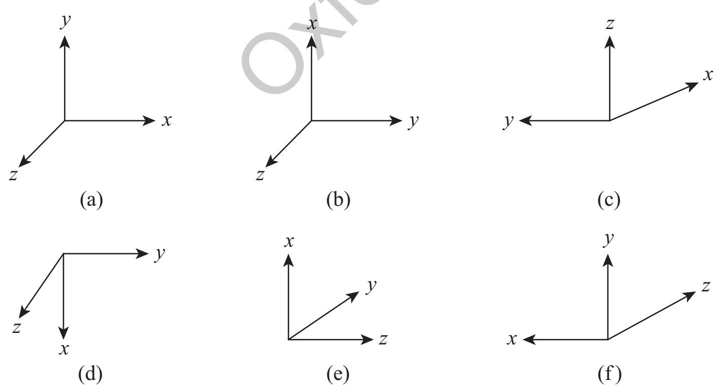

Figure 1.14 For Review Question 1.3.

1.4 Which of these is correct?
- (a) $\mathbf{A} \times \mathbf{A} = |\mathbf{A}|^2$
- (b) $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{A} = \mathbf{0}$
- (c) $\mathbf{A} \cdot \mathbf{B} \cdot \mathbf{C} = \mathbf{B} \cdot \mathbf{C} \cdot \mathbf{A}$
- (d) $\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_z$
- (e) $\mathbf{a}_k = \mathbf{a}_x - \mathbf{a}_y$, where $\mathbf{a}_k$ is a unit vector

1.5 Which of the following identities is not valid?

(a) $\mathbf{a}(\mathbf{b}+\mathbf{c}) = \mathbf{ab}+\mathbf{bc}$

(b) $\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b}+\mathbf{a}\times\mathbf{c}$

(c) $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$

(d) $\mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = -\mathbf{b}\cdot(\mathbf{a}\times\mathbf{c})$

(e) $\mathbf{a}_A\cdot\mathbf{a}_B = \cos\theta_{AB}$

1.6 Which of the following statements are meaningless?

(a) $\mathbf{A}\cdot\mathbf{B}+2\mathbf{A}=0$

(b) $\mathbf{A}\cdot\mathbf{B}+5=2\mathbf{A}$

(c) $\mathbf{A}+(\mathbf{A}+\mathbf{B})+2=0$

(d) $\mathbf{A}\cdot\mathbf{A}+\mathbf{B}\cdot\mathbf{B}=0$

1.7 Let $\mathbf{F} = 2\mathbf{a}_x - 6\mathbf{a}_y + 10\mathbf{a}_z$ and $\mathbf{G} = \mathbf{a}_x + G_y\mathbf{a}_y + 5\mathbf{a}_z$. If **F** and **G** have the same unit vector, G_y *is*

(a) 6

(b) −3

(c) 0

(d) 6

1.8 Given that $\mathbf{A} = \mathbf{a}_x + \alpha\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = \alpha\mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, if **A** and **B** are normal to each other, α is

(a) −2

(b) −1/2

(c) 0

(d) 2

(e) 1

1.9 The component of $6\mathbf{a}_x + 2\mathbf{a}_y - 3\mathbf{a}_z$ along $3\mathbf{a}_x - 4\mathbf{a}_y$ is

(a) $-12\mathbf{a}_x - 9\mathbf{a}_y - 3\mathbf{a}_z$

(b) $30\mathbf{a}_x - 40\mathbf{a}_y$

(c) 10/7

(d) 2

(e) 10

1.10 Given $\mathbf{A} = -6\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$, the projection of **A** along $\mathbf{a}_y$ is

(a) −12

(b) −4

(c) 3

(d) 7

(e) 12

Answers 1.1d, 1.2a, 1.3b, e, 1.4b, 1.5a, 1.6a, b, c, 1.7b, 1.8b, 1.9d, 1.10c.

PROBLEMS

Section 1.4—Unit Vector

1.1 Determine the unit vector along the direction *OP*, where *O* is the origin and *P* is point (4, −5, 1).

1.2 Find the unit vector along the line joining point (2, 4, 4) to point (−3, 2, 2).

Sections 1.5–1.7—Vector Addition, Subtraction, and Multiplication

1.3 Given vectors $\mathbf{A} = 4\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -\mathbf{a}_x + 8\mathbf{a}_y + 5\mathbf{a}_z$, find (a) $\mathbf{A}-2\mathbf{B}$, (b) $\mathbf{A}\cdot\mathbf{B}$, (c) $\mathbf{A}\times\mathbf{B}$.

1.4 Let $\mathbf{A} = 4\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, $\mathbf{B} = 3\mathbf{a}_x + 5\mathbf{a}_y + \mathbf{a}_z$, and $\mathbf{C} = \mathbf{a}_y - 7\mathbf{a}_z$. Find $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$.

1.5 Let $\mathbf{A} = \mathbf{a}_x - \mathbf{a}_z$, $\mathbf{B} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$, $\mathbf{C} = \mathbf{a}_y + 2\mathbf{a}_z$, find:

(a) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$

(b) $(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$

(c) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})$

(d) $(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$

1.6 If the position vectors of points *T* and *S* are $3\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z$ and $4\mathbf{a}_x + 6\mathbf{a}_y + 2\mathbf{a}_z$, respectively, find (a) coordinates of *T* and *S*, (b) the distance vector from *T* to *S*, (c) the distance between *T* and *S*.

1.7 Let $\mathbf{A} = \alpha\mathbf{a}_x + 3\mathbf{a}_y - 2\mathbf{a}_z$ and $\mathbf{B} = 4\mathbf{a}_x + \beta\mathbf{a}_y + 8\mathbf{a}$.

(a) Find α and β if **A** and **B** are parallel.

(b) Determine the relationship between α and β if **B** is perpendicular to **A.**

1.8 (a) Show that

$$(\mathbf{A}\cdot\mathbf{B})^2 + |\mathbf{A}\times\mathbf{B}|^2 = (AB)^2$$

(b) Show that

$$\mathbf{a}_x = \frac{\mathbf{a}_y \times \mathbf{a}_z}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \mathbf{a}_y = \frac{\mathbf{a}_z \times \mathbf{a}_x}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}, \mathbf{a}_z = \frac{\mathbf{a}_x \times \mathbf{a}_y}{\mathbf{a}_x \cdot \mathbf{a}_y \times \mathbf{a}_z}$$

1.9 Given that

$$\mathbf{P} = 2\mathbf{a}_x - \mathbf{a}_y - 2\mathbf{a}_z$$
$$\mathbf{Q} = 4\mathbf{a}_x + 3\mathbf{a}_y + 2\mathbf{a}_z$$
$$\mathbf{R} = -\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z$$

find: (a) $|\mathbf{P}+\mathbf{Q}-\mathbf{R}|$, (b) $\mathbf{P}\cdot\mathbf{Q}\times\mathbf{R}$, (c) $\mathbf{Q}\times\mathbf{P}\cdot\mathbf{R}$, (d) $(\mathbf{P}\times\mathbf{Q})\cdot(\mathbf{Q}\times\mathbf{R})$, (e) $(\mathbf{P}\times\mathbf{Q})\times(\mathbf{Q}\times\mathbf{R})$, (f) $\cos\theta_{PR}$, (g) $\sin\theta_{PQ}$.

1.10 Show that vectors $\mathbf{A} = \mathbf{a}_x - 2\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{B} = -2\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z$ are parallel.

1.11 Simplify the following expressions:
(a) $\mathbf{A}\times(\mathbf{A}\times\mathbf{B})$ (b) $\mathbf{A}\times[\mathbf{A}\times(\mathbf{A}\times\mathbf{B})]$

1.12 A right angle triangle has its corners located at $P_1(5, -3, 1)$, $P_2(1, -2, 4)$, and $P_3(3, 3, 5)$. (a) Which corner is a right angle? (b) Calculate the area of the triangle.

1.13 Points P, Q, and R are located at (−1, 4, 8), (2, −1, 3), and (−1, 2, 3), respectively. Determine (a) the distance between P and Q, (b) the distance vector from P to R, (c) the angle between QP and QR, (d) the area of triangle PQR, (e) the perimeter of triangle PQR.

1.14 Two points $P(2, 4, -1)$ and $Q(12, 16, 9)$ form a straight line. Calculate the time taken for a sonar signal traveling at 300 m/s to get from the origin to the midpoint of PQ.

1.15 Show that the dot and cross in the triple scalar product may be interchanged, that is, $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$.

***1.16** (a) Prove that $\mathbf{P} = \cos\theta_1\mathbf{a}_x + \sin\theta_1\mathbf{a}_y$ and $\mathbf{Q} = \cos\theta_2\mathbf{a}_x + \sin\theta_2\mathbf{a}_y$ are unit vectors in the xy-plane, respectively, making angles θ_1 and θ_2 with the x-axis.
(b) By means of dot product, obtain the formula for $\cos(\theta_2 - \theta_1)$. By similarly formulating $\mathbf{P}$ and $\mathbf{Q}$, obtain the formula for $\cos(\theta_2 + \theta_1)$.
(c) If θ is the angle between $\mathbf{P}$ and $\mathbf{Q}$, find $\frac{1}{2}|\mathbf{P}-\mathbf{Q}|$ in terms of θ.

1.17 Consider a rigid body rotating with a constant angular velocity ω radians per second about a fixed axis through O as in Figure 1.15. Let $\mathbf{r}$ be the distance vector from O to P, the position of a particle in the body. The magnitude of the velocity $\mathbf{u}$ of the body at P is $|\mathbf{u}| = d|\omega| = |\mathbf{r}|\sin\theta|\omega|$. or $\mathbf{u} = \omega\times\mathbf{r}$ If the rigid body is rotating at 3 rad/s about an axis parallel to $\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$ and passing through point, (2, −3, 1) determine the velocity of the body at (1, 3, 4).

1.18 A cube of side 1 m has one corner placed at the origin. Determine the angle between the diagonals of the cube.

1.19 Given vectors $\mathbf{T} = 2\mathbf{a}_x - 6\mathbf{a}_y + 3\mathbf{a}_z$ and $\mathbf{S} = \mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$, find (a) the scalar projection of $\mathbf{T}$ on $\mathbf{S}$, (b) the vector projection of $\mathbf{S}$ on $\mathbf{T}$, (c) the smaller angle between $\mathbf{T}$ and $\mathbf{S}$.

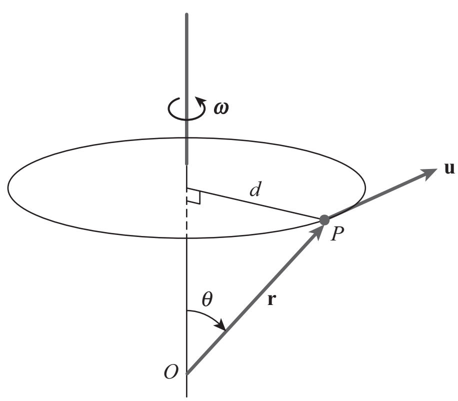

Figure 1.15 For Problem 1.17.

Section 1.8—Components of a Vector

1.20 Given two vectors $\mathbf{A}$ and $\mathbf{B}$, show that the vector component of $\mathbf{A}$ perpendicular to $\mathbf{B}$ is

$$\mathbf{C} = \mathbf{A} - \frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{B}\cdot\mathbf{B}}\mathbf{B}$$

*Single asterisks indicate problems of intermediate difficulty

1.21 If $\mathbf{H} = 2xy\mathbf{a}_x - (x+z)\mathbf{a}_y + z^2\mathbf{a}_z$, find:
(a) A unit vector parallel to **H** at $P(1, 3, -2)$
(b) The equation of the surface on which $|\mathbf{H}| = 10$

1.22 Given three vectors

$$\mathbf{A} = 4\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$$
$$\mathbf{B} = \mathbf{a}_x - \mathbf{a}_y$$
$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

Find: (a) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$, (b) the vector component of **A** along **B**.

1.23 Let $\mathbf{G} = x^2\mathbf{a}_x - y\mathbf{a}_y + 2z\mathbf{a}_z$ and $\mathbf{H} = yz\mathbf{a}_x + 3\mathbf{a}_y + xz\mathbf{a}_z$. At point $(1, -2, 3)$, (a) calculate the magnitude of **G** and **H**, (b) determine $\mathbf{G}\cdot\mathbf{H}$, (c) find the angle between **G** and **H**.

1.24 Determine the scalar component of vector $\mathbf{H} = y\mathbf{a}_x - x\mathbf{a}_z$ at point $P(1, 0, 3)$ that is directed toward point $Q(-2, 1, 4)$.

1.25 Given two vector fields

$$\mathbf{D} = yz\mathbf{a}_x + xz\mathbf{a}_y + xy\mathbf{a}_z \text{ and } \mathbf{E} = 5xy\mathbf{a}_x + 6(x^2+3)\mathbf{a}_y + 8x^2\mathbf{a}_z$$

(a) Evaluate $\mathbf{C} = \mathbf{D} + \mathbf{E}$ at point $P(-1, 2, 4)$.
(b) Find the angle **C** makes with the x-axis at P.

1.26 **E** and **F** are vector fields given by $\mathbf{E} = 2x\mathbf{a}_x + \mathbf{a}_y + yz\mathbf{a}_z$ and $\mathbf{F} = xy\mathbf{a}_x - y^2\mathbf{a}_y + xyz\mathbf{a}_z$. Determine:
(a) $|\mathbf{E}|$ at $(1, 2, 3)$
(b) The component of **E** along **F** at $(1, 2, 3)$
(c) A vector perpendicular to both **E** and **F** at $(0, 1, -3)$ whose magnitude is unity

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The Accreditation Board for Engineering and Technology (ABET) establishes eleven criteria for accrediting engineering, technology, and computer science programs. The criteria are as follows:

A. Ability to apply mathematics science and engineering principles
B. Ability to design and conduct experiments and interpret data
C. Ability to design a system, component, or process to meet desired needs
D. Ability to function on multidisciplinary teams
E. Ability to identify, formulate, and solve engineering problems
F. Ability to understand professional and ethical responsibility
G. Ability to communicate effectively
H. Ability to understand the impact of engineering solutions in a global context
I. Ability to recognize the need for and to engage in lifelong learning
J. Ability to know of contemporary issues
K. Ability to use the techniques, skills, and modern engineering tools necessary for engineering practice

Criterion A applies directly to electromagnetics. As students, you are expected to study mathematics, science, and engineering with the purpose of being able to apply that knowledge to the solution of engineering problems. The skill needed here is the ability to apply the fundamentals of EM in solving a problem. The best approach is to attempt as many problems as you can. This will help you to understand how to use formulas and assimilate the material. Keep nearby all your basic mathematics, science, and engineering textbooks. You may need to consult them from time to time.