# Microelectronic Circuits Theory and Applications <br> $7^{\text {TH }}$ EDITION South Asia Edition 

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## Preface

This adapted version of Microelectronic Circuits, international seventh edition, is intended as a text for the core courses in electronic circuits taught to students pursuing an undergraduate or a postgraduate degree in electrical and electronics engineering. It should also prove useful to engineers and other professionals wishing to update their knowledge through self-study.

As was the case with the first six editions, the objective of this book is to develop in the reader the ability to analyze and design electronic circuits, both analog and digital, discrete and integrated. While the application of integrated circuits is covered, emphasis is placed on transistor circuit design. This is done because of our belief that even if the majority of those studying this book were not to pursue a career in IC design, knowledge of what is inside the IC package would enable intelligent and innovative application of such chips. Furthermore, with the advances in VLSI technology and design methodology, IC design itself has become accessible to an increasing number of engineers.

## Prerequisites

The prerequisite for studying the material in this book is a first course in circuit analysis. As a review, some linear circuits material is included here in the appendices: specifically, two-port network parameters in Appendix C; some useful network theorems in Appendix D; single-time-constant circuits in Appendix E; and $s$-domain analysis in Appendix F. In addition, a number of relevant circuit analysis problems are included at the beginning of the end-of-chapter problems section of Chapter 1 . No prior knowledge of physical electronics is assumed. All required semiconductor device physics is included, and Appendix A provides a brief description of IC fabrication. All these appendices can be found on the Oxford University Press India (OUPI)'s website.

## Emphasis on Design

It has been our philosophy that circuit design is best taught by pointing out the various tradeoffs available in selecting a circuit configuration and in selecting component values for a given configuration. The emphasis on design has been retained in this edition. In addition to design examples, and designoriented exercises and end-of-chapter problems (indicated with a D ), the book includes an extensive appendix (Appendix B on the OUPI website) where a large number of simulation and design examples are presented. These emphasize the use of SPICE, the most valuable circuit-design aid.

## New to This Edition

While maintaining the philosophy and pedagogical approach of the first six editions, several changes have been made to both organization and coverage. Our goal in making structural changes has been to
increase modularity and thus flexibility for the instructor, without causing disturbance to courses currently using the sixth edition. Changes in coverage are necessitated by the continuing advances in technology which make some topics of greater relevance and others of less interest. As well, advances in IC process technology require that the numbers used in the examples, exercises and end-of-chapter problems be updated to reflect the parameters of newer generations of IC technologies (e.g., some problems utilize the parameters of the $65-\mathrm{nm}$ CMOS process). This ensures that students are acquiring a real-world perspective on technology.

To improve presentation, a number of chapters and sections have been rewritten for greater clarity. Specific, noteworthy changes are:

1. New End-of-Chapter Problems and a New Instructor's Solutions Manual. The number of the end-of-chapter problems has been increased. The new Instructor's Solutions Manual is written by Adel Sedra.
2. Expand-Your-Perspective Notes. This is a new feature providing historical and application perspectives. About two such notes are included in each chapter. Most are focused on notable circuit engineers and key inventions.
3. Improved Structural Organization. The content of the book has been restructured by dividing it into four parts. In Part I, Operational Amplifiers (Chapter 2) is discussed before diodes and transistors. This provides readers with opportunity to work and experiment with nontrivial circuits. Part II of the book focuses on analog integrated circuits and Part IV on digital integrated circuits. Part III contains specialized application of the material presented in Parts I and II.
4. Greater Flexibility in Presenting the MOSFET and the BJT. Two short and completely parallel chapters present the BJT (Chapter 4) and the MOSFET (Chapter 5). Here the focus is on the device structure and its physical operation, its current-voltage characteristics, and its application in de circuits. The order of coverage of these two chapters is entirely at the instructor's discretion as they have been written to be completely independent of each other.
5. A Unified Treatment of Transistor Amplifiers. The heart of a first course in electronics is the study of transistor amplifiers. The seventh edition provides a new approach to this subject: A new Chapter 6 begins with the basic principles that underlie the operation of a transistor of either type as an amplifier, and presents such concepts as small-signal operation and modeling. This is followed by the classical configurations of transistor amplifiers, biasing methods, and practical discrete-circuit amplifiers. The combined presentation emphasizes the unity of the basic principles while allowing for separate treatment of the two device types where this is warranted. Very importantly, we are able to compare the two devices and to draw conclusions about their unique areas of application.
6. Improved Presentation of Cascoding. Chapter 7 dealing with the basic building blocks of IC amplifiers has been rewritten to improve presentation. Specifically, the development of cascoding and the key circuit building blocks, the cascode amplifier and the cascode current source, is now much clearer.
7. Streamlined Presentation of Frequency Response. In order to keep the treatment of frequency response all together, a new Chapter 9 has been introduced which provides comprehensive coverage of frequency response analysis of amplifiers.
8. Clearer and Simplified Study of Feedback. The feedback chapter has been rewritten to improve, simplify and clarify the presentation of this key subject.
9. Updated Treatment of Output Stages and Power Amplifiers. Here, we have updated the material on MOS power transistors and added a new section on the increasingly important class D switching power amplifier.
10. A More Contemporary Approach to Operational Amplifier Circuits. While maintaining coverage of some of the enduring features and subcircuits of the classical 741 op amp , its total coverage is somewhat reduced to make room for modern IC op-amp design techniques.
11. Increased Emphasis on Integrated-Circuit Filters and Oscillators. A section on a popular approach to integrated-circuit filter design, namely, transconductance-C filters, has been added to Chapter 13. To make room for this new material, the subsection on stagger-tuned amplifiers has been removed and placed in Appendix H, on the OUPI website. The cross-coupled LC oscillator, popular in IC design, has been added to Chapter 14. The section on precision diode circuits has been removed but is still made available on the website.
12. Better Organized and Modernized Coverage of Digital IC Design. Significant improvements have been made to the brief but comprehensive coverage of digital IC design in Part IV. These include a better motivated study of CMOS logic circuits (Chapter 15) which now begins with logic gate circuits. The material on logic circuit technologies and design methodologies as well as the advanced topic of technology scaling and its implications has been moved to Chapter 16. This modularly structured chapter now deals with a selection of advanced and somewhat specialized topics. Since bipolar is hardly ever used in new digital design, coverage of ECL has been significantly reduced. Similarly, BiCMOS has become somewhat of a specialty topic and its coverage has been correspondingly reduced. Nevertheless, the complete material on both ECL and BiCMOS is now available on the website. Finally, we have added a new section on image sensors to Chapter 17 (Memory Circuits).
13. A Useful and Insightful Comparison of the MOSFET and the BJT. This is now included in Appendix G, available on the website.
14. PSpice ${ }^{\circledR}$ Simulations (earlier available on the $C D$ that accompanied the sixth edition). The PSpice solutions have been uploaded on the OUPI website from where these can be accessed easily.
15. Multisim ${ }^{\mathrm{TM}}$ Simulations. The website also provides Multisim solutions to the examples and end -of-chapter problems identified with a SIM icon.

## Exercises and End-of-Chapter Problems

Over 475 exercises are integrated throughout the text. The answer to each exercise is given below the exercise so students can check their understanding of the material as they read. Solving these exercises should enable the reader to gauge his or her grasp of the preceding material. In addition, nearly 1400 end-ofchapter problems, $65 \%$ of which are new or revised in this edition, are provided. The problems are keyed to the individual chapter sections and their degree of difficulty is indicated by a rating system: difficult problems are marked with an asterisk $(*)$; more difficult problems with two asterisks ( ${ }^{* *}$ ); and very difficult (and/or time-consuming) problems with three asterisks $\left({ }^{* * *}\right)$. We must admit, however, that this classification is by no means exact. Our rating no doubt depended to some degree on our thinking (and mood!) at the time a particular problem was created. Answers to selected problems are given in Appendix L, so students have a checkpoint to tell if they are working out the problems correctly. Complete solutions for all exercises and problems are included in the Instructor's Solutions Manual, which is available from the publisher to those instructors who adopt the book.

As in the previous six editions, many examples are included. The examples, and indeed most of the problems and exercises, are based on real circuits and anticipate the applications encountered in
designing real-life circuits. This edition continues the use of numbered solution steps in the figures for many examples, as an attempt to recreate the dynamics of the classroom.

## An Outline for the Reader

Part I, Devices and Basic Circuits, includes the most fundamental and essential topics for the study of electronic circuits. At the same time, it constitutes a complete package for a first course on the subject.

The book starts with an introduction to the basic concepts of electronics in Chapter 1. Signals, their frequency spectra, and their analog and digital forms are presented. Amplifiers are introduced as circuit building blocks and their various types and models are studied. This chapter also establishes some of the terminology and conventions used throughout the text.

The second part of the chapter provides an overview of semiconductor concepts at a level sufficient for understanding the operation of diodes and transistors in later chapters. Coverage of this material is useful in particular for students who have had no prior exposure to device physics. Even those with such a background would find a review of this material beneficial as a refresher. The instructor can choose to cover this material in class or assign it for outside reading.

Chapter 2 deals with operational amplifiers, their terminal characteristics, simple applications, and practical limitations. We chose to discuss the op amp as a circuit building block at this early stage simply because it is easy to deal with and because the student can experiment with op-amp circuits that perform nontrivial tasks with relative ease and with a sense of accomplishment. We have found this approach to be highly motivating to the student. We should point out, however, that part or all of this chapter can be skipped and studied at a later stage (for instance, in conjunction with Chapter 8, Chapter 10, and/or Chapter 12) with no loss of continuity.

The first electronic device, the diode, is studied in Chapter 3. The diode terminal characteristics, the circuit models that are used to represent it, and its circuit applications are presented. Depending on the time available in the course, some of the diode applications (e.g. Section 3.6) can be skipped. Also, the brief description of special diode types (Section 3.7) can be left for the student to read.

The foundation of electronic circuits is established by the study of the two transistor types in use today: the bipolar transistor in Chapter 4 and the MOS transistor in Chapter 5. These two chapters have been written to be completely independent of one another and thus can be studied in either order, as desired. Furthermore, the two chapters have the same structure, making it easier and faster to study the second device, as well as to draw comparisons between the two device types.

Each of Chapters 4 and 5 begins with a study of the device structure and its physical operation, leading to a description of its terminal characteristics. Then, to allow the student to become very familiar with the operation of the transistor as a circuit element, a large number of examples are presented of dc circuits utilizing the device. The last section of each of Chapters 4 and 5 deals with second-order effects that are included for completeness, but that can be skipped if time does not permit detailed coverage.

The heart of a first course in electronics is the study of transistor amplifiers. Chapter 6 (new to this edition) presents a unified treatment of the subject. It begins with the basic principles that underlie the operation of a transistor, of either type, as an amplifier, and proceeds to present the important concepts of small-signal operation and modeling. This is followed by a study of the basic configurations of single-transistor amplifiers. After a presentation of dc biasing methods, the chapter concludes with practical examples of discrete-circuit amplifiers. The combined presentation emphasizes the unity of the basic principles while allowing for separate treatment of the two device types where this is warranted.

Very importantly, we are able to compare the two devices and to draw conclusions about their unique areas of application.

After the study of Part I, the reader will be fully prepared to study either integrated-circuit amplifiers in Part II, or digital integrated circuits in Part IV.

Part II, Integrated-Circuit Amplifiers, is devoted to the study of practical amplifier circuits that can be fabricated in the integrated-circuit (IC) form. Its four chapters constitute a coherent treatment of IC amplifier design and can thus serve as the major part of a second course in electronic circuits.

Throughout Part II, both MOS and bipolar circuits are presented side-by-side. Because the MOSFET is by far the dominant device, its circuits are presented first. Bipolar circuits are discussed to the same depth but occasionally more briefly.

Beginning with a brief introduction to the philosophy of IC design, Chapter 7 presents the basic circuit building blocks that are used in the design of IC amplifiers. These include current mirrors, current sources, gain cells, and cascode amplifiers.

The most important IC building block, the differential pair, is the main topic of Chapter 8. The last section of Chapter 8 is devoted to the study of multistage amplifiers.

Chapter 9 (new to this edition) presents a comprehensive treatment of the important subject of amplifier frequency response. Here, Sections 9.1, 9.2, and 9.3 contain essential material; Section 9.4 provides an in-depth treatment of very useful new tools; and Sections 9.5 through 9.8 present the frequency response analysis of a variety of amplifier configurations that can be studied as and when needed. A selection of the latter sections can be made depending on the time available and the instructor's preference.

The fourth of the essential topics of Part II, feedback, is the subject of Chapter 10. Both the theory of negative feedback and its application in the design of practical feedback amplifiers are presented. We also discuss the stability problem in feedback amplifiers and treat frequency compensation in some detail.

Part III, Analog Integrated Circuits, consists of four chapters, each of which represents a somewhat specialized application of the basic material presented in Parts I and II.

In Chapter 11 we switch gears from dealing with small-signal amplifiers to those that are required to handle large signals and large amounts of power. Here we study the different amplifier classes-A, B, and AB - and their realization in bipolar and CMOS technologies. We also consider power BJTs and power MOSFETs, and study representative IC power amplifiers. A brief study of the increasingly popular class D amplifier is also presented. Depending on the availability of time, some of the later sections can be skipped in a first reading.

Chapter 12 brings together all the topics of Chapters 7 through 11 in an important application; namely, the design of operational-amplifier circuits. We study both CMOS and bipolar op amps. In the latter category, besides the classical and still timely 741 circuit, we present modern techniques for the design of low-voltage op amps (Section 12.4).

Chapter 13 deals with the design of filters, which are important building blocks of communication and instrumentation systems. A comprehensive, design-oriented treatment of the subject is presented. The material provided should allow the reader to perform a complete filter design, starting from specification and ending with a complete circuit realization. A wealth of design tables is included.

Chapter 14 deals with circuits for the generation of signals with a variety of waveforms: sinusoidal, square, and triangular. We also present circuits for the nonlinear shaping of waveforms.

Part IV, Digital Integrated Circuits, provides a brief but nonetheless comprehensive and sufficiently detailed study of digital IC design. Our treatment is almost self-contained, requiring for the most part only a thorough understanding of the MOSFET material presented in Chapter 5. Thus, Part IV can be
studied right after Chapter 5 . The only exception to this is the last section in Chapter 16 which requires knowledge of the BJT (Chapter 4). Also, knowledge of the MOSFET internal capacitances (Section 9.2 .2 ) will be needed.

Chapter 15 is the foundation of Part IV. It begins with the motivating topic of CMOS logic-gate circuits. Then, following a detailed study of digital logic inverters, we concentrate on the CMOS inverter; its static and dynamic characteristics and its design. Transistor sizing and power dissipation round out the topics of Chapter 15. The material covered in this chapter is the minimum needed to learn something meaningful about digital circuits.

Chapter 16 has a modular structure and presents six topics of somewhat advanced nature. It begins with a presentation of Moore's law and the technology scaling that has made the multi billion-transistor chip possible. This is followed by an overview of digital IC technologies, and the design methodologies that make the design of super-complex digital ICs feasible. Four different logic-circuit types are then presented. Only the last of these includes bipolar transistors.

Digital circuits can be broadly divided into logic and memory circuits. Chapter 17 deals with the latter.

Appendices. The twelve appendices contain much useful background and supplementary material. We wish to draw the reader's attention in particular to the first two: Appendix A provides a concise introduction to the important topic of IC fabrication technology including IC layout. Appendix B provides SPICE device models as well as a large number of design and simulation examples in PSpice ${ }^{\circledR}$ and Multisim ${ }^{\mathrm{TM}}$. The examples are keyed to the book chapters. These appendices and a great deal more material on these simulation examples can be found on the OUPI website.

## Online Resources

The following resources are available at https://india.oup.com/orcs/9780199476299 to support the faculty and students using this book.

## For Faculty

- Instructor's Manual
- PowerPoint-based Figure Slides
- Simulations in Multisim ${ }^{\text {TM }}$ and PSpice ${ }^{\circledR}$


## For Students

- Appendices and Additional Material
- Data Sheets
- Simulations in Multisim ${ }^{\text {TM }}$ and PSpice ${ }^{\circledR}$

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## CHAPTER 6

## Transistor Amplifiers

## IN THIS CHAPTER YOU WILL LEARN

1. How the transistor (a MOSFET or a BJT) can be used to make an amplifier.
2. How to obtain linear amplification from the fundamentally nonlinear MOS and bipolar transistor.
3. How to model the linear operation of a transistor around a bias point by an equivalent circuit that can be used in the analysis and design of transistor amplifiers.
4. The three basic ways to connect a MOSFET or a BJT to construct amplifiers with different properties.
5. Practical circuits for MOS and bipolar transistor amplifiers that can be constructed using discrete components.

## Introduction

Having studied the two major transistor types, the BJT (Chapter 4) and the MOSFET (Chapter 5), we now begin the study of their application. There are two distinctly different kinds of transistor application: as a switch, in the design of digital circuits (Chapters 15-17) and as a controlled source, in the design of amplifiers for analog circuits. This chapter and the subsequent six focus on the latter application, namely, the use of the transistor in the design of a variety of amplifier types.

Since the basic principles that underlie the use of the MOSFET and the BJT in amplifier design are the same, the two devices are studied together in this chapter. Besides providing some economy in presentation, this unified study enables us to make important comparisons between MOS and bipolar amplifiers.

The bulk of this chapter is concerned with the fundamental principles and concepts that are the basis for the application of transistors in amplifier design: We study in detail the models that are used to represent both transistor types in the analysis and design of small-signal linear amplifiers. We also study the three basic configurations in which each of the two transistor types can be connected to realize an amplifier.

The chapter concludes with examples of discrete-circuit amplifiers. These are circuits that can be assembled using discrete transistors, resistors, and capacitors on printed-circuit boards (PCBs). They predominantly use BJTs, and their design differs in significant ways from the design of integrated-circuit (IC) amplifiers. The latter predominantly use MOSFETs, and their study begins in Chapter 7. However, the
fundamental principles and concepts introduced in this chapter apply equally well to both discrete and integrated amplifiers.

### 6.1 Basic Principles

### 6.1.1 The Basis for Amplifier Operation

The basis for the application of the transistor (a MOSFET or a BJT) in amplifier design is that when the device is operated in the active region, a voltage-controlled current source is realized. Specifically, when a MOSFET is operated in the saturation or pinch-off region, also referred to in this chapter as the active region, the voltage between gate and source, $v_{G S}$, controls the drain current $i_{D}$ according to the square-law relationship which, for an NMOS transistor, is expressed as

$$
\begin{equation*}
i_{D}=\frac{1}{2} k_{n}\left(v_{G S}-V_{t n}\right)^{2} \tag{6.1}
\end{equation*}
$$

We note that in this first-order model of MOSFET operation, the drain current $i_{D}$ does not depend on the drain voltage $v_{D S}$ because the channel is pinched off at the drain end, ${ }^{1}$ thus "isolating" the drain.

Similarly, when a BJT is operated in the active region, the base-emitter voltage $v_{B E}$ controls the collector current $i_{C}$ according to the exponential relationship which, for an npn transistor, is expressed as

$$
\begin{equation*}
i_{C}=I_{S} e^{v_{B E} / V_{T}} \tag{6.2}
\end{equation*}
$$

Here, this first-order model of BJT operation indicates that the collector current $i_{C}$ does not depend on the collector voltage $v_{\text {CE }}$ because the collector-base junction is reverse biased, thus "isolating" the collector.


Figure 6.1 Operating (a) an NMOS transistor and (b) an $n p n$ transistor in the active mode. Note that $v_{G S}=V_{t n}+v_{O V}$ and $v_{D S} \geq v_{O V}$; thus $v_{G D} \leq$ $V_{t n}$, which ensures channel pinch-off at the drain end. Similarly, $v_{B E} \simeq$ 0.7 V , and $v_{C E} \geq 0.3 \mathrm{~V}$ results in $v_{B C} \leq 0.4 \mathrm{~V}$, which is sufficient to keep the CBJ from conducting.

Figure 6.1 shows an NMOS transistor and an $n p n$ transistor operating in the active mode. Observe that for the NMOS transistor, the pinch-off condition is ensured by keeping $v_{D S} \geq v_{O V}$. Since the overdrive voltage $v_{O V}=$ $v_{G S}-V_{t n}$, this condition implies that $v_{G D} \leq$ $V_{t n}$, which indeed ensures channel pinch-off at the drain end.

Similarly, for the $n p n$ transistor in Fig. 6.1(b), the CBJ reverse-bias condition is ensured by keeping $v_{C E} \geq 0.3 \mathrm{~V}$. Since $v_{B E}$ is usually in the vicinity of $0.7 \mathrm{~V}, v_{B C}$ is thus kept smaller than 0.4 V , which is sufficient to prevent this relatively large-area junction from conducting.

Although we used NMOS and npn transistors to illustrate the conditions for activemode operation, similar conditions apply for PMOS and pnp transistors, as studied in Chapters 5 and 4, respectively.
Finally, we note that the control relationships in Eqs. (6.1) and (6.2) are nonlinear. Nevertheless, we shall shortly devise a technique for obtaining almost-linear amplification from these fundamentally nonlinear devices.

[^0]
### 6.1.2 Obtaining a Voltage Amplifier

From the above we see that the transistor is basically a transconductance amplifier: that is, an amplifier whose input signal is a voltage and whose output signal is a current. More commonly, however, one is interested in voltage amplifiers. A simple way to convert a transconductance amplifier to a voltage amplifier is to pass the output current through a resistor and take the voltage across the resistor as the output. Doing this for a MOSFET results in the simple amplifier circuit shown in Fig. 6.2(a). Here $v_{G S}$ is the input voltage, $R_{D}$ (known as a load resistance) converts the drain current $i_{D}$ to a voltage ( $i_{D} R_{D}$ ), and $V_{D D}$ is the supply voltage that powers up the amplifier and, together with $R_{D}$, establishes operation in the active region, as will be shown shortly.

In the amplifier circuit of Fig. 6.2(a) the output voltage is taken between the drain and ground, rather than simply across $R_{D}$. This is done because of the need to maintain a common ground reference between the input and the output. The output voltage $v_{D S}$ is given by

$$
\begin{equation*}
v_{D S}=V_{D D}-i_{D} R_{D} \tag{6.3}
\end{equation*}
$$

Thus it is an inverted version (note the minus sign) of $i_{D} R_{D}$ that is shifted by the constant value of the supply voltage $V_{D D}$.

An exactly similar arrangement applies for the BJT amplifier, as illustrated in Fig. 6.2(c). Here the output voltage $v_{C E}$ is given by

$$
\begin{equation*}
v_{C E}=V_{C C}-i_{C} R_{C} \tag{6.4}
\end{equation*}
$$

### 6.1.3 The Voltage-Transfer Characteristic (VTC)

A useful tool that provides insight into the operation of an amplifier circuit is its voltage-transfer characteristic (VTC). This is simply a plot (or a clearly labeled sketch) of the output voltage versus the input voltage. For the MOS amplifier in Fig. 6.2(a), this is the plot of $v_{D S}$ versus $v_{G S}$ shown in Fig. 6.2(b).

Observe that for $v_{G S}<V_{t}$, the transistor is cut off, $i_{D}=0$ and, from Eq. (6.3), $v_{D S}=V_{D D}$. As $v_{G S}$ exceeds $V_{t}$, the transistor turns on and $v_{D S}$ decreases. However, since initially $v_{D S}$ is still high, the MOSFET will be operating in saturation or the active region. This continues as $v_{G S}$ is increased until the value of $v_{G S}$ is reached that results in $v_{D S}$ becoming lower than $v_{G S}$ by $V_{t}$ volts (point B on the VTC in Fig. 6.2b). For $v_{G S}$ greater than that at point B , the transistor operates in the triode region and $v_{D S}$ decreases more slowly.


Figure 6.2 (a) An NMOS amplifier and (b) its VTC; and (c) an $n p n$ amplifier and (d) its VTC.

(c)

(d)

Figure 6.2 continued

The VTC in Fig. 6.2(b) indicates that the segment of greatest slope (hence potentially the largest amplifier gain) is that labeled $A B$, which corresponds to operation in the active region. When a MOSFET is operated as an amplifier, its operating point is confined to the segment AB at all times. An expression for the segment AB can be obtained by substituting for $i_{D}$ in Eq. (6.3) by its active-region value from Eq. (6.1), thus

$$
\begin{equation*}
v_{D S}=V_{D D}-\frac{1}{2} k_{n} R_{D}\left(v_{G S}-V_{t}\right)^{2} \tag{6.5}
\end{equation*}
$$

This is obviously a nonlinear relationship. Nevertheless, linear (or almost-linear) amplification can be obtained by using the technique of biasing the MOSFET. Before considering biasing, however, it is useful to determine the coordinates of point B , which is at the boundary between the saturation and the triode regions of operation. These can be obtained by substituting in Eq. (6.5), $v_{G S}=\left.V_{G S}\right|_{\mathrm{B}}$ and $v_{D S}=\left.V_{D S}\right|_{\mathrm{B}}=\left.V_{G S}\right|_{\mathrm{B}}-V_{t}$. The result is

$$
\begin{equation*}
\left.V_{G S}\right|_{\mathrm{B}}=V_{t}+\frac{\sqrt{2 k_{n} R_{D} V_{D D}+1}-1}{k_{n} R_{D}} \tag{6.6}
\end{equation*}
$$

Point B can be alternatively characterized by the overdrive voltage

$$
\begin{equation*}
\left.\left.V_{O V}\right|_{\mathrm{B}} \equiv V_{G S}\right|_{\mathrm{B}}-V_{t}=\frac{\sqrt{2 k_{n} R_{D} V_{D D}+1}-1}{k_{n} R_{D}} \tag{6.7}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.V_{D S}\right|_{\mathrm{B}}=\left.V_{O V}\right|_{\mathrm{B}} \tag{6.8}
\end{equation*}
$$

An exactly similar development applies to the BJT case. This is illustrated in Fig. 6.2(c) and (d). In this case, over the active-region or amplifier segment AB , the output voltage $v_{C E}$ is related to the input voltage $v_{B E}$ by

$$
\begin{equation*}
v_{C E}=V_{C C}-R_{C} I_{S} e^{v_{B E} / V_{T}} \tag{6.9}
\end{equation*}
$$

Here also, the input-output relationship is nonlinear. Nevertheless, linear (or almost-linear) amplification can be obtained by using the biasing technique discussed next.

## EXERCISE

6.1 Consider the amplifier of Fig. 6.2(a) with $V_{D D}=1.8 \mathrm{~V}, R_{D}=17.5 \mathrm{k} \Omega$, and with a MOSFET specified to have $V_{t}=0.4 \mathrm{~V}, k_{n}=4 \mathrm{~mA} / \mathrm{V}^{2}$, and $\lambda=0$. Determine the coordinates of the end points of the active-region segment of the VTC. Also, determine $\left.V_{D S}\right|_{C}$ assuming $\left.V_{G S}\right|_{C}=V_{D D}$.

$$
\text { Ans. A: } 0.4 \mathrm{~V}, 1.8 \mathrm{~V} ; \mathrm{B}: 0.613 \mathrm{~V}, 0.213 \mathrm{~V} ;\left.V_{D S}\right|_{C}=18 \mathrm{mV}
$$

### 6.1.4 Obtaining Linear Amplification by Biasing the Transistor

Biasing enables us to obtain almost-linear amplification from the MOSFET and the BJT. The technique is illustrated for the MOSFET case in Fig. 6.3(a). A dc voltage $V_{G S}$ is selected to obtain operation at a point Q on the segment AB of the VTC. How to select an appropriate location for the bias point Q will be discussed shortly. For the time being, observe that the coordinates of Q are the dc voltages $V_{G S}$ and $V_{D S}$, which are related by

$$
\begin{equation*}
V_{D S}=V_{D D}-\frac{1}{2} k_{n} R_{D}\left(V_{G S}-V_{t}\right)^{2} \tag{6.10}
\end{equation*}
$$

Point Q is known as the bias point or the dc operating point. Also, since at Q no signal component is present, it is also known as the quiescent point (which is the origin of the symbol Q).

Next, the signal to be amplified, $v_{g s}$, a function of time $t$, is superimposed on the bias voltage $V_{G S}$, as shown in Fig. 6.4(a). Thus the total instantaneous value of $v_{G S}$ becomes

$$
v_{G S}(t)=V_{G S}+v_{g s}(t)
$$


(a)

(b)

Figure 6.3 Biasing the MOSFET amplifier at a point Q located on the segment AB of the VTC.

The resulting $v_{D S}(t)$ can be obtained by substituting for $v_{G S}(t)$ into Eq. (6.5). Graphically, we can use the VTC to obtain $v_{D S}(t)$ point by point, as illustrated in Fig. 6.4(b). Here we show the case of $v_{g s}$ being a triangular wave of "small" amplitude. Specifically, the amplitude of $v_{g s}$ is small enough to restrict the excursion of the instantaneous operating point to a short, almost-linear segment of the VTC around the bias point Q . The shorter the segment, the greater the linearity achieved, and the closer to an ideal triangular wave the signal component at the output, $v_{d s}$, will be. This is the essence of obtaining linear amplification from the nonlinear MOSFET.

Before leaving Fig. 6.4(b) we wish to draw the reader's attention to the consequence of increasing the amplitude of the signal $v_{g s}$. As the instantaneous operating point will no longer be confined to the almost-linear segment of the VTC, the output signal $v_{d s}$ will deviate from its ideal triangular shape; that is, it will exhibit nonlinear distortion. Worse yet, if the input signal amplitude becomes sufficiently large, the instantaneous operating point may leave the segment AB altogether. If this happens at the negative peaks of $v_{g s}$, the transistor will cut off for a portion of the cycle and the positive peaks of $v_{d s}$ will be "clipped off." If it occurs at the positive peaks of $v_{g s}$, the transistor will enter the triode region for a


Figure 6.4 The MOSFET amplifier with a small time-varying signal $v_{g s}(t)$ superimposed on the dc bias voltage $V_{G S}$. The MOSFET operates on a short almost-linear segment of the VTC around the bias point Q and provides an output voltage $v_{d s}=A_{v} v_{g s}$.
portion of the cycle, and the negative peaks of $v_{d s}$ will become flattened. It follows that the selection of the location of the bias point Q can have a profound effect on the maximum allowable amplitude of $v_{d s}$, referred to as the allowable signal swing at the output. We will have more to say later on this important point.

An exactly parallel development can be applied to the BJT amplifier. In fact, all we need to do is replace the NMOS transistor in Figs. 6.3 and 6.4 with an $n p n$ transistor and change the voltage and current symbols to their BJT counterparts. The resulting bias point Q will be characterized by dc voltages $V_{B E}$ and $V_{C E}$, which are related by

$$
\begin{equation*}
V_{C E}=V_{C C}-R_{C} I_{S} e^{V_{B E} / V_{T}} \tag{6.11}
\end{equation*}
$$

and a dc current $I_{C}$,

$$
\begin{equation*}
I_{C}=I_{S} e^{V_{B E} / V_{T}} \tag{6.12}
\end{equation*}
$$

Also, superimposing a small-signal $v_{b e}$ on the dc bias voltage $V_{B E}$ results in

$$
v_{B E}(t)=V_{B E}+v_{b e}(t)
$$

which can be substituted into Eq. (6.9) to obtain the total instantaneous value of the output voltage $v_{C E}(t)$. Here again, almost-linear operation is obtained by keeping $v_{b e}$ small enough to restrict the excursion of the instantaneous operating point to a short, almost-linear segment of the VTC around the bias point Q. Similar comments also apply to the maximum allowable signal swing at the output.

### 6.1.5 The Small-Signal Voltage Gain

The MOSFET Case Consider the MOSFET amplifier in Fig. 6.4(a). If the input signal $v_{g s}$ is kept small, the corresponding signal at the output $v_{d s}$ will be nearly proportional to $v_{g s}$ with the constant of proportionality being the slope of the almost-linear segment of the VTC around Q . This is the voltage gain of the amplifier, and its value can be determined by evaluating the slope of the tangent to the VTC at the bias point Q ,

$$
\begin{equation*}
A_{v}=\left.\frac{d v_{D S}}{d v_{G S}}\right|_{v_{G S}=V_{G S}} \tag{6.13}
\end{equation*}
$$

Utilizing Eq. (6.5) we obtain

$$
\begin{equation*}
A_{v}=-k_{n}\left(V_{G S}-V_{t}\right) R_{D} \tag{6.14}
\end{equation*}
$$

which can be expressed in terms of the overdrive voltage at the bias point, $V_{O V}$, as

$$
\begin{equation*}
A_{v}=-k_{n} V_{O V} R_{D} \tag{6.15}
\end{equation*}
$$

We make the following observations on this expression for the voltage gain.

1. The gain is negative, which signifies that the amplifier is inverting; that is, there is a $180^{\circ}$ phase shift between the input and the output. This inversion is obvious in Fig. 6.4(b) and should have been anticipated from Eq. (6.5).
2. The gain is proportional to the load resistance $R_{D}$, to the transistor transconductance parameter $k_{n}$, and to the overdrive voltage $V_{O V}$. This all makes intuitive sense.

Another simple and insightful expression for the voltage gain $A_{v}$ can be derived by recalling that the dc current in the drain at the bias point is related to $V_{O V}$ by

$$
I_{D}=\frac{1}{2} k_{n} V_{O V}^{2}
$$

This equation can be combined with Eq. (6.15) to obtain

$$
\begin{equation*}
A_{v}=-\frac{I_{D} R_{D}}{V_{O V} / 2} \tag{6.16}
\end{equation*}
$$

That is, the gain is simply the ratio of the dc voltage drop across the load resistance $R_{D}$ to $V_{O V} / 2$. It can be expressed in the alternative form

$$
\begin{equation*}
A_{v}=-\frac{V_{D D}-V_{D S}}{V_{O V} / 2} \tag{6.17}
\end{equation*}
$$

Since the maximum slope of the VTC in Fig. 6.4(b) occurs at point B, the maximum gain magnitude $\left|A_{v \text { max }}\right|$ is obtained by biasing the transistor at point B ,

$$
\left|A_{v \max }\right|=\frac{V_{D D}-\left.V_{D S}\right|_{\mathrm{B}}}{\left.V_{O V}\right|_{\mathrm{B}} / 2}
$$

and since $\left.V_{D S}\right|_{\mathrm{B}}=\left.V_{O V}\right|_{\mathrm{B}}$,

$$
\begin{equation*}
\left|A_{v \max }\right|=\frac{V_{D D}-\left.V_{O V}\right|_{\mathrm{B}}}{\left.V_{O V}\right|_{\mathrm{B}} / 2} \tag{6.18}
\end{equation*}
$$

where $\left.V_{O V}\right|_{\mathrm{B}}$ is given by Eq. (6.7). Of course, this result is only of theoretical importance since biasing at B would leave no room for negative signal swing at the output. Nevertheless, the result in Eq. (6.18) is valuable as it provides an upper bound on the magnitude of voltage gain achievable from this basic amplifier circuit. As an example, for a discrete-circuit amplifier operated with $V_{D D}=5 \mathrm{~V}$ and $\left.V_{O V}\right|_{\mathrm{B}}=0.5 \mathrm{~V}$, the maximum achievable gain is $18 \mathrm{~V} / \mathrm{V}$. An integrated-circuit amplifier utilizing a modern submicron MOSFET operated with $V_{D D}=1.3 \mathrm{~V}$ and with $\left.V_{O V}\right|_{\mathrm{B}}=0.2 \mathrm{~V}$ realizes a maximum gain of $11 \mathrm{~V} / \mathrm{V}$.

Finally, note that to maximize the gain, the bias point Q should be as close to point B as possible, consistent with the required signal swing at the output. This point will be explored further in the end-of-chapter problems.

## Example 6.1

Consider the amplifier circuit shown in Fig. 6.4(a). The transistor is specified to have $V_{t}=0.4 \mathrm{~V}$, $k_{n}^{\prime}=0.4 \mathrm{~mA} / \mathrm{V}^{2}, W / L=10$, and $\lambda=0$. Also, let $V_{D D}=1.8 \mathrm{~V}, R_{D}=17.5 \mathrm{k} \Omega$, and $V_{G S}=0.6 \mathrm{~V}$.
(a) For $v_{g s}=0$ (and hence $v_{d s}=0$ ), find $V_{O V}, I_{D}, V_{D S}$, and $A_{v}$.
(b) What is the maximum symmetrical signal swing allowed at the drain? Hence, find the maximum allowable amplitude of a sinusoidal $v_{g s}$.

## Example 6.1 continued

## Solution

(a) With $V_{G S}=0.6 \mathrm{~V}, V_{O V}=0.6-0.4=0.2 \mathrm{~V}$. Thus,

$$
\begin{aligned}
I_{D} & =\frac{1}{2} k_{n}^{\prime}\left(\frac{W}{L}\right) V_{O V}^{2} \\
& =\frac{1}{2} \times 0.4 \times 10 \times 0.2^{2}=0.08 \mathrm{~mA} \\
V_{D S} & =V_{D D}-R_{D} I_{D} \\
& =1.8-17.5 \times 0.08=0.4 \mathrm{~V}
\end{aligned}
$$

Since $V_{D S}$ is greater than $V_{O V}$, the transistor is indeed operating in saturation. The voltage gain can be found from Eq. (6.15),

$$
\begin{aligned}
A_{v} & =-k_{n} V_{O V} R_{D} \\
& =-0.4 \times 10 \times 0.2 \times 17.5=-14 \mathrm{~V} / \mathrm{V}
\end{aligned}
$$

An identical result can be found using Eq. (6.17).
(b) Since $V_{O V}=0.2 \mathrm{~V}$ and $V_{D S}=0.4 \mathrm{~V}$, we see that the maximum allowable negative signal swing at the drain is 0.2 V . In the positive direction, a swing of +0.2 V would not cause the transistor to cut off (since the resulting $v_{D S}$ would be still lower than $V_{D D}$ ) and thus is allowed. Thus the maximum symmetrical signal swing allowable at the drain is $\pm 0.2 \mathrm{~V}$. The corresponding amplitude of $v_{g s}$ can be found from

$$
\hat{v}_{g s}=\frac{\hat{v}_{d s}}{\left|A_{v}\right|}=\frac{0.2 \mathrm{~V}}{14}=14.2 \mathrm{mV}
$$

Since $\hat{v}_{g s} \ll V_{O V}$, the operation will be reasonably linear (more on this in later sections).
Greater insight into the issue of allowable signal swing can be obtained by examining the signal waveforms shown in Fig. 6.5. Note that for the MOSFET to remain in saturation at the negative peak of $v_{d s}$, we must ensure that

$$
v_{D S \text { min }} \geq v_{G S \text { max }}-V_{t}
$$

that is,

$$
0.4-\left|A_{v}\right| \hat{v}_{g s} \geq 0.6+\hat{v}_{g s}-0.4
$$

which results in

$$
\hat{v}_{g s} \leq \frac{0.2}{\left|A_{v}\right|+1}=13.3 \mathrm{mV}
$$

This result differs slightly from the one obtained earlier.

## Example 6.1 continued



Figure 6.5 Signal waveforms at gate and drain for the amplifier in Example 6.1. Note that to ensure operation in the saturation region at all times, $v_{D S \text { min }} \geq v_{G S \text { max }}-V_{t}$.

## EXERCISE

D6.2 For the amplifier circuit studied in Example 6.1, create two alternative designs, each providing a voltage gain of -10 by (a) changing $R_{D}$ while keeping $V_{O V}$ constant and (b) changing $V_{O V}$ while keeping $R_{D}$ constant. For each design, specify $V_{G S}, I_{D}, R_{D}$, and $V_{D S}$.

Ans. (a) $0.6 \mathrm{~V}, 0.08 \mathrm{~mA}, 12.5 \mathrm{k} \Omega, 0.8 \mathrm{~V}$; (b) $0.54 \mathrm{~V}, 0.04 \mathrm{~mA}, 17.5 \mathrm{k} \Omega, 1.1 \mathrm{~V}$

The BJT Case A similar development can be used to obtain the small-signal voltage gain of the BJT amplifier shown in Fig. 6.6,

$$
\begin{equation*}
A_{v}=\left.\frac{d v_{C E}}{d v_{B E}}\right|_{v_{B E}=V_{B E}} \tag{6.19}
\end{equation*}
$$

Utilizing Eq. (6.9) together with Eq. (6.12), we obtain

$$
\begin{equation*}
A_{v}=-\left(\frac{I_{C}}{V_{T}}\right) R_{C} \tag{6.20}
\end{equation*}
$$



Figure 6.6 BJT amplifier biased at a point Q , with a small voltage signal $v_{b e}$ superimposed on the dc bias voltage $V_{B E}$. The resulting output signal $v_{c e}$ appears superimposed on the dc collector voltage $V_{C E}$. The amplitude of $v_{c e}$ is larger than that of $v_{b e}$ by the voltage gain $A_{v}$.

We make the following observations on this expression for the voltage gain:

1. The gain is negative, which signifies that the amplifier is inverting; that is, there is a $180^{\circ}$ phase shift between the input and the output. This inversion should have been anticipated from Eq. (6.9).
2. The gain is proportional to the collector bias current $I_{C}$ and to the load resistance $R_{C}$.

Additional insight into the voltage gain $A_{v}$ can be obtained by expressing Eq. (6.20) as

$$
\begin{equation*}
A_{v}=-\frac{I_{C} R_{C}}{V_{T}} \tag{6.21}
\end{equation*}
$$

That is, the gain is the ratio of the dc voltage drop across the load resistance $R_{C}$ to the physical constant $V_{T}$ (recall that the thermal voltage $V_{T} \simeq 25 \mathrm{mV}$ at room temperature). This relationship is similar in form to that for the MOSFET (Eq. 6.16) except that here the denominator is a physical constant $\left(V_{T}\right)$ rather than a design parameter $\left(V_{O V} / 2\right)$. Usually, $V_{O V} / 2$ is larger than $\left(V_{T}\right)$, thus we can obtain higher voltage gain from the BJT amplifier than from the MOSFET amplifier. This should not be surprising, as the exponential $i_{C}-v_{B E}$ relationship is much steeper than the square-law relationship $i_{D}-v_{G S}$.
The gain $A_{v}$ in Eq. (6.21) can be expressed alternately as

$$
\begin{equation*}
A_{v}=-\frac{V_{C C}-V_{C E}}{V_{T}} \tag{6.22}
\end{equation*}
$$

from which we see that maximum gain is achieved when $V_{C E}$ is at its minimum value of about 0.3 V ,

$$
\begin{equation*}
\left|A_{v \max }\right|=\frac{V_{C C}-0.3}{V_{T}} \tag{6.23}
\end{equation*}
$$

Here again, this is only a theoretical maximum, since biasing the BJT at the edge of saturation leaves no room for negative signal swing at the output. Equation (6.23) nevertheless provides an upper bound on the voltage gain achievable from the basic BJT amplifier. As an example, for $V_{C C}=5 \mathrm{~V}$, the maximum gain is $188 \mathrm{~V} / \mathrm{V}$, considerably larger than in the MOSFET case. For modern low-voltage technologies, a $V_{C C}$ of 1.3 V provides a gain of $40 \mathrm{~V} / \mathrm{V}$, again much larger than the MOSFET case. The reader should not, however, jump to the conclusion that the BJT is preferred to the MOSFET in the design of modern integrated-circuit amplifiers; in fact, the opposite is true, as we shall see in Chapter 7 and beyond.

Finally, we conclude from Eq. (6.22) that to maximize $\left|A_{v}\right|$ the transistor should be biased at the lowest possible $V_{C E}$ consistent with the desired value of negative signal swing at the output.

## Example 6.2

Consider an amplifier circuit using a BJT having $I_{S}=10^{-15} \mathrm{~A}$, a collector resistance $R_{C}=6.8 \mathrm{k} \Omega$, and a power supply $V_{C C}=10 \mathrm{~V}$.

## Example 6.2 continued

(a) Determine the value of the bias voltage $V_{B E}$ required to operate the transistor at $V_{C E}=3.2 \mathrm{~V}$. What is the corresponding value of $I_{C}$ ?
(b) Find the voltage gain $A_{v}$ at this bias point. If an input sine-wave signal of $5-\mathrm{mV}$ peak amplitude is superimposed on $V_{B E}$, find the amplitude of the output sine-wave signal (assume linear operation).
(c) Find the positive increment in $v_{B E}$ (above $V_{B E}$ ) that drives the transistor to the edge of saturation, where $v_{C E}=0.3 \mathrm{~V}$.
(d) Find the negative increment in $v_{B E}$ that drives the transistor to within $1 \%$ of cutoff (i.e., to $v_{C E}=0.99 V_{C C}$ ).

## Solution

(a)

$$
I_{C}=\frac{V_{C C}-V_{C E}}{R_{C}}=\frac{10-3.2}{6.8}=1 \mathrm{~mA}
$$

The value of $V_{B E}$ can be determined from

$$
1 \times 10^{-3}=10^{-15} e^{V_{B E} / V_{T}}
$$

which results in
(b)

$$
\begin{aligned}
& A_{v}=-\frac{V_{C C}-V_{C E}}{V_{T}}=\frac{10-3.2}{0.025}=-272 \mathrm{~V} / \mathrm{V} \\
& \hat{v}_{c e}=272 \times 0.005=1.36 \mathrm{~V}
\end{aligned}
$$

(c) For $v_{C E}=0.3 \mathrm{~V}$,

$$
i_{C}=\frac{10-0.3}{6.8}=1.617 \mathrm{~mA}
$$

To increase $i_{C}$ from 1 mA to $1.617 \mathrm{~mA}, v_{B E}$ must be increased by

$$
\Delta v_{B E}=V_{T} \ln \left(\frac{1.617}{1}\right)=12 \mathrm{mV}
$$

(d) For $v_{C E}=0.99 V_{C C}=9.9 \mathrm{~V}$,

$$
i_{C}=\frac{10-9.9}{6.8}=0.0147 \mathrm{~mA}
$$

To decrease $i_{C}$ from 1 mA to $0.0147 \mathrm{~mA}, v_{B E}$ must change by

$$
\Delta v_{B E}=V_{T} \ln \left(\frac{0.0147}{1}\right)=-105.5 \mathrm{mV}
$$

## EXERCISE

6.3 For the situation described in Example 6.2, while keeping $I_{C}$ unchanged at 1 mA , find the value of $R_{C}$ that will result in a voltage gain of $-320 \mathrm{~V} / \mathrm{V}$. What is the largest negative signal swing allowed at the output (assume that $v_{C E}$ is not to decrease below 0.3 V )? What (approximately) is the corresponding input signal amplitude? (Assume linear operation.)


Figure 6.7 Graphical construction to determine the voltage-transfer characteristic of the amplifier in Fig. 6.4(a).

### 6.1.6 Determining the VTC by Graphical Analysis

Figure 6.7 shows a graphical method for determining the VTC of the amplifier of Fig. 6.4(a). Although graphical analysis of transistor circuits is rarely employed in practice, it is useful to us at this stage for gaining greater insight into circuit operation, especially in answering the question of where to locate the bias point Q .

The graphical analysis is based on the observation that for each value of $v_{G S}$, the circuit will be operating at the point of intersection of the $i_{D}-v_{D S}$ graph corresponding to the particular value of $v_{G S}$ and the straight line representing Eq. (6.3), which can be rewritten in the form

$$
\begin{equation*}
i_{D}=\frac{V_{D D}}{R_{D}}-\frac{1}{R_{D}} v_{D S} \tag{6.24}
\end{equation*}
$$

The straight line representing this relationship is superimposed on the $i_{D}-v_{D S}$ characteristics in Fig. 6.7. It intersects the horizontal axis at $v_{D S}=V_{D D}$ and has a slope of $-1 / R_{D}$. Since this straight line represents in effect the load resistance $R_{D}$, it is called the load line. The VTC is then determined point by point. Note that we have labeled four important points: point A at which $v_{G S}=V_{t}$, point Q at which the MOSFET can be biased for amplifier operation $\left(v_{G S}=V_{G S}\right.$ and $\left.v_{D S}=V_{D S}\right)$, point B at which the MOSFET leaves saturation and enters the triode region, and point C , which is deep into the triode region and for which $v_{G S}=V_{D D}$. If the MOSFET is to be used as a switch, then operating points A and C are applicable: At A the transistor is off (open switch), and at C the transistor operates as a low-valued resistance $r_{D S}$ and has a small voltage drop (closed switch). The incremental resistance at point C is also known as the closure resistance. The operation of the MOSFET as a switch is illustrated in Fig. 6.8. A detailed study of the application of the MOSFET as a switch is undertaken in Chapter 15, dealing with CMOS digital logic circuits.


Figure 6.8 Operation of the MOSFET in Fig. 6.4(a) as a switch: (a) open, corresponding to point A in Fig. 6.7; (b) closed, corresponding to point C in Fig. 6.7. The closure resistance is approximately equal to $r_{D S}$ because $V_{D S}$ is usually very small.

The graphical analysis method above can be applied to determine the VTC of the BJT amplifier in Fig. 6.2(c). Here point A, Fig. 6.2(d), corresponds to the BJT just turning on ( $v_{B E} \simeq 0.5 \mathrm{~V}$ ) and point B corresponds to the BJT leaving the active region and entering the saturation region. If the BJT is to be operated as a switch, the two modes of operation are cutoff (open switch) and saturation (closed switch). As discussed in Section 4.2, in saturation, the BJT has a small closure resistance $R_{\text {CEsat }}$ as well as an offset voltage. More seriously, switching the BJT out of its saturation region can require a relatively long delay time to ensure the removal of the charge stored in the BJT base region. This phenomenon has made the BJT much less attractive in digital logic applications relative to the MOSFET. ${ }^{2}$

### 6.1.7 Deciding on a Location for the Bias Point $\mathbf{Q}$

For the MOSFET amplifier, the bias point Q is determined by the value of $V_{G S}$ and that of the load resistance $R_{D}$. Two important considerations in deciding on the location of Q are the required gain and the desired signal swing at the output. To illustrate, consider the VTC shown in Fig. 6.4(b). Here the value of $R_{D}$ is fixed and the only variable remaining is the value of $V_{G S}$. Since the slope increases as we move closer to point B , we obtain higher gain by locating Q as close to B as possible. However, the closer Q is to the boundary point B , the smaller the allowable magnitude of negative signal swing. Thus, as often happens in engineering design, we encounter a situation requiring a trade-off. The answer here is relatively simple: For a given $R_{D}$, locate Q as close to the triode region (point B) as possible to obtain high gain but sufficiently distant to allow for the required negative signal swing.

In deciding on a value for $R_{D}$, it is useful to refer to the $i_{D}-v_{D S}$ plane. Figure 6.9 shows two load lines resulting in two extreme bias points: Point $\mathrm{Q}_{1}$ is too close to $V_{D D}$, resulting in a severe constraint on the positive signal swing of $v_{d s}$. Exceeding the allowable positive maximum results in the positive peaks of the signal being clipped off, since the MOSFET will turn off for the part of each cycle near the positive peak. We speak of this situation by saying that the circuit does not have sufficient "headroom." Similarly, point $Q_{2}$ is too close to the boundary of the triode region, thus severely limiting the allowable negative signal swing of $v_{d s}$. Exceeding this limit would result in the transistor entering the triode region for part of each cycle near the negative peaks, resulting in a distorted output signal. In this situation we say that the circuit does not have sufficient "legroom." We will have more to say on bias design in Section 6.4.

Finally, we note that exactly similar considerations apply to the case of the BJT amplifier.

### 6.2 Small-Signal Operation and Models

In our study of the operation of the MOSFET and BJT amplifiers in Section 6.1 we learned that linear amplification can be obtained by biasing the transistor to operate in the active region and by keeping the input signal small. In this section, we explore the small-signal operation in greater detail.

[^1]

Figure 6.9 Two load lines and corresponding bias points. Bias point $Q_{1}$ does not leave sufficient room for positive signal swing at the drain (too close to $V_{D D}$ ). Bias point $\mathrm{Q}_{2}$ is too close to the boundary of the triode region and might not allow for sufficient negative signal swing.

### 6.2.1 The MOSFET Case

Consider the conceptual amplifier circuit shown in Fig. 6.10. Here the MOS transistor is biased by applying a dc voltage ${ }^{3} V_{G S}$, and the input signal to be amplified, $v_{g s}$, is superimposed on the dc bias voltage $V_{G S}$. The output voltage is taken at the drain.

The DC Bias Point The de bias current $I_{D}$ can be found by setting the signal $v_{g s}$ to zero; thus,

$$
\begin{equation*}
I_{D}=\frac{1}{2} k_{n}\left(V_{G S}-V_{t}\right)^{2}=\frac{1}{2} k_{n} V_{O V}^{2} \tag{6.25}
\end{equation*}
$$

where we have neglected channel-length modulation (i.e., we have assumed $\lambda=0$ ). Here $V_{O V}=V_{G S}-V_{t}$ is the overdrive voltage at which the MOSFET is biased to operate. The dc voltage at the drain, $V_{D S}$, will be

$$
\begin{equation*}
V_{D S}=V_{D D}-R_{D} I_{D} \tag{6.26}
\end{equation*}
$$

To ensure saturation-region operation, we must have

$$
V_{D S}>V_{O V}
$$

Furthermore, since the total voltage at the drain will have a signal component superimposed on $V_{D S}, V_{D S}$ has to be sufficiently greater than $V_{O V}$ to allow for the required negative signal swing.

[^2]The Signal Current in the Drain Terminal Next, consider the situation with the input signal $v_{g s}$ applied. The total instantaneous gate-to-source voltage will be

$$
\begin{equation*}
v_{G S}=V_{G S}+v_{g s} \tag{6.27}
\end{equation*}
$$

resulting in a total instantaneous drain current $i_{D}$,

$$
\begin{align*}
i_{D} & =\frac{1}{2} k_{n}\left(V_{G S}+v_{g s}-V_{t}\right)^{2} \\
& =\frac{1}{2} k_{n}\left(V_{G S}-V_{t}\right)^{2}+k_{n}\left(V_{G S}-V_{t}\right) v_{g s}+\frac{1}{2} k_{n} v_{g s}^{2} \tag{6.28}
\end{align*}
$$

The first term on the right-hand side of Eq. (6.28) can be recognized as the dc bias current $I_{D}$ (Eq. 6.25). The second term represents a current component that is directly proportional to the input signal $v_{g s}$. The third term is a current component that is proportional to the square of the input signal. This last component is undesirable because it represents nonlinear distortion. To reduce the nonlinear distortion introduced by the MOSFET, the input signal should be kept small so that

$$
\frac{1}{2} k_{n} v_{g s}^{2} \ll k_{n}\left(V_{G S}-V_{t}\right) v_{g s}
$$

resulting in

$$
\begin{equation*}
v_{g s} \ll 2\left(V_{G S}-V_{t}\right) \tag{6.29}
\end{equation*}
$$

or, equivalently,

$$
\begin{equation*}
v_{g s} \ll 2 V_{o v} \tag{6.30}
\end{equation*}
$$

If this small-signal condition is satisfied, we may neglect the last term in Eq. (6.28) and express $i_{D}$ as

$$
\begin{equation*}
i_{D} \simeq I_{D}+i_{d} \tag{6.31}
\end{equation*}
$$



Figure 6.10 Conceptual circuit utilized to study the operation of the MOSFET as a small-signal amplifier.


Figure 6.11 Small-signal operation of the MOSFET amplifier.
where

$$
i_{d}=k_{n}\left(V_{G S}-V_{t}\right) v_{g s}
$$

The parameter that relates $i_{d}$ and $v_{g s}$ is the MOSFET transconductance $g_{m}$,

$$
\begin{equation*}
g_{m} \equiv \frac{i_{d}}{v_{g s}}=k_{n}\left(V_{G S}-V_{t}\right) \tag{6.32}
\end{equation*}
$$

or in terms of the overdrive voltage $V_{O V}$,

$$
\begin{equation*}
g_{m}=k_{n} V_{O V} \tag{6.33}
\end{equation*}
$$

Figure 6.11 presents a graphical interpretation of the small-signal operation of the MOSFET amplifier. Note that $g_{m}$ is equal to the slope of the $i_{D}-v_{G S}$ characteristic at the bias point,

$$
\begin{equation*}
\left.g_{m} \equiv \frac{\partial i_{D}}{\partial v_{G S}}\right|_{v_{G S}=V_{G S}} \tag{6.34}
\end{equation*}
$$

This is the formal definition of $g_{m}$, which can be shown to yield the expressions given in Eqs. (6.32) and (6.33).

The Voltage Gain Returning to the circuit of Fig. 6.10, we can express the total instantaneous drain voltage $v_{D S}$ as follows:

$$
v_{D S}=V_{D D}-R_{D} i_{D}
$$

Under the small-signal condition, we have

$$
v_{D S}=V_{D D}-R_{D}\left(I_{D}+i_{d}\right)
$$

which can be rewritten as

$$
v_{D S}=V_{D S}-R_{D} i_{d}
$$

Thus the signal component of the drain voltage is

$$
\begin{equation*}
v_{d s}=-i_{d} R_{D}=-g_{m} v_{g s} R_{D} \tag{6.35}
\end{equation*}
$$

which indicates that the voltage gain is given by

$$
\begin{equation*}
A_{v} \equiv \frac{v_{d s}}{v_{g s}}=-g_{m} R_{D} \tag{6.36}
\end{equation*}
$$

The minus sign in Eq. (6.36) indicates that the output signal $v_{d s}$ is $180^{\circ}$ out of phase with respect to the input signal $v_{g s}$. This is illustrated in Fig. 6.12, which shows $v_{G S}$ and $v_{D S}$. The input signal is assumed to have a triangular waveform with an amplitude much smaller than $2\left(V_{G S}-V_{t}\right)$, the small-signal condition in Eq. (6.29), to ensure linear operation. For operation in the saturation (active) region at all times, the minimum value of $v_{D S}$ should not fall below the corresponding value of $v_{G S}$ by more than $V_{t}$. Also, the maximum value of $v_{D S}$ should be smaller than $V_{D D}$; otherwise the FET will enter the cutoff region and the peaks of the output signal waveform will be clipped off.

Finally, we note that by substituting for $g_{m}$ from Eq. (6.33) the voltage-gain expression in Eq. (6.36) becomes identical to that derived in Section 6.1-namely, Eq. (6.15).

Separating the DC Analysis and the Signal Analysis From the preceding analysis, we see that under the small-signal approximation, signal quantities are superimposed on dc quantities. For instance, the total drain current $i_{D}$ equals the dc current $I_{D}$ plus the signal current $i_{d}$, the total drain voltage $v_{D S}=V_{D S}+v_{d s}$, and so on. It follows that the analysis and design can be greatly simplified by separating dc or bias calculations from small-signal calculations. That is, once a stable dc operating point has been established and all dc quantities calculated, we may then perform signal analysis ignoring de quantities.


Figure 6.12 Total instantaneous voltages $v_{G S}$ and $v_{D S}$ for the circuit in Fig. 6.10.

Small-Signal Equivalent-Circuit Models From a signal point of view, the FET behaves as a voltage-controlled current source. It accepts a signal $v_{g s}$ between gate and source and provides a current $g_{m} v_{g s}$ at the drain terminal. The input resistance of this controlled source is very high-ideally, infinite. The output resistance-that is, the resistance looking into the drain-also is high, and we have assumed it to be infinite thus far. Putting all of this together, we arrive at the circuit in Fig. 6.13(a), which represents the small-signal operation of the MOSFET and is thus a small-signal model or a small-signal equivalent circuit.

In the analysis of a MOSFET amplifier circuit, the transistor can be replaced by the equivalent-circuit model shown in Fig. 6.13(a). The rest of the circuit remains unchanged except that ideal constant dc voltage sources are replaced by short circuits. This is a result of the fact that the voltage across an ideal constant dc voltage source does not change, and thus there will always be a zero voltage signal across a constant dc voltage source. A dual statement applies for constant dc current sources; namely, the signal current of an ideal constant dc current source will always be zero, and thus an ideal constant dc current source can be replaced by an open circuit in the small-signal equivalent circuit of the amplifier. The circuit resulting can then be used to perform any required signal analysis, such as calculating voltage gain.

The most serious shortcoming of the small-signal model of Fig. 6.13(a) is that it assumes the drain current in saturation to be independent of the drain voltage. From our study of the MOSFET characteristics in saturation, we know that the drain current does in fact depend on $v_{D S}$ in a linear manner. Such dependence was modeled by a finite resistance $r_{o}$ between drain and source, whose value was given by Eq. (5.27) in Section 5.2.4, which we repeat here (with the prime on $I_{D}$ dropped) as

$$
\begin{equation*}
r_{o}=\frac{\left|V_{A}\right|}{I_{D}} \tag{6.37}
\end{equation*}
$$

where $V_{A}=1 / \lambda$ is a MOSFET parameter that either is specified or can be measured. It should be recalled that for a given process technology, $V_{A}$ is proportional to the MOSFET channel length. The current $I_{D}$ is the value of the dc drain current without the channel-length modulation taken into account; that is,

$$
\begin{equation*}
I_{D}=\frac{1}{2} k_{n} V_{O V}^{2} \tag{6.38}
\end{equation*}
$$

Typically, $r_{o}$ is in the range of $10 \mathrm{k} \Omega$ to $1000 \mathrm{k} \Omega$. It follows that the accuracy of the small-signal model can be improved by including $r_{o}$ in parallel with the controlled source, as shown in Fig. 6.13(b).

It is important to note that the small-signal model parameters $g_{m}$ and $r_{o}$ depend on the dc bias point of the MOSFET.

Returning to the amplifier of Fig. 6.10, we find that replacing the MOSFET with the small-signal model of Fig. 6.13(b) results in the voltage-gain expression

$$
\begin{equation*}
A_{v}=\frac{v_{d s}}{v_{g s}}=-g_{m}\left(R_{D} \| r_{o}\right) \tag{6.39}
\end{equation*}
$$

Thus, the finite output resistance $r_{o}$ results in a reduction in the magnitude of the voltage gain.
Although the analysis above is performed on an NMOS transistor, the results, and the equivalent-circuit models of Fig. 6.13, apply equally well to PMOS devices, except for using $\left|V_{G S}\right|,\left|V_{t}\right|,\left|V_{O V}\right|$, and $\left|V_{A}\right|$ and replacing $k_{n}$ with $k_{p}$.

The Transconductance $\boldsymbol{g}_{\boldsymbol{m}}$ We shall now take a closer look at the MOSFET transconductance given by Eq. (6.32), which we rewrite with $k_{n}=k_{n}^{\prime}(W / L)$ as follows:

$$
\begin{equation*}
g_{m}=k_{n}^{\prime}(W / L)\left(V_{G S}-V_{t}\right)=k_{n}^{\prime}(W / L) V_{O V} \tag{6.40}
\end{equation*}
$$

This relationship indicates that $g_{m}$ is proportional to the process transconductance parameter $k_{n}^{\prime}=\mu_{n} C_{o x}$ and to the $W / L$ ratio of the MOS transistor; hence to obtain relatively large transconductance the device must be short and wide. We also observe that for a given device the transconductance is proportional to the overdrive voltage, $V_{O V}=V_{G S}-V_{t}$, the amount by which the bias voltage $V_{G S}$ exceeds the threshold voltage $V_{t}$. Note, however, that increasing $g_{m}$ by biasing the device at a larger $V_{G S}$ has the disadvantage of reducing the allowable voltage signal swing at the drain.

Another useful expression for $g_{m}$ can be obtained by substituting for $V_{O V}$ in Eq. (6.40) by $\sqrt{2 I_{D} /\left(k_{n}^{\prime}(W / L)\right)}$ [from Eq. (6.25)]:

$$
\begin{equation*}
g_{m}=\sqrt{2 k_{n}^{\prime}} \sqrt{W / L} \sqrt{I_{D}} \tag{6.41}
\end{equation*}
$$



Figure 6.13 Small-signal models for the MOSFET: (a) neglecting the dependence of $i_{D}$ on $v_{D S}$ in the active region (the channel-length modulation effect) and (b) including the effect of channel-length modulation, modeled by output resistance $r_{o}=\left|V_{A}\right| / I_{D}$. These models apply equally well for both NMOS and PMOS transistors.


Figure 6.14 The slope of the tangent at the bias point Q intersects the $v_{O V}$ axis at $\frac{1}{2} V_{O V}$. Thus, $g_{m}=I_{D} /\left(\frac{1}{2} V_{O V}\right)$.

This expression shows two things:

1. For a given MOSFET, $g_{m}$ is proportional to the square root of the dc bias current.
2. At a given bias current, $g_{m}$ is proportional to $\sqrt{W / L}$.

In contrast, as we shall see shortly, the transconductance of the bipolar junction transistor (BJT) is proportional to the bias current and is independent of the physical size and geometry of the device.

To gain some insight into the values of $g_{m}$ obtained in MOSFETs consider an integrated-circuit device operating at $I_{D}=0.5 \mathrm{~mA}$ and having $k_{n}^{\prime}=120 \mu \mathrm{~A} / \mathrm{V}^{2}$. Equation (6.41) shows that for $W / L=1, g_{m}=$ $0.35 \mathrm{~mA} / \mathrm{V}$, whereas a device for which $W / L=100$ has $g_{m}=3.5 \mathrm{~mA} / \mathrm{V}$. In contrast, a BJT operating at a collector current of 0.5 mA has $g_{m}=20 \mathrm{~mA} / \mathrm{V}$.

Yet another useful expression for $g_{m}$ of the MOSFET can be obtained by substituting for $k_{n}^{\prime}(W / L)$ in Eq. (6.40) by $2 I_{D} /\left(V_{G S}-V_{t}\right)^{2}$ :

$$
\begin{equation*}
g_{m}=\frac{2 I_{D}}{V_{G S}-V_{t}}=\frac{2 I_{D}}{V_{O V}} \tag{6.42}
\end{equation*}
$$

A convenient graphical construction that clearly illustrates this relationship is shown in Fig. 6.14.
In summary, there are three different relationships for determining $g_{m}$ —Eqs. (6.40), (6.41), and (6.42)—and there are three design parameters- $(W / L), V_{O V}$, and $I_{D}$, any two of which can be chosen independently. That is, the designer may choose to operate the MOSFET with a certain overdrive voltage $V_{O V}$ and at a particular current $I_{D}$; the required $W / L$ ratio can then be found and the resulting $g_{m}$ determined. ${ }^{4}$

## Example 6.3

Figure 6.15(a) shows a discrete MOSFET amplifier utilizing a drain-to-gate resistance $R_{G}$ for biasing purposes. Such a biasing arrangement will be studied in Section 6.4. The input signal $v_{i}$ is coupled to the gate via a large capacitor, and the output signal at the drain is coupled to the load resistance $R_{L}$ via another large capacitor. We wish to analyze this amplifier circuit to determine its small-signal voltage gain, its input resistance, and the largest allowable input signal. The transistor has $V_{t}=1.5 \mathrm{~V}, k_{n}^{\prime}(W / L)$ $=0.25 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{A}=50 \mathrm{~V}$. Assume the coupling capacitors to be sufficiently large so as to act as short circuits at the signal frequencies of interest.

[^3]
## Example 6.3 continued



Figure 6.15 Example 6.3: (a) amplifier circuit; (b) circuit for determining the dc operating point; (c) the amplifier small-signal equivalent circuit; (d) a simplified version of the circuit in (c).

## Solution

We first determine the dc operating point. For this purpose, we eliminate the input signal $v_{i}$, and open-circuit the two coupling capacitors (since they block dc currents). The result is the circuit shown in Fig. 6.14(b). We note that since $I_{G}=0$, the dc voltage drop across $R_{G}$ will be zero, and

$$
\begin{equation*}
V_{G S}=V_{D S}=V_{D D}-R_{D} I_{D} \tag{6.43}
\end{equation*}
$$

With $V_{D S}=V_{G S}$, the NMOS transistor will be operating in saturation. Thus,

$$
\begin{equation*}
I_{D}=\frac{1}{2} k_{n}\left(V_{G S}-V_{t}\right)^{2} \tag{6.44}
\end{equation*}
$$

where, for simplicity, we have neglected the effect of channel-length modulation on the dc operating point. Substituting $V_{D D}=15 \mathrm{~V}, R_{D}=10 \mathrm{k} \Omega, k_{n}=0.25 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{t}=1.5 \mathrm{~V}$ in Eqs. (6.43) and (6.44), and substituting for $V_{G S}$ from Eq. (6.43) into Eq. (6.44) results in a quadratic equation in $I_{D}$.

## Example 6.3 continued

Solving the latter and discarding the root that is not physically meaningful yields the solution

$$
I_{D}=1.06 \mathrm{~mA}
$$

which corresponds to

$$
V_{G S}=V_{D S}=4.4 \mathrm{~V}
$$

and

$$
V_{O V}=4.4-1.5=2.9 \mathrm{~V}
$$

Next we proceed with the small-signal analysis of the amplifier. Toward that end we replace the MOSFET with its small-signal model to obtain the small-signal equivalent circuit of the amplifier, shown in Fig. 6.15(c). Observe that we have replaced the coupling capacitors with short circuits. The dc voltage supply $V_{D D}$ has also been replaced with a short circuit to ground.

The values of the transistor small-signal parameters $g_{m}$ and $r_{o}$ can be determined by using the dc bias quantities found above, as follows:

$$
\begin{aligned}
g_{m} & =k_{n} V_{O V}=0.25 \times 2.9=0.725 \mathrm{~mA} / \mathrm{V} \\
r_{o} & =\frac{V_{A}}{I_{D}}=\frac{50}{1.06}=47 \mathrm{k} \Omega
\end{aligned}
$$

Next we use the equivalent circuit of Fig. 6.15(c) to determine the input resistance $R_{\text {in }} \equiv v_{i} / i_{i}$ and the voltage gain $A_{v}=v_{o} / v_{i}$. Toward that end we simplify the circuit by combining the three parallel resistances $r_{o}, R_{D}$, and $R_{L}$ in a single resistance $R_{L}^{\prime}$,

$$
R_{L}^{\prime}=R_{L}\left\|R_{D}\right\| r_{o}=10\|10\| 47=4.52 \mathrm{k} \Omega
$$

as shown in Fig. 6.15(d). For the latter circuit we can write the two equations

$$
\begin{equation*}
v_{o}=\left(i_{i}-g_{m} v_{g s}\right) R_{L}^{\prime} \tag{6.45}
\end{equation*}
$$

and

$$
\begin{equation*}
i_{i}=\frac{v_{g s}-v_{o}}{R_{G}} \tag{6.46}
\end{equation*}
$$

Substituting for $i_{i}$ from Eq. (6.46) into Eq. (6.45) results in the following expression for the voltage gain $A_{v} \equiv v_{o} / v_{i}=v_{o} / v_{g s}$ :

$$
A_{v}=-g_{m} R_{L}^{\prime} \frac{1-\left(1 / g_{m} R_{G}\right)}{1+\left(R_{L}^{\prime} / R_{G}\right)}
$$

Since $R_{G}$ is very large, $g_{m} R_{G} \gg 1$ and $R_{L}^{\prime} / R_{G} \ll 1$ (the reader can easily verify this), and the gain expression can be approximated as

$$
\begin{equation*}
A_{v} \simeq-g_{m} R_{L}^{\prime} \tag{6.47}
\end{equation*}
$$

Substituting $g_{m}=0.725 \mathrm{~mA} / \mathrm{V}$ and $R_{L}^{\prime}=4.52 \mathrm{k} \Omega$ yields $A_{v}=-3.3 \mathrm{~V} / \mathrm{V}$
To obtain the input resistance, we substitute in Eq. (6.46) for $v_{o}=A_{v} v_{g s}=-g_{m} R_{L}^{\prime} v_{g s}$, then use $R_{\mathrm{in}} \equiv v_{i} / i_{i}=v_{g s} / i_{i}$ to obtain

## Example 6.3 continued

$$
\begin{equation*}
R_{\mathrm{in}}=\frac{R_{G}}{1+g_{m} R_{L}^{\prime}} \tag{6.48}
\end{equation*}
$$

This is an interesting relationship: The input resistance decreases as the gain $\left(g_{m} R_{L}^{\prime}\right)$ is increased. The value of $R_{\text {in }}$ can now be determined; it is

$$
R_{\mathrm{in}}=\frac{10 \mathrm{M} \Omega}{1+3.3}=2.33 \mathrm{M} \Omega
$$

which is still very large.
The largest allowable input signal $\hat{v}_{i}$ is constrained by the need to keep the transistor in saturation at all times; that is, $v_{D S} \geq v_{G S}-V_{t}$

Enforcing this condition with equality at the point $v_{G S}$ is maximum and $v_{D S}$ is minimum, we write

$$
\begin{gathered}
v_{D S \text { min }}=v_{G S \text { max }}-V_{t} \\
V_{D S}-\left|A_{v}\right| \hat{v}_{i}=V_{G S}+\hat{v}_{i}-V_{t}
\end{gathered}
$$

Since $V_{D S}=V_{G S}$, we obtain

$$
\hat{v}_{i}=\frac{V_{t}}{\left|A_{v}\right|+1}
$$

This is a general relationship that applies to this circuit irrespective of the component values. Observe that it simply states that the maximum signal swing is determined by the fact that the bias arrangement makes $V_{D}=V_{G}$ and thus, to keep the MOSFET out of the triode region, the signal between D and G is constrained to be equal to $V_{t}$. For our particular design,

$$
\hat{v}_{i}=\frac{1.5}{3.3+1}=0.35 \mathrm{~V}
$$

Since $V_{O V}=2.9 \mathrm{~V}$, a $v_{i}$ of 0.35 is indeed much smaller than $2 V_{O V}=5.8 \mathrm{~V}$; thus the assumption of linear operation is justified.

A modification of this circuit that increases the allowable signal swing is investigated in Problem 6.94.

## EXERCISE

D6.4 Consider the amplifier circuit of Fig. 6.15(a) without the load resistance $R_{L}$ and with channel-length modulation neglected. Let $V_{D D}=5 \mathrm{~V}, V_{t}=0.7 \mathrm{~V}$, and $k_{n}=1 \mathrm{~mA} / \mathrm{V}^{2}$. Find $V_{O V}, I_{D}, R_{D}$, and $R_{G}$ to obtain a voltage gain of $-25 \mathrm{~V} / \mathrm{V}$ and an input resistance of $0.5 \mathrm{M} \Omega$. What is the maximum allowable input signal, $\hat{v}_{i}$ ?

Ans. $0.319 \mathrm{~V} ; 50.9 \mu \mathrm{~A} ; 78.5 \mathrm{k} \Omega ; 13 \mathrm{M} \Omega ; 27 \mathrm{mV}$

The T Equivalent-Circuit Model Through a simple circuit transformation it is possible to develop an alternative equivalent-circuit model for the MOSFET. The development of such a model, known as the T model, is illustrated in Fig. 6.16. Figure 6.16(a) shows the equivalent circuit studied above without $r_{o}$. In Fig. 6.16(b) we have added a second $g_{m} v_{g s}$ current source in series with the original controlled source. This addition obviously does not change the terminal currents and is thus allowed. The newly created circuit node, labeled X , is joined to the gate terminal G in Fig. 6.16(c). Observe that the gate current does not change-that is, it remains equal to zero-and thus this connection does not alter the terminal characteristics. We now note that we have a controlled current source $g_{m} v_{g s}$ connected across its control voltage $v_{g s}$. We can replace this controlled source by a resistance as long as this resistance draws an equal current as the source. (See the source-absorption theorem in Appendix D.) Thus the value of the resistance is $v_{g s} / g_{m} v_{g s}=1 / g_{m}$. This replacement is shown in Fig. 6.16(d), which depicts the alternative model. Observe that $i_{g}$ is still zero, $i_{d}=$ $g_{m} v_{g s}$, and $i_{s}=v_{g s} /\left(1 / g_{m}\right)=g_{m} v_{g s}$, all the same as in the original model in Fig. 6.16(a).


Figure 6.16 Development of the T equivalent-circuit model for the MOSFET. For simplicity, $r_{o}$ has been omitted; however, it may be added between D and S in the T model of (d).


Figure 6.17 (a) The T model of the MOSFET augmented with the drain-to-source resistance $r_{o}$. (b) An alternative representation of the T model.

The model of Fig. 6.16(d) shows that the resistance between gate and source looking into the source is $1 / g_{m}$. This observation and the T model prove useful in many applications. Note that the resistance between gate and source, looking into the gate, is infinite.

In developing the T model we did not include $r_{o}$. If desired, this can be done by incorporating in the circuit of Fig. 6.16(d) a resistance $r_{o}$ between drain and source, as shown in Fig. 6.17(a). An alternative representation of the T model, in which the voltage-controlled current source is replaced with a current-controlled current source, is shown in Fig. 6.17(b).

Finally, we should note that in order to distinguish the model of Fig. 6.13(b) from the equivalent T model, the former is sometimes referred to as the hybrid- $\boldsymbol{\pi}$ model, a carryover from the bipolar transistor literature. The origin of this name will be explained shortly.

## Example 6.4

Figure 6.18(a) shows a MOSFET amplifier biased by a constant-current source $I$. Assume that the values of $I$ and $R_{D}$ are such that the MOSFET operates in the saturation region. The input signal $v_{i}$ is coupled to the source terminal by utilizing a large capacitor $C_{C 1}$. Similarly, the output signal at the drain is taken through a large coupling capacitor $C_{C 2}$. Find the input resistance $R_{\text {in }}$ and the voltage gain $v_{o} / v_{i}$. Neglect channel-length modulation.

## Solution

Replacing the MOSFET with its T equivalent-circuit model results in the amplifier equivalent circuit shown in Fig. 6.18(b). Observe that the dc current source $I$ is replaced with an open circuit and the dc voltage source $V_{D D}$ is replaced by a short circuit. The large coupling capacitors have been replaced by short circuits. From the equivalent-circuit model we determine

$$
R_{\mathrm{in}}=\frac{v_{i}}{-i}=1 / g_{m}
$$

and

$$
v_{o}=-i R_{D}=\left(\frac{v_{i}}{1 / g_{m}}\right) R_{D}=g_{m} R_{D} v_{i}
$$

Thus,

$$
A_{v} \equiv \frac{v_{o}}{v_{i}}=g_{m} R_{D}
$$

## Example 6.4 continued

We note that this amplifier, known as the common-gate amplifier because the gate at ground potential is common to both the input and output ports, has a low input resistance $\left(1 / g_{m}\right)$ and a noninverting gain. We shall study this amplifier type in Section 6.3.5.


Figure 6.18 (a) Amplifier circuit for Example 6.4. (b) Small-signal equivalent circuit of the amplifier in (a).

## EXERCISE

6.5 Use the T model of Fig. 6.17 (b) to show that a MOSFET whose drain is connected to its gate exhibits an incremental resistance equal to $\left[\left(1 / g_{m}\right) \| r_{o}\right]$.

Ans. See Fig. E6.5.


Figure E6.5 Circuits for Exercise 6.5. Note that the bias arrangement of $Q$ is not shown.

Modeling the Body Effect As mentioned earlier (see Section 5.4), the body effect occurs in a MOSFET when the source is not tied to the substrate (which is always connected to the most negative power supply in the integrated circuit for $n$-channel devices and to the most positive for $p$-channel devices). Thus the substrate (body) will be at signal ground, but since the source is not, a signal voltage $v_{b s}$ develops between the body (B) and the source (S). The substrate then acts as a "second gate" or a backgate for the MOSFET. Thus the signal $v_{b s}$ gives rise to a drain-current component, which we shall write as $g_{m b} v_{b s}$, where $g_{m b}$ is the body transconductance, defined as

$$
\begin{equation*}
\left.g_{m b} \equiv \frac{\partial i_{D}}{\partial v_{B S}}\right|_{\substack{v_{G S}=\text { constant } \\ v_{D S}=\text { constant }}} \tag{6.49}
\end{equation*}
$$

Recalling that $i_{D}$ depends on $v_{B S}$ through the dependence of $V_{t}$ on $V_{B S}$, we can show that

$$
\begin{equation*}
g_{m b}=\chi g_{m} \tag{6.50}
\end{equation*}
$$

where

$$
\begin{equation*}
\chi \equiv \frac{\partial V_{t}}{\partial V_{S B}}=\frac{\gamma}{2 \sqrt{2 \phi_{f}+V_{S B}}} \tag{6.51}
\end{equation*}
$$

Typically the value of $\chi$ lies in the range 0.1 to 0.3 .
Figure 6.19 shows the MOSFET model augmented to include the controlled source $g_{m b} v_{b s}$ that models the body effect. Ideally, this is the model to be used whenever the source is not connected to the substrate. It has been found, however, that except in some very particular situations, the body effect can generally be ignored in the initial, pencil-and-paper design of MOSFET amplifiers.

Finally, although the analysis above was performed on an NMOS transistor, the results and the equivalent circuit of Fig. 6.19 apply equally well to PMOS transistors, except for using $\left|V_{G S}\right|,\left|V_{t}\right|,\left|V_{O V}\right|,\left|V_{A}\right|,\left|V_{S B}\right|,|\gamma|$, and $|\lambda|$ and replacing $k_{n}^{\prime}$ with $k_{p}^{\prime}$ in the appropriate formula.

(a)

(b)

Figure 6.19 Small-signal, equivalent-circuit model of a MOSFET in which the source is not connected to the body.

## EXERCISES

6.6 For the amplifier in Fig. 6.4, let $V_{D D}=5 \mathrm{~V}, R_{D}=10 \mathrm{k} \Omega, V_{t}=1 \mathrm{~V}, k_{n}^{\prime}=20 \mu \mathrm{~A} / \mathrm{V}^{2}, W / L=20, V_{G S}=2 \mathrm{~V}$, and $\lambda=0$.
(a) Find the dc current $I_{D}$ and the dc voltage $V_{D S}$.
(b) Find $g_{m}$.
(c) Find the voltage gain.
(d) If $v_{g s}=0.2 \sin \omega t$ volts, find $v_{d s}$ assuming that the small-signal approximation holds. What are the minimum and maximum values of $v_{D S}$ ?
(e) Use Eq. (6.28) to determine the various components of $i_{D}$. Using the identity $\left(\sin ^{2} \omega t=\frac{1}{2}-\frac{1}{2} \cos 2 \omega t\right)$, show that there is a slight shift in $I_{D}$ (by how much?) and that there is a second-harmonic component (i.e., a component with frequency $2 \omega$ ). Express the amplitude of the second-harmonic component as a percentage of the amplitude of the fundamental. (This value is known as the second-harmonic distortion.)

Ans. (a) $200 \mu \mathrm{~A}, 3 \mathrm{~V}$; (b) $0.4 \mathrm{~mA} / \mathrm{V}$; (c) $-4 \mathrm{~V} / \mathrm{V}$; (d) $v_{d s}=-0.8 \sin \omega t$ volts, $2.2 \mathrm{~V}, 3.8 \mathrm{~V}$;
(e) $i_{D}=(204+80 \sin \omega t-4 \cos 2 \omega t) \mu \mathrm{A}, 5 \%$
6.7 An NMOS transistor has $\mu_{n} C_{o x}=60 \mu \mathrm{~A} / \mathrm{V}^{2}, W / L=40, V_{t}=1 \mathrm{~V}$, and $V_{A}=15 \mathrm{~V}$. Find $g_{m}$ and $r_{o}$ when (a) the bias voltage $V_{G S}=1.5 \mathrm{~V}$, (b) the bias current $I_{D}=0.5 \mathrm{~mA}$.

Ans. (a) $1.2 \mathrm{~mA} / \mathrm{V}, 50 \mathrm{k} \Omega$; (b) $1.55 \mathrm{~mA} / \mathrm{V}, 30 \mathrm{k} \Omega$
6.8 A MOSFET is to operate at $I_{D}=0.1 \mathrm{~mA}$ and is to have $g_{m}=1 \mathrm{~mA} / \mathrm{V}$. If $k_{n}^{\prime}=50 \mu \mathrm{~A} / \mathrm{V}^{2}$, find the required $W / L$ ratio and the overdrive voltage.

Ans. 100; 0.2 V
6.9 For a fabrication process for which $\mu_{p} \simeq 0.4 \mu_{n}$, find the ratio of the width of a PMOS transistor to the width of an NMOS transistor so that the two devices have equal $g_{m}$ for the same bias conditions. The two devices have equal channel lengths.

Ans. 2.5
6.10 A PMOS transistor has $V_{t}=-1 \mathrm{~V}, k_{p}^{\prime}=60 \mu \mathrm{~A} / \mathrm{V}^{2}$, and $W / L=16 \mu \mathrm{~m} / 0.8 \mu \mathrm{~m}$. Find $I_{D}$ and $g_{m}$ when the device is biased at $V_{G S}=-1.6 \mathrm{~V}$. Also, find the value of $r_{o}$ if $\lambda($ at $L=1 \mu \mathrm{~m})=-0.04 \mathrm{~V}^{-1}$.

Ans. $216 \mu \mathrm{~A} ; 0.72 \mathrm{~mA} / \mathrm{V} ; 92.6 \mathrm{k} \Omega$
6.11 Derive an expression for $\left(g_{m} r_{o}\right)$ in terms of $V_{A}$ and $V_{O V}$. As we shall see in Chapter 7, this is an important transistor parameter and is known as the intrinsic gain. Evaluate the value of $g_{m} r_{o}$ for an NMOS transistor fabricated in a $0.8-\mu \mathrm{m}$ CMOS process for which $V_{A}^{\prime}=12.5 \mathrm{~V} / \mu \mathrm{m}$ of channel length. Let the device have minimum channel length and be operated at an overdrive voltage of 0.2 V .

Ans. $g_{m} r_{o}=2 V_{A} / V_{O V} ; 100 \mathrm{~V} / \mathrm{V}$

### 6.2.2 The BJT Case

We next consider the small-signal operation of the BJT and develop small-signal equivalent-circuit models that represent its operation at a given bias point. The following development parallels what we used for the MOSFET except that here we have an added complication: The BJT draws a finite base current. As will be seen shortly, this phenomenon (finite $\beta$ ) manifests itself as a finite input resistance looking into the base of the BJT (as compared to the infinite input resistance looking into the gate of the MOSFET).

Consider the conceptual amplifier circuit shown in Fig. 6.20(a). Here the base-emitter junction is forward biased by a dc voltage $V_{B E}$. The reverse bias of the collector-base junction is established by connecting the collector to another power supply of voltage $V_{C C}$ through a resistor $R_{C}$. The input signal to be amplified is represented by the voltage source $v_{b e}$ that is superimposed on $V_{B E}$.

The DC Bias Point We consider first the dc bias conditions by setting the signal $v_{b e}$ to zero. The circuit reduces to that in Fig. 6.20 (b), and we can write the following relationships for the dc currents and voltages:


Figure 6.20 (a) Conceptual circuit to illustrate the operation of the transistor as an amplifier. (b) The circuit of (a) with the signal source $v_{b e}$ eliminated for dc (bias) analysis.

$$
\begin{align*}
I_{C} & =I_{S} e^{V_{B E} V_{T}}  \tag{6.52}\\
I_{E} & =I_{C} / \alpha  \tag{6.53}\\
I_{B} & =I_{C} / \beta  \tag{6.54}\\
V_{C E} & =V_{C C}-I_{C} R_{C} \tag{6.55}
\end{align*}
$$

For active-mode operation, $V_{C E}$ should be greater than $\left(V_{B E}-0.4\right)$ by an amount that allows for the required negative signal swing at the collector.

The Collector Current and the Transconductance If a signal $v_{b e}$ is applied as shown in Fig. 6.20(a), the total instantaneous base-emitter voltage $v_{B E}$ becomes

$$
v_{B E}=V_{B E}+v_{b e}
$$

Correspondingly, the collector current becomes

$$
\begin{aligned}
i_{C} & =I_{S} e^{v_{B E} / V_{T}}=I_{S} e^{\left(V_{B E}+v_{b e}\right) / V_{T}} \\
& =I_{S} e^{V_{B E} / V_{T}} e^{v_{b e} / V_{T}}
\end{aligned}
$$

Use of Eq. (6.52) yields

$$
\begin{equation*}
i_{C}=I_{C} e^{v_{b} / V_{T}} \tag{6.56}
\end{equation*}
$$

Now, if $v_{b e} \ll V_{T}$, we may approximate Eq. (6.56) as

$$
\begin{equation*}
i_{C} \simeq I_{C}\left(1+\frac{v_{b e}}{V_{T}}\right) \tag{6.57}
\end{equation*}
$$

Here we have expanded the exponential in Eq. (6.56) in a series and retained only the first two terms. That is, we have assumed that

$$
\begin{equation*}
v_{b e} \ll V_{T} \tag{6.58}
\end{equation*}
$$

so that we can neglect the higher-order terms in the exponential series expansion. The condition in Eq. (6.58) is the small-signal approximation for the BJT and corresponds to that in Eq. (6.29) for the MOSFET case. The small-signal approximation for the BJT is valid only for $v_{b e}$ less than 5 mV or 10 mV , at most. Under this approximation, the total collector current is given by Eq. (6.57) and can be rewritten

$$
\begin{equation*}
i_{C}=I_{C}+\frac{I_{C}}{V_{T}} v_{b e} \tag{6.59}
\end{equation*}
$$

Thus the collector current is composed of the dc bias value $I_{C}$ and a signal component $i_{c}$,

$$
\begin{equation*}
i_{c}=\frac{I_{C}}{V_{T}} v_{b e} \tag{6.60}
\end{equation*}
$$

This equation relates the signal current in the collector to the corresponding base-emitter signal voltage. It can be rewritten as

$$
\begin{equation*}
i_{c}=g_{m} v_{b e} \tag{6.61}
\end{equation*}
$$

where $g_{m}$ is the transconductance, and from Eq. (6.60), it is given by

$$
\begin{equation*}
g_{m}=\frac{I_{C}}{V_{T}} \tag{6.62}
\end{equation*}
$$



Figure 6.21 Linear operation of the transistor under the small-signal condition: A small-signal $v_{b e}$ with a triangular waveform is superimposed on the dc voltage $V_{B E}$. It gives rise to a collector-signal current $i_{c}$, also of triangular waveform, superimposed on the dc current $I_{C}$. Here, $i_{c}=g_{m} v_{b e}$, where $g_{m}$ is the slope of the $i_{C}-v_{B E}$ curve at the bias point Q .

We observe that the transconductance of the BJT is directly proportional to the collector bias current $I_{C}$. Thus to obtain a constant predictable value for $g_{m}$, we need a constant predictable $I_{C}$. Also, we note that BJTs have relatively high transconductance in comparison to MOSFETs: for instance, at $I_{C}=1 \mathrm{~mA}, g_{m} \simeq 40 \mathrm{~mA} / \mathrm{V}$. Finally, unlike the MOSFET, whose $g_{m}$ depends on the device dimensions ( $W$ and $L$ ), $g_{m}$ of a BJT depends only on the dc collector current at which it is biased to operate.

A graphical interpretation for $g_{m}$ is given in Fig. 6.21, where it is shown that $g_{m}$ is equal to the slope of the tangent to the $i_{C}-v_{B E}$ characteristic curve at $i_{C}=I_{C}$ (i.e., at the bias point Q ). Thus,

$$
\begin{equation*}
g_{m}=\left.\frac{\partial i_{C}}{\partial v_{B E}}\right|_{i_{C}=I_{C}} \tag{6.63}
\end{equation*}
$$

The small-signal approximation implies keeping the signal amplitude sufficiently small that operation is restricted to an almost-linear segment of the $i_{C}-v_{B E}$ exponential curve. Increasing the signal amplitude will result in the collector current having components nonlinearly related to $v_{b e}$.

## EXERCISES

6.12 Use Eq. (6.63) to derive the expression for $g_{m}$ in Eq. (6.62).
6.13 Calculate the value of $g_{m}$ for a BJT biased at $I_{C}=0.5 \mathrm{~mA}$.

Ans. $20 \mathrm{~mA} / \mathrm{V}$

The Base Current and the Input Resistance at the Base To determine the resistance seen by $v_{b e}$, we first evaluate the total base current $i_{B}$ using Eq. (6.59), as follows:

$$
i_{B}=\frac{i_{C}}{\beta}=\frac{I_{C}}{\beta}+\frac{1}{\beta} \frac{I_{C}}{V_{T}} v_{b e}
$$

Thus,

$$
\begin{equation*}
i_{B}=I_{B}+i_{b} \tag{6.64}
\end{equation*}
$$

where $I_{B}$ is equal to $I_{C} / \beta$ and the signal component $i_{b}$ is given by

$$
\begin{equation*}
i_{b}=\frac{1}{\beta} \frac{I_{C}}{V_{T}} v_{b e} \tag{6.65}
\end{equation*}
$$

Substituting for $I_{C} / V_{T}$ by $g_{m}$ gives

$$
\begin{equation*}
i_{b}=\frac{g_{m}}{\beta} v_{b e} \tag{6.66}
\end{equation*}
$$

The small-signal input resistance between base and emitter, looking into the base, is denoted by $r_{\pi}$ and is defined as

$$
\begin{equation*}
r_{\pi} \equiv \frac{v_{b e}}{i_{b}} \tag{6.67}
\end{equation*}
$$

Using Eq. (6.66) gives

$$
\begin{equation*}
r_{\pi}=\frac{\beta}{g_{m}} \tag{6.68}
\end{equation*}
$$

Thus $r_{\pi}$ is directly dependent on $\beta$ and is inversely proportional to the bias current $I_{C}$. Substituting for $g_{m}$ in Eq. (6.68) from Eq. (6.62) and replacing $I_{C} / \beta$ by $I_{B}$ gives an alternative expression for $r_{\pi}$,

$$
\begin{equation*}
r_{\pi}=\frac{V_{T}}{I_{B}} \tag{6.69}
\end{equation*}
$$

Here, we recall that because the gate current of the MOSFET is zero (at dc and low frequencies) the input resistance at the gate is infinite; that is, in the MOSFET there is no counterpart to $r_{\pi}{ }^{5}$.

## EXERCISE

6.14 A BJT amplifier is biased to operate at a constant collector current $I_{C}=0.5 \mathrm{~mA}$ irrespective of the value $\beta$. If the transistor manufacturer specifies $\beta$ to range from 50 to 200 , give the expected range of $g_{m}, I_{B}$, and $r_{\pi}$.

Ans. $g_{m}$ is constant at $20 \mathrm{~mA} / \mathrm{V} ; I_{B}=10 \mu \mathrm{~A}$ to $2.5 \mu \mathrm{~A} ; r_{\pi}=2.5 \mathrm{k} \Omega$ to $10 \mathrm{k} \Omega$

The Emitter Current and the Input Resistance at the Emitter The total emitter current $i_{E}$ can be determined using Eq. (6.59) as

$$
i_{E}=\frac{i_{C}}{\alpha}=\frac{I_{C}}{\alpha}+\frac{i_{c}}{\alpha}
$$

Thus,

$$
\begin{equation*}
i_{E}=I_{E}+i_{e} \tag{6.70}
\end{equation*}
$$

where $I_{E}$ is equal to $I_{C} / \alpha$ and the signal current $i_{e}$ is given by

$$
\begin{equation*}
i_{e}=\frac{i_{c}}{\alpha}=\frac{I_{C}}{\alpha V_{T}} v_{b e}=\frac{I_{E}}{V_{T}} v_{b e} \tag{6.71}
\end{equation*}
$$

[^4]If we denote the small-signal resistance between base and emitter looking into the emitter by $r_{e}$, it can be defined as

$$
\begin{equation*}
r_{e} \equiv \frac{v_{b e}}{i_{e}} \tag{6.72}
\end{equation*}
$$

Using Eq. (6.71) we find that $r_{e}$, called the emitter resistance, is given by

$$
\begin{equation*}
r_{e}=\frac{V_{T}}{I_{E}} \tag{6.73}
\end{equation*}
$$

Comparison with Eq. (6.62) reveals that

$$
\begin{equation*}
r_{e}=\frac{\alpha}{g_{m}} \simeq \frac{1}{g_{m}} \tag{6.74}
\end{equation*}
$$

The relationship between $r_{\pi}$ and $r_{e}$ can be found by combining their respective definitions in Eqs. (6.67) and (6.72) as

$$
v_{b e}=i_{b} r_{\pi}=i_{e} r_{e}
$$

Thus,

$$
r_{\pi}=\left(i_{e} / i_{b}\right) r_{e}
$$

which yields

$$
\begin{equation*}
r_{\pi}=(\beta+1) r_{e} \tag{6.75}
\end{equation*}
$$

Figure 6.22 illustrates the definition of $r_{\pi}$ and $r_{e}$.
Finally, a comparison with the MOSFET would be useful: For the MOSFET, $\alpha=1$ and the resistance looking into the source is simply $1 / g_{m}$.

## SHOCKLEY AND SILICON VALLEY

In 1956 William Bradford Shockley started a new company, Shockley Semiconductor Laboratory in Mountain View, California (near Stanford, his birthplace). While at Bell Labs, together with John Bardeen and Walter Brattain, he had invented the BJT. At Shockley, the initial concentration was on developing semiconductor devices, particularly a new four-layer diode. But Shockley's scientific genius and ability to select and attract good team members, first demonstrated at Bell Labs, was not accompanied by comparable talent for management. Consequently, in 1957, eight of his team members (the so-called Traitorous Eight, including Gordon Moore and Robert Noyce) left Shockley to create Fairchild Semiconductor. It was a propitious time: The first Sputnik was launched a month later, and the ensuing space race accelerated demand for solid-state circuits. Decades passed, and in 2002 a group of some 30 individuals who had been associated with Silicon Valley since 1956 met at Stanford University to reminisce about Shockley's contributions to the information technology age. They unanimously concluded that Shockley was the man who brought silicon to Silicon Valley!

## EXERCISE

6.15 A BJT having $\beta=100$ is biased at a dc collector current of 1 mA . Find the value of $g_{m}, r_{e}$, and $r_{\pi}$ at the bias point.

Ans. $40 \mathrm{~mA} / \mathrm{V} ; 25 \Omega ; 2.5 \mathrm{k} \Omega$


Figure 6.22 Illustrating the definition of $r_{\pi}$ and $r_{e}$.


Figure 6.23 The amplifier circuit of Fig. 6.20(a) with the dc sources ( $V_{B E}$ and $V_{C C}$ ) eliminated (short-circuited). Thus only the signal components are present. Note that this is a representation of the signal operation of the BJT and not an actual amplifier circuit.

The Voltage Gain The total collector voltage $v_{C E}$ is

$$
\begin{align*}
v_{C E} & =V_{C C}-i_{C} R_{C} \\
& =V_{C C}-\left(I_{C}+i_{c}\right) R_{C}  \tag{6.76}\\
& =\left(V_{C C}-I_{C} R_{C}\right)-i_{c} R_{C} \\
& =V_{C E}-i_{c} R_{C}
\end{align*}
$$

Thus, superimposed on the collector bias voltage $V_{C E}$ we have signal voltage $v_{c e}$ given by

$$
\begin{align*}
v_{c e} & =-i_{c} R_{C}=-g_{m} v_{b e} R_{C}  \tag{6.77}\\
& =\left(-g_{m} R_{C}\right) v_{b e}
\end{align*}
$$

from which we find the voltage gain $A_{v}$ of this amplifier as

$$
\begin{equation*}
A_{v} \equiv \frac{v_{c e}}{v_{b e}}=-g_{m} R_{C} \tag{6.78}
\end{equation*}
$$

Here again we note that because $g_{m}$ is directly proportional to the collector bias current, the gain will be as stable as the collector bias current is made. Substituting for $g_{m}$ from Eq. (6.62) enables us to express the gain in the form

$$
\begin{equation*}
A_{v}=-\frac{I_{C} R_{C}}{V_{T}} \tag{6.79}
\end{equation*}
$$

which is identical to the expression we derived in Section 6.1 (Eq. 6.21). Finally, we note that the gain expression in Eq. (6.78) is identical in form to that for the MOSFET amplifier (namely, $-g_{m} R_{D}$ ).

## EXERCISE

6.16 In the circuit of Fig. 6.20(a), $V_{B E}$ is adjusted to yield a dc collector current of 1 mA . Let $V_{C C}=15 \mathrm{~V}$, $R_{C}=10 \mathrm{k} \Omega$, and $\beta=100$. Find the voltage gain $v_{c e} / v_{b e}$. If $v_{b e}=0.005 \sin \omega t$ volt, find $v_{C}(t)$ and $i_{B}(t)$. Ans. $-400 \mathrm{~V} / \mathrm{V} ; 5-2 \sin \omega t$ volts; $10+2 \sin \omega t \mu \mathrm{~A}$

Separating the Signal and the DC Quantities The analysis above indicates that every current and voltage in the amplifier circuit of Fig. 6.20(a) is composed of two components: a dc component and a signal component. For instance, $v_{B E}=V_{B E}+v_{b e}, I_{C}=I_{C}+i_{c}$, and so on. The dc components are determined from the dc circuit given in Fig. 6.20(b) and from the relationships imposed by the transistor (Eqs. 6.52 through 6.54). On the other hand, a representation of the signal operation of the BJT can be obtained by eliminating the dc sources, as shown in Fig. 6.23. Observe that since the voltage of an ideal dc supply does not change, the signal voltage across it will be zero. For this reason we have replaced $V_{C C}$ and $V_{B E}$ with short circuits. Had the circuit contained ideal dc current sources, these would have been replaced by open circuits. Note, however, that the circuit of Fig. 6.23 is useful only insofar as it shows the various signal currents and voltages; it is not an actual amplifier circuit, since the dc bias circuit is not shown.

Figure 6.23 also shows the expressions for the current increments ( $i_{c}, i_{b}$, and $i_{e}$ ) obtained when a small signal $v_{b e}$ is applied. These relationships can be represented by a circuit. Such a circuit should have three terminals-C, B, and E-and should yield the same terminal currents indicated in Fig. 6.23. The resulting circuit is then equivalent to the transistor as far as small-signal operation is concerned, and thus it can be considered an equivalent small-signal circuit model.

The Hybrid- $\boldsymbol{\pi}$ Model An equivalent-circuit model for the BJT is shown in Fig. 6.24(a). This model represents the BJT as a voltage-controlled current source and explicitly includes the input resistance looking into the base, $r_{\pi}$. The model obviously yields $i_{c}=g_{m} v_{b e}$ and $i_{b}=v_{b e} / r_{\pi}$. Not so obvious, however, is the fact that the model also yields the correct expression for $i_{e}$. This can be shown as follows: At the emitter node we have

$$
\begin{aligned}
i_{e} & =\frac{v_{b e}}{r_{\pi}}+g_{m} v_{b e}=\frac{v_{b e}}{r_{\pi}}\left(1+g_{m} r_{\pi}\right) \\
& =\frac{v_{b e}}{r_{\pi}}(1+\beta)=v_{b e} /\left(\frac{r_{\pi}}{1+\beta}\right)=v_{b e} / r_{e}
\end{aligned}
$$

A slightly different equivalent-circuit model can be obtained by expressing the current of the controlled source $\left(g_{m} v_{b e}\right)$ in terms of the base current $i_{b}$ as follows:

$$
g_{m} v_{b e}=g_{m}\left(i_{b} r_{\pi}\right)=\left(g_{m} r_{\pi}\right) i_{b}=\beta i_{b}
$$

This results in the alternative equivalent-circuit model shown in Fig. 6.24(b). Here the transistor is represented as a current-controlled current source, with the control current being $i_{b}$.

(a)

(b)

Figure 6.24 Two slightly different versions of the hybrid- $\pi$ model for the small-signal operation of the BJT. The equivalent circuit in (a) represents the BJT as a voltage-controlled current source (a transconductance amplifier), and that in (b) represents the BJT as a current-controlled current source (a current amplifier).


Figure 6.25 The hybrid- $\pi$ small-signal model, in its two versions, with the resistance $r_{o}$ included.

As we have done in the case of the MOSFET's small-signal models, we can account for the Early effect (the slight dependence of $i_{C}$ on $v_{C E}$ due to basewidth modulation) by adding the resistance $r_{o}=V_{A} / I_{C}$ between collector and emitter, as shown in Fig. 6.25. Note that to conform with the literature, we have renamed $v_{b e}$ as $v_{\pi}$. The two models of Fig. 6.25 are versions of the hybrid $-\pi$ model, the most widely used model for the BJT. The equivalent circuit of Fig. 6.25(a) corresponds to that of the MOSFET (Fig. 6.13b) except for $r_{\pi}$, which accounts for the finite base current (or finite $\beta$ ) of the BJT. However, the equivalent circuit of Fig. 6.25(b) has no MOS counterpart.

It is important to note that the small-signal equivalent circuits of Fig. 6.25 model the operation of the BJT at a given bias point. This should be obvious from the fact that the model parameters $g_{m}, r_{\pi}$, and $r_{o}$ depend on the value of the dc bias current $I_{C}$, as indicated in Fig. 6.25. That is, these equivalent circuits model the incremental operation of the BJT around the bias point.

As in the case of the MOSFET amplifier, including $r_{o}$ in the BJT model causes the voltage gain of the conceptual amplifier of Fig. 6.20(a) to become

$$
\begin{equation*}
\frac{v_{o}}{v_{b e}}=-g_{m}\left(R_{C} \| r_{o}\right) \tag{6.80}
\end{equation*}
$$

Thus, the magnitude of the gain is reduced somewhat.

## EXERCISE

6.17 For the model in Fig. 6.24(b) show that $i_{c}=g_{m} v_{b e}$ and $i_{e}=v_{b e} / r_{e}$.

The T Model Although the hybrid- $\pi$ model (in one of its two variants shown in Fig. 6.24) can be used to carry out small-signal analysis of any transistor circuit, there are situations in which an alternative model, shown in Fig. 6.26, is much more convenient. This model, called, as in the case of the MOSFET, the T model, is shown in two versions in Fig. 6.26. The model of Fig. 6.26(a) represents the BJT as a voltage-controlled current source with the control voltage being $v_{b e}$. Here, however, the resistance between base and emitter, looking into the emitter, is explicitly shown. From Fig. 6.26(a) we see clearly that the model yields the correct expressions for $i_{c}$ and $i_{e}$. It can also be shown to yield the correct expression for $i_{b}$ (see Exercise 6.18).

If in the model of Fig. 6.26(a) the current of the controlled source is expressed in terms of the emitter current as

$$
g_{m} v_{b e}=g_{m}\left(i_{e} r_{e}\right)=\left(g_{m} r_{e}\right) i_{e}=\alpha i_{e}
$$


(a)

(b)

Figure 6.26 Two slightly different versions of what is known as the T model of the BJT. The circuit in (a) is a voltage-controlled current source representation and that in (b) is a current-controlled current source representation. These models explicitly show the emitter resistance $r_{e}$ rather than the base resistance $r_{\pi}$ featured in the hybrid- $\pi$ model.

(a)

(b)

Figure 6.27 The T models of the BJT.
we obtain the alternative T model shown in Fig. 6.26(b). Here the BJT is represented as a current-controlled current source but with the control signal being $i_{e}$.

Finally, the T models can be augmented by $r_{o}$ to account for the dependence of $i_{c}$ to $v_{c e}$ (the Early effect) to obtain the equivalent circuits shown in Fig. 6.27.

## EXERCISE

6.18 Show that for the T model in Fig. 6.24(a), $i_{b}=v_{b e} / r_{\pi}$.

Small-Signal Models of the pnp Transistor Although the small-signal models in Figs. 6.25 and 6.27 were developed for the case of the $n p n$ transistor, they apply equally well to the $p n p$ transistor with no change in polarities.

## Example 6.5

We wish to analyze the transistor amplifier shown in Fig. 6.28(a) to determine its voltage gain $v_{o} / v_{i}$. Assume $\beta=100$ and neglect the Early effect.


Figure 6.28 Example 6.5: (a) amplifier circuit; (b) circuit for dc analysis; (c) amplifier circuit with dc sources replaced by short circuits; (d) amplifier circuit with transistor replaced by its hybrid- $\pi$, small-signal model.

## Example 6.5 continued

## Solution

We shall follow a five-step process:

1. The first step in the analysis consists of determining the quiescent operating point. For this purpose we assume that $v_{i}=0$ and thus obtain the dc circuit shown in Fig. 6.28(b). The dc base current will be

$$
I_{B}=\frac{V_{B B}-V_{B E}}{R_{B B}} \simeq \frac{3-0.7}{100}=0.023 \mathrm{~mA}
$$

The dc collector current will be

$$
I_{C}=\beta I_{B}=100 \times 0.023=2.3 \mathrm{~mA}
$$

The dc voltage at the collector will be

$$
V_{C}=V_{C C}-I_{C} R_{C}=+10-2.3 \times 3=+3.1 \mathrm{~V}
$$

Since $V_{B}$ at +0.7 V is less than $V_{C}$, it follows that in the quiescent condition the transistor will be operating in the active mode. The dc analysis is illustrated in Fig. 6.28(b).
2. Having determined the operating point, we can now proceed to determine the small-signal model parameters:

$$
\begin{aligned}
r_{e} & =\frac{V_{T}}{I_{E}}=\frac{25 \mathrm{mV}}{(2.3 / 0.99) \mathrm{mA}}=10.8 \Omega \\
g_{m} & =\frac{I_{C}}{V_{T}}=\frac{2.3 \mathrm{~mA}}{25 \mathrm{mV}}=92 \mathrm{~mA} / \mathrm{V} \\
r_{\pi} & =\frac{\beta}{g_{m}}=\frac{100}{92}=1.09 \mathrm{k} \Omega
\end{aligned}
$$

3. Replacing $V_{B B}$ and $V_{C C}$ with short circuits results in the circuit in Fig. 6.28(c).
4. To carry out the small-signal analysis it is equally convenient to employ either of the two hybrid- $\pi$, equivalent-circuit models of Fig. 6.24 to replace the transistor in the circuit of Fig. 6.28(c). Using the first results in the amplifier equivalent circuit given in Fig. 6.28(d).
5. Analysis of the equivalent circuit in Fig. 6.28(d) proceeds as follows:

$$
\begin{equation*}
v_{b e}=v_{i} \frac{r_{\pi}}{r_{\pi}+R_{B B}}=v_{i} \frac{1.09}{101.09}=0.011 v_{i} \tag{6.81}
\end{equation*}
$$

The output voltage $v_{o}$ is given by

$$
v_{o}=-g_{m} v_{b e} R_{C}=-92 \times 0.011 v_{i} \times 3=-3.04 v_{i}
$$

Thus the voltage gain will be

$$
\begin{equation*}
A_{v}=\frac{v_{o}}{v_{i}}=-3.04 \mathrm{~V} / \mathrm{V} \tag{6.82}
\end{equation*}
$$

## Example 6.6

To gain more insight into the operation of transistor amplifiers, we wish to consider the waveforms at various points in the circuit analyzed in the previous example. For this purpose assume that $v_{i}$ has a triangular waveform. First determine the maximum amplitude that $v_{i}$ is allowed to have. Then, with the amplitude of $v_{i}$ set to this value, give the waveforms of the total quantities $i_{B}(t), v_{B E}(t), i_{C}(t)$, and $v_{C}(t)$.

## Solution

One constraint on signal amplitude is the small-signal approximation, which stipulates that $v_{b e}$ should not exceed about 10 mV . If we take the triangular waveform $v_{b e}$ to be 20 mV peak-to-peak and work backward, Eq. (6.81) can be used to determine the maximum possible peak of $v_{i}$,

$$
\hat{v}_{i}=\frac{\hat{v}_{b e}}{0.011}=\frac{10}{0.011}=0.91 \mathrm{~V}
$$

To check whether the transistor remains in the active mode with $v_{i}$ having a peak value $\hat{v}_{i}=0.91 \mathrm{~V}$, we have to evaluate the collector voltage. The voltage at the collector will consist of a triangular wave $v_{o}$ superimposed on the dc value $V_{C}=3.1 \mathrm{~V}$. The peak voltage of the triangular waveform will be

$$
\hat{v}_{o}=\hat{v}_{i} \times \text { gain }=0.91 \times 3.04=2.77 \mathrm{~V}
$$

It follows that when the output swings negative, the collector voltage reaches a minimum of $3.1-$ $2.77=0.33 \mathrm{~V}$, which is lower than the base voltage by less than 0.4 V . Thus the transistor will remain in the active mode with $v_{i}$ having a peak value of 0.91 V . Nevertheless, to be on the safe side, we will use a somewhat lower value for $\hat{v}_{i}$ of approximately 0.8 V , as shown in Fig. 6.29(a), and complete the analysis of this problem utilizing the equivalent circuit in Fig. 6.28(d). The signal current in the base will be triangular, with a peak value $\hat{i}_{b}$ of

$$
\hat{i}_{b}=\frac{\hat{v}_{i}}{R_{B B}+r_{\pi}}=\frac{0.8}{100+1.09}=0.008 \mathrm{~mA}
$$

This triangular-wave current will be superimposed on the quiescent base current $I_{B}$, as shown in Fig. 6.29(b). The base-emitter voltage will consist of a triangular-wave component superimposed on the dc $V_{B E}$ that is approximately 0.7 V . The peak value of the triangular waveform will be

$$
\hat{v}_{b e}=\hat{v}_{i} \frac{r_{\pi}}{r_{\pi}+R_{B B}}=0.8 \frac{1.09}{100+1.09}=8.6 \mathrm{mV}
$$

The total $v_{B E}$ is sketched in Fig. 6.29(c).
The signal current in the collector will be triangular in waveform, with a peak value $\hat{i}_{c}$ given by

$$
\hat{i}_{c}=\beta \hat{i}_{b}=100 \times 0.008=0.8 \mathrm{~mA}
$$

This current will be superimposed on the quiescent collector current $I_{C}(=2.3 \mathrm{~mA})$, as shown in Fig. 6.29(d).

The signal voltage at the collector can be obtained by multiplying $v_{i}$ by the voltage gain; that is,

$$
\hat{v}_{o}=3.04 \times 0.8=2.43 \mathrm{~V}
$$

## Example 6.6 continued

Figure 6.29(e) shows a sketch of the total collector voltage $v_{C}$ versus time. Note the phase reversal between the input signal $v_{i}$ and the output signal $v_{o}$.

Finally, we observe that each of the total quantities is the sum of a dc quantity (found from the dc circuit in Fig. 6.28b), and a signal quantity (found from the circuit in Fig. 6.28d).

(a)

(b)

(c)

(d)

Figure 6.29 Signal waveforms in the circuit of Fig. 6.28.

## Example 6.6 continued



Figure 6.29 continued

## Example 6.7

We need to analyze the circuit of Fig. 6.30(a) to determine the voltage gain and the signal waveforms at various points. The capacitor $C_{C 1}$ is a coupling capacitor whose purpose is to couple the signal $v_{i}$ to the emitter while blocking dc. In this way the dc bias established by $V^{+}$and $V^{-}$together with $R_{E}$ and $R_{C}$ will not be disturbed when the signal $v_{i}$ is connected. For the purpose of this example, $C_{C 1}$ will be assumed to be very large so as to act as a perfect short circuit at signal frequencies of interest. Similarly, another very large capacitor $C_{C 2}$ is used to couple the output signal $v_{o}$ to other parts of the system. You may neglect the Early effect.

(a)

(b)

Figure 6.30 Example 6.7: (a) circuit; (b) dc analysis; (c) circuit with the dc sources eliminated; (d) small-signal analysis using the T model for the BJT.

## Example 6.7 continued


(c)

(d)

Figure 6.30 continued

## Solution

Here again we shall follow a five-step process:

1. Figure 6.30 (b) shows the circuit with the signal source and the coupling capacitors eliminated. The dc operating point can be determined as follows:

$$
I_{E}=\frac{+10-V_{E}}{R_{E}} \simeq \frac{+10-0.7}{10}=0.93 \mathrm{~mA}
$$

Assuming $\beta=100$, then $\alpha=0.99$, and

$$
\begin{aligned}
I_{C} & =0.99 I_{E}=0.92 \mathrm{~mA} \\
V_{C} & =-10+I_{C} R_{C}=-10+0.92 \times 5=-5.4 \mathrm{~V}
\end{aligned}
$$

Thus the transistor is in the active mode.
2. We now determine the small-signal parameters as follows:

$$
\begin{aligned}
& g_{m}=\frac{I_{C}}{V_{T}}=\frac{0.92}{0.025}=36.8 \mathrm{~mA} / \mathrm{V} \\
& r_{e}=\frac{V_{T}}{I_{E}}=\frac{0.025}{0.92}=27.2 \Omega \\
& \beta=100 \quad \alpha=0.99 \\
& r_{\pi}=\frac{\beta}{g_{m}}=\frac{100}{36.8}=2.72 \mathrm{k} \Omega
\end{aligned}
$$

## Example 6.7 continued

3. To prepare the circuit for small-signal analysis, we replace the dc sources with short circuits. The resulting circuit is shown in Fig. 6.30(c). Observe that we have also eliminated the two coupling capacitors, since they are assumed to be acting as perfect short circuits.
4. We are now ready to replace the BJT with one of the four equivalent-circuit models of Figs. 6.24 and 6.26. Although any of the four will work, the T models of Fig. 6.26 will be more convenient because the base is grounded. Selecting the version in Fig. 6.26(b) results in the amplifier equivalent circuit shown in Fig. 6.30(d).
5. Analysis of the circuit in Fig. 6.30(d) to determine the output voltage $v_{o}$ and hence the voltage gain $v_{o} / v_{i}$ is straightforward and is given in the figure. The result is

$$
A_{v}=\frac{v_{o}}{v_{i}}=\frac{\alpha R_{C}}{r_{e}}=\frac{0.99 \times 5}{0.0272}=182 \mathrm{~V} / \mathrm{V}
$$

Note that the voltage gain is positive, indicating that the output is in phase with the input signal. This property is due to the fact that the input signal is applied to the emitter rather than to the base, as was done in Example 6.5. We should emphasize that the positive gain has nothing to do with the fact that the transistor used in this example is of the pnp type.

Returning to the question of allowable signal magnitude, we observe from Fig. 6.30(d) that $v_{e b}=v_{i}$. Thus, if small-signal operation is desired (for linearity), then the peak of $v_{i}$ should be limited to approximately 10 mV . With $\hat{V}_{i}$ set to this value, as shown for a sine-wave input in Fig. 6.31, the peak amplitude at the collector, $\hat{V}_{o}$, will be

$$
\hat{V}_{o}=182 \times 0.01=1.82 \mathrm{~V}
$$



Figure 6.31 Input and output waveforms for the circuit of Fig. 6.30. Observe that this amplifier is noninverting, a property of the grounded-base configuration.

## EXERCISE

6.19 To increase the voltage gain of the amplifier analyzed in Example 6.7, the collector resistance $R_{C}$ is increased to $7.5 \mathrm{k} \Omega$. Find the new values of $V_{C}, A_{v}$, and the peak amplitude of the output sine wave corresponding to an input sine wave $v_{i}$ of $10-\mathrm{mV}$ peak.

Ans. $-3.1 \mathrm{~V} ; 276 \mathrm{~V} / \mathrm{V} ; 2.76 \mathrm{~V}$

Performing Small-Signal Analysis Directly on the Circuit Diagram In most cases one should explicitly replace each BJT with its small-signal model and analyze the resulting circuit, as we have done in the examples above. This systematic procedure is particularly recommended for beginning students. Experienced circuit designers, however, often perform a first-order analysis directly on the circuit. Figure 6.32 illustrates this process for the two circuits we analyzed in Examples 6.5 and 6.7. The reader is urged to follow this direct analysis procedure (the steps are numbered). Observe that the equivalent-circuit model is implicitly utilized; we are only saving the step of drawing the circuit with the BJT replaced by its model. Direct analysis, however, has an additional very important benefit: It provides insight regarding the signal transmission through the circuit. Such insight can prove invaluable in design, particularly at the stage of selecting a circuit configuration appropriate for a given application. Direct analysis can be utilized also for MOS amplifier circuits.


Figure 6.32 Performing signal analysis directly on the circuit diagram with the BJT small-signal model implicitly employed: (a) circuit for Example 6.5; (b) circuit for Example 6.7.

## EXERCISE

6.20 The transistor in Fig. E6.20 is biased with a constant current source $I=1 \mathrm{~mA}$ and has $\beta=100$ and $V_{A}=100 \mathrm{~V}$.
(a) Neglecting the Early effect, find the dc voltages at the base, emitter, and collector.
(b) Find $g_{m}, r_{\pi}$, and $r_{o}$.
(c) If terminal Z is connected to ground, X to a signal source $v_{\text {sig }}$ with a source resistance $R_{\text {sig }}=2 \mathrm{k} \Omega$, and Y to an $8-\mathrm{k} \Omega$ load resistance, use the hybrid- $\pi$ model shown earlier (Fig. 6.25) to draw the small-signal equivalent circuit of the amplifier. (Note that the current source $I$ should be replaced with an open circuit.) Calculate the overall voltage gain $v_{y} / v_{\text {sig }}$. If $r_{o}$ is neglected, what is the error in estimating the gain magnitude? (Note: An infinite capacitance is used to indicate that the capacitance is sufficiently large that it acts as a short circuit at all signal frequencies of interest. However, the capacitor still blocks dc.)


Figure E6.20

Ans. (a) $-0.1 \mathrm{~V},-0.8 \mathrm{~V},+2.1 \mathrm{~V}$; (b) $40 \mathrm{~mA} / \mathrm{V}, 2.5 \mathrm{k} \Omega, 100 \mathrm{k} \Omega$; (c) $-77 \mathrm{~V} / \mathrm{V},+3.9 \%$

### 6.2.3 Summary Tables

We conclude this section by presenting three useful summary tables: Table 6.1 lists the five steps to be followed in the analysis of a MOSFET or a BJT amplifier circuit. Table 6.2 presents the MOSFET small-signal, equivalent-circuit models, together with the formulas for calculating the parameter values of the models. Finally, Table 6.3 supplies the corresponding data for the BJT.

Table 6.1 Systematic Procedure for the Analysis of Transistor Amplifier Circuits

1. Eliminate the signal source and determine the dc operating point of the transistor.
2. Calculate the values of the parameters of the small-signal model.
3. Eliminate the dc sources by replacing each dc voltage source by a short circuit and each dc current source by an open circuit.
4. Replace the transistor with one of its small-signal, equivalent-circuit models. Although any of the models can be used, one might be more convenient than the others for the particular circuit being analyzed. This point will be made clearer in the next section.
5. Analyze the resulting circuit to determine the required quantities (e.g., voltage gain, input resistance).

Table 6.2 Small-Signal Models of the MOSFET

## Small-Signal Parameters

NMOS transistors

- Transconductance:

$$
g_{m}=\mu_{n} C_{o x} \frac{W}{L} V_{O V}=\sqrt{2 \mu_{n} C_{o x} \frac{W}{L} I_{D}}=\frac{2 I_{D}}{V_{O V}}
$$

- Output resistance:
$r_{o}=V_{A} / I_{D}=1 / \lambda I_{D}$


## PMOS transistors

Same formulas as for NMOS except using $\left|V_{O V}\right|,\left|V_{A}\right|,|\lambda|$ and replacing $\mu_{n}$ with $\mu_{p}$.

Small-Signal, Equivalent-Circuit Models


Hybrid- $\pi$ model

### 6.3 Basic Configurations

It is useful at this point to take stock of where we are and where we are going in our study of transistor amplifiers. In Section 6.1 we examined the underlying principle for the application of the MOSFET, and of the BJT, as an amplifier. There we found that almost-linear amplification can be obtained by dc biasing the transistor at an appropriate point in its active region of operation, and by keeping the input signal ( $v_{g s}$ or $v_{b e}$ ) small. We then developed, in Section 6.2, circuit models that represent the small-signal operation of each of the two transistor types (Tables 6.2 and 6.3), thus providing a systematic procedure (Table 6.1) for the analysis of transistor amplifiers.

We are now ready to consider the various possible configurations of MOSFET and BJT amplifiers, and we will do that in the present section. To focus our attention on the salient features of the various configurations, we shall present them in their most simple, or "stripped-down," version. Thus, we will not show the de biasing arrangements, leaving the study of bias design to the next section. Finally, in Section 6.5 we will

## Table 6.3 Small-Signal Models of the BJT

Hybrid- $\pi$ Model

■ $\left(g_{m} v_{\pi}\right)$ Version


- ( $\beta i_{b}$ ) Version



## T Model

- $\left(g_{m} v_{\pi}\right)$ Version


Model Parameters in Terms of DC Bias Currents
$g_{m}=\frac{I_{C}}{V_{T}}$
$r_{e}=\frac{V_{T}}{I_{E}}=\alpha \frac{V_{T}}{I_{C}}$
$r_{\pi}=\frac{V_{T}}{I_{B}}=\beta \frac{V_{T}}{I_{C}}$
$r_{o}=\frac{\left|V_{A}\right|}{I_{C}}$

In Terms of $g_{m}$
$r_{e}=\frac{\alpha}{g_{m}} \quad r_{\pi}=\frac{\beta}{g_{m}}$
In Terms of $r_{e}$
$g_{m}=\frac{\alpha}{r_{e}}$
$r_{\pi}=(\beta+1) r_{e}$
$g_{m}+\frac{1}{r_{\pi}}=\frac{1}{r_{e}}$

Relationships between $\alpha$ and $\beta$
$\beta=\frac{\alpha}{1-\alpha}$
$\beta+1=\frac{1}{1-\alpha}$
bring everything together and present practical discrete-circuit amplifiers, namely, amplifier circuits that can be constructed using discrete components. The study of integrated-circuit amplifiers begins in Chapter 7.

### 6.3.1 The Three Basic Configurations

There are three basic configurations for connecting a MOSFET or a BJT as an amplifier. Each of these configurations is obtained by connecting one of the device terminals to ground, thus creating a two-port network with the grounded terminal being common to the input and output ports. The resulting configurations are shown in Fig. 6.33(a-c) for the MOSFET and in Fig. 6.33(d-f) for the BJT.

In the circuit of Fig. 6.33(a) the source terminal is connected to ground, the input voltage signal $v_{i}$ is applied between the gate and ground, and the output voltage signal $v_{o}$ is taken between the drain and ground, across the resistance $R_{D}$. This configuration, therefore, is called the grounded-source or common-source (CS) amplifier. It is by far the most popular MOS amplifier configuration, and we utilized it in Sections 6.1 and 6.2 to study MOS amplifier operation. A parallel set of remarks apply to the BJT counterpart, the grounded-emitter or common-emitter (CE) amplifier in Fig. 6.33(d).

The common-gate (CG) or grounded-gate amplifier is shown in Fig. 6.33(b), and its BJT counterpart, the common-base (CB) or grounded-base amplifier in Fig. 6.33(e). Here the gate (base) is grounded, the input signal $v_{i}$ is applied to the source (emitter), and the output signal $v_{o}$ is taken at the drain (collector) across the resistance $R_{D}\left(R_{C}\right)$. We encountered a CG amplifier in Example 6.4 and a CB amplifier in Example 6.7.

Finally, Fig. 6.33(c) shows the common drain (CD) or grounded-drain amplifier, and Fig. 6.33(f) shows its BJT counterpart, the common-collector (CC) or grounded collector amplifier. Here the drain (collector)

(a) Common Source (CS)

(d) Common-Emitter (CE)

(b) Common Gate (CG)

(e) Common-Base (CB)

(c) Common Drain (CD) or Source Follower

(f) Common-Collector (CC) or Emitter Follower

Figure 6.33 The basic configurations of transistor amplifiers. (a)-(c) For the MOSFET; (d)-(f) for the BJT.
terminal is grounded, the input signal $v_{i}$ is applied between gate (base) and ground, and the output voltage $v_{o}$ is taken between the source (emitter) and ground, across a resistance $R_{L}$. For reasons that will become apparent shortly, this pair of configurations is more commonly called the source follower and the emitter follower.

Our study of the three basic amplifier configurations of the MOSFET and of the BJT will reveal that each has distinctly different attributes, hence areas of application. As well, it will be shown that although each pair of configurations (e.g., CS and CE) has many common attributes, important differences remain.

Our next step is to replace the transistor in each of the six circuits in Fig. 6.33 by an appropriate equivalent-circuit model (from Tables 6.2 and 6.3 ) and analyze the resulting circuits to determine important characteristic parameters of the particular amplifier configuration. To simplify matters, we shall not include $r_{o}$ in the initial analysis. At the end of the section we will offer a number of comments about when to include $r_{o}$ in the analysis, and on the expected magnitude of its effect.

### 6.3.2 Characterizing Amplifiers

Before we begin our study of the different transistor amplifier configurations, we consider how to characterize the performance of an amplifier as a circuit building block. An introduction to this topic was presented in Section 1.5.

Figure 6.34(a) shows an amplifier fed with a signal source having an open-circuit voltage $v_{\text {sig }}$ and an internal resistance $R_{\text {sig }}$. These can be the parameters of an actual signal source or, in a cascade amplifier, the Thévenin equivalent of the output circuit of another amplifier stage preceding the one under study. The amplifier is shown with a load resistance $R_{L}$ connected to the output terminal. Here, $R_{L}$ can be an actual load resistance or the input resistance of a succeeding amplifier stage in a cascade amplifier.

Figure 6.34(b) shows the amplifier circuit with the amplifier block replaced by its equivalent-circuit model. The input resistance $R_{\text {in }}$ represents the loading effect of the amplifier input on the signal source. It is found from

$$
R_{\mathrm{in}} \equiv \frac{v_{i}}{i_{i}}
$$

and together with the resistance $R_{\text {sig }}$ forms a voltage divider that reduces $v_{\text {sig }}$ to the value $v_{i}$ that appears at the amplifier input,

$$
\begin{equation*}
v_{i}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} v_{\mathrm{sig}} \tag{6.83}
\end{equation*}
$$

Most of the amplifier circuits studied in this section are unilateral. That is, they do not contain internal feedback, and thus $R_{\text {in }}$ will be independent of $R_{L}$. However, in general $R_{\text {in }}$ may depend on the load resistance $R_{L}$. Indeed one of the six configurations studied in this section, the emitter follower, exhibits such dependence.

The second parameter in characterizing amplifier performance is the open-circuit voltage gain $A_{v o}$, defined as

$$
\left.A_{v o} \equiv \frac{v_{o}}{v_{i}}\right|_{R_{L}=\infty}
$$

The third and final parameter is the output resistance $R_{o}$. Observe from Fig. 6.34(b) that $R_{o}$ is the resistance seen looking back into the amplifier output terminal with $v_{i}$ set to zero. Thus $R_{o}$ can be determined, at least conceptually, as indicated in Fig. 6.34(c) with

$$
R_{o}=\frac{v_{x}}{i_{x}}
$$



Figure 6.34 Characterization of the amplifier as a functional block: (a) An amplifier fed with a voltage signal $v_{\text {sig }}$ having a source resistance $R_{\text {sig }}$, and feeding a load resistance $R_{L}$; (b) equivalent-circuit representation of the circuit in (a); (c) determining the amplifier output resistance $R_{o}$.

Because $R_{o}$ is determined with $v_{i}=0$, the value of $R_{o}$ does not depend on $R_{\text {sig }}$.
The controlled source $A_{v o} v_{i}$ and the output resistance $R_{o}$ represent the Thévenin equivalent of the amplifier output circuit, and the output voltage $v_{o}$ can be found from

$$
\begin{equation*}
v_{o}=\frac{R_{L}}{R_{L}+R_{o}} A_{v o} v_{i} \tag{6.84}
\end{equation*}
$$

Thus the voltage gain of the amplifier proper, $A_{v}$, can be found as

$$
\begin{equation*}
A_{v} \equiv \frac{v_{o}}{v_{i}}=A_{v o} \frac{R_{L}}{R_{L}+R_{o}} \tag{6.85}
\end{equation*}
$$

and the overall voltage gain, $G_{v}$,

$$
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}
$$

can be determined by combining Eqs. (6.83) and (6.85):

$$
\begin{equation*}
G_{v}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} A_{v o} \frac{R_{L}}{R_{L}+R_{o}} \tag{6.86}
\end{equation*}
$$

### 6.3.3 The Common-Source (CS) and Common-Emitter (CE) Amplifiers

Of the three basic transistor amplifier configurations, the common-source (common-emitter, for BJT), is the most widely used. Typically, in an amplifier formed by cascading a number of gain stages, the bulk of the voltage gain is obtained by using one or more common-source (or common-emitter, for BJT) stages in cascade.

Characteristic Parameters of the CS Amplifier Figure 6.35(a) shows a common-source amplifier (with the biasing arrangement omitted) fed with a signal source $v_{\text {sig }}$ having a source resistance $R_{\text {sig }}$. We wish to analyze this circuit to determine $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$. For this purpose, we assume that $R_{D}$ is part of the amplifier; thus if a load resistance $R_{L}$ is connected to the amplifier output, $R_{L}$ appears in parallel with $R_{D}$. In such a case, we wish to determine $A_{v}$ and $G_{v}$ as well.

Replacing the MOSFET with its hybrid- $\pi$ model (without $r_{o}$ ), we obtain the CS amplifier equivalent circuit in Fig. 6.35(b) for which, tracing the signal from input to output, we can write by inspection

$$
\begin{align*}
R_{\mathrm{in}} & =\infty  \tag{6.87}\\
v_{i} & =v_{\mathrm{sig}} \\
v_{g s} & =v_{i} \\
v_{o} & =-g_{m} v_{g s} R_{D}
\end{align*}
$$

Thus,

$$
\begin{gather*}
A_{v o} \equiv \frac{v_{o}}{v_{i}}=-g_{m} R_{D}  \tag{6.88}\\
R_{o}=R_{D} \tag{6.89}
\end{gather*}
$$


(b)

Figure 6.35 (a) Common-source amplifier fed with a signal $v_{\text {sig }}$ from a generator with a resistance $R_{\text {sig }}$. The bias circuit is omitted. (b) The common-source amplifier with the MOSFET replaced with its hybrid- $\pi$ model.

If a load resistance $R_{L}$ is connected across $R_{D}$, the voltage gain $A_{v}$ can be obtained from

$$
\begin{equation*}
A_{v}=A_{v o} \frac{R_{L}}{R_{L}+R_{o}} \tag{6.90}
\end{equation*}
$$

where $A_{v o}$ is given by Eq. (6.88) and $R_{o}$ by Eq. (6.89), or alternatively by simply adding $R_{L}$ in parallel with $R_{D}$ in Eq. (6.88), thus

$$
\begin{equation*}
A_{v}=-g_{m}\left(R_{D} \| R_{L}\right) \tag{6.91}
\end{equation*}
$$

The reader can easily show that the expression obtained from Eq. (6.90) is identical to that in Eq. (6.91). Finally, since $R_{\mathrm{in}}=\infty$ and thus $v_{i}=v_{\mathrm{sig}}$, the overall voltage gain $G_{v}$ is equal to $A_{v}$,

$$
\begin{equation*}
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=-g_{m}\left(R_{D} \| R_{L}\right) \tag{6.92}
\end{equation*}
$$

## EXERCISE

6.21 A CS amplifier utilizes a MOSFET biased at $I_{D}=0.25 \mathrm{~mA}$ with $V_{O V}=0.25 \mathrm{~V}$ and $R_{D}=20 \mathrm{k} \Omega$. The amplifier is fed with a signal source having $R_{\text {sig }}=100 \mathrm{k} \Omega$, and a $20-\mathrm{k} \Omega$ load is connected to the output. Find $R_{\mathrm{in}}, A_{v o}, R_{o}, A_{v}$, and $G_{v}$. If, to maintain reasonable linearity, the peak of the input sine-wave signal is limited to $10 \%$ of $2 V_{O V}$, what is the peak of the sine-wave voltage at the output?

Ans. $\infty$; $-40 \mathrm{~V} / \mathrm{V} ; 20 \mathrm{k} \Omega ;-20 \mathrm{~V} / \mathrm{V} ;-20 \mathrm{~V} / \mathrm{V} ; 1 \mathrm{~V}$

Characteristic Parameters of the CE Amplifier Figure 6.36(a) shows a common-emitter amplifier. Its equivalent circuit, obtained by replacing the BJT with its hybrid- $\pi$ model (without $r_{o}$ ), is shown in Fig. 6.36(b). The latter circuit can be analyzed to obtain the characteristic parameters of the CE amplifier. The analysis parallels that for the MOSFET above except that here we have the added complexity of a finite input resistance $r_{\pi}$. Tracing the signal through the amplifier from input to output, we can write by inspection

$$
R_{\mathrm{in}}=r_{\pi}
$$

Then we write

$$
\begin{align*}
v_{i} & =\frac{r_{\pi}}{r_{\pi}+R_{\mathrm{sig}}} v_{\mathrm{sig}}  \tag{6.93}\\
v_{\pi} & =v_{i} \\
v_{o} & =-g_{m} v_{\pi} R_{C}
\end{align*}
$$

Thus,

$$
\begin{gather*}
A_{v o} \equiv \frac{v_{o}}{v_{i}}=-g_{m} R_{C}  \tag{6.94}\\
R_{o}=R_{C} \tag{6.95}
\end{gather*}
$$

With a load resistance $R_{L}$ connected across $R_{C}$,

$$
\begin{equation*}
A_{v}=-g_{m}\left(R_{C} \| R_{L}\right) \tag{6.96}
\end{equation*}
$$



Figure 6.36 (a) Common-emitter amplifier fed with a signal $v_{\text {sig }}$ from a generator with a resistance $R_{\text {sig }}$. The bias circuit is omitted. (b) The common-emitter amplifier circuit with the BJT replaced by its hybrid- $\pi$ model.
and the overall voltage gain $G_{v}$ can be found from

$$
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=\frac{v_{i}}{v_{\text {sig }}} \frac{v_{o}}{v_{i}}
$$

Thus,

$$
\begin{equation*}
G_{v}=-\frac{r_{\pi}}{r_{\pi}+R_{\mathrm{sig}}} g_{m}\left(R_{C} \| R_{L}\right) \tag{6.97}
\end{equation*}
$$

It is important to note here the effect of the finite input resistance $\left(r_{\pi}\right)$ in reducing the magnitude of the voltage gain by the voltage-divider ratio $r_{\pi} /\left(r_{\pi}+R_{\text {sig }}\right)$. The extent of the gain reduction depends on the relative values of $r_{\pi}$ and $R_{\text {sig }}$. However, there is a compensating effect in the CE amplifier: $g_{m}$ of the BJT is usually much higher than the corresponding value of the MOSFET.

## Example 6.8

A CE amplifier utilizes a BJT with $\beta=100$ is biased at $I_{C}=1 \mathrm{~mA}$ and has a collector resistance $R_{C}=5 \mathrm{k} \Omega$. Find $R_{\mathrm{in}}, R_{o}$, and $A_{v o}$. If the amplifier is fed with a signal source having a resistance of $5 \mathrm{k} \Omega$, and a load resistance $R_{L}=5 \mathrm{k} \Omega$ is connected to the output terminal, find the resulting $A_{v}$ and $G_{v}$. If $\hat{v}_{\pi}$ is to be limited to 5 mV , what are the corresponding $\hat{v}_{\text {sig }}$ and $\hat{v}_{o}$ with the load connected?

## Example 6.8 continued

## Solution

At $I_{C}=1 \mathrm{~mA}$,

$$
\begin{aligned}
g_{m} & =\frac{I_{C}}{V_{T}}=\frac{1 \mathrm{~mA}}{0.025 \mathrm{~V}}=40 \mathrm{~mA} / \mathrm{V} \\
r_{\pi} & =\frac{\beta}{g_{m}}=\frac{100}{40 \mathrm{~mA} / \mathrm{V}}=2.5 \mathrm{k} \Omega
\end{aligned}
$$

The amplifier characteristic parameters can now be found as

$$
\begin{aligned}
R_{\mathrm{in}} & =r_{\pi}=2.5 \mathrm{k} \Omega \\
A_{v o} & =-g_{m} R_{C}=-40 \mathrm{~mA} / \mathrm{V} \times 5 \mathrm{k} \Omega=-200 \mathrm{~V} / \mathrm{V} \\
R_{o} & =R_{C}=5 \mathrm{k} \Omega
\end{aligned}
$$

With a load resistance $R_{L}=5 \mathrm{k} \Omega$ connected at the output, we can find $A_{v}$ by either of the following two approaches:

$$
A_{v}=A_{v o} \frac{R_{L}}{R_{L}+R_{o}}=-200 \times \frac{5}{5+5}=-100 \mathrm{~V} / \mathrm{V}
$$

or

$$
A_{v}=-g_{m}\left(R_{C} \| R_{L}\right)=-40(5 \| 5)=-100 \mathrm{~V} / \mathrm{V}
$$

The overall voltage gain $G_{v}$ can now be determined as

$$
G_{v}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} A_{v}=\frac{2.5}{2.5+5} \times-100=-33.3 \mathrm{~V} / \mathrm{V}
$$

If the maximum amplitude of $v_{\pi}$ is to be 5 mV , the corresponding value of $\hat{v}_{\text {sig }}$ will be

$$
\hat{v}_{\mathrm{sig}}=\left(\frac{R_{\mathrm{in}}+R_{\mathrm{sig}}}{R_{\mathrm{in}}}\right) \hat{v}_{\pi}=\frac{2.5+5}{2.5} \times 5=15 \mathrm{mV}
$$

and the amplitude of the signal at the output will be

$$
\hat{v}_{o}=G_{v} \hat{v}_{\text {sig }}=33.3 \times 0.015=0.5 \mathrm{~V}
$$

## EXERCISE

6.22 The designer of the amplifier in Example 6.8 decides to lower the bias current to half its original value in order to raise the input resistance and hence increase the fraction of $v_{\text {sig }}$ that appears at the input of the amplifier proper. In an attempt to maintain the voltage gain, the designer decides to double the value of $R_{C}$. For the new design, determine $R_{\mathrm{in}}, A_{v o}, R_{o}, A_{v}$, and $G_{v}$. If the peak amplitude of $v_{\pi}$ is to be limited to 5 mV , what are the corresponding values of $\hat{v}_{\text {sig }}$ and $\hat{v}_{o}$ (with the load connected)?

Ans. $5 \mathrm{k} \Omega ;-200 \mathrm{~V} / \mathrm{V} ; 10 \mathrm{k} \Omega ;-66.7 \mathrm{~V} / \mathrm{V} ;-33.3 \mathrm{~V} / \mathrm{V} ; 10 \mathrm{mV} ; 0.33 \mathrm{~V}$
Comment: Although a larger fraction of the input signal reaches the amplifier input, linearity considerations cause the output signal to be in fact smaller than in the original design!

## Final Remarks

1. The CS and CE amplifiers are the most useful of all transistor amplifier configurations. They exhibit a moderate to high input resistance (infinite for the CS), a moderate to high output resistance, and reasonably high voltage gain.
2. The input resistance of the CE amplifier, $R_{\mathrm{in}}=r_{\pi}=\beta / g_{m}$, is inversely proportional to the dc bias current $I_{C}$. To increase $R_{\text {in }}$ one is tempted to lower the bias current $I_{C}$; however, this also lowers $g_{m}$ and hence the voltage gain. This is a significant design trade-off. If a much higher input resistance is desired, then a modification of the CE configuration (to be discussed in Section 6.3.4) can be applied, or an emitter-follower stage can be inserted between the signal source and the CE amplifier (see Section 6.3.6).
3. Reducing $R_{D}$ or $R_{C}$ to lower the output resistance of the CS or CE amplifier, respectively, is usually not a viable proposition because the voltage gain is also reduced. Alternatively, if a very low output resistance (in the ohms or tens-of-ohms range) is needed, a source-follower or an emitter-follower stage can be utilized between the output of the CS or CE amplifier and the load resistance (see Section 6.3.6).
4. Although the CS and the CE configurations are the workhorses of transistor amplifiers, both suffer from a limitation on their high-frequency response. As will be shown in Chapter 9, combining the CS (CE) amplifier with a CG (CB) amplifier can extend the bandwidth considerably. The CG and CB amplifiers are studied in Section 6.3.5.

### 6.3.4 The Common-Source (Common-Emitter) Amplifier with a Source (Emitter) Resistance

It is often beneficial to insert a resistance $R_{s}$ (a resistance $R_{e}$ ) in the source lead (the emitter lead) of a common-source (common-emitter) amplifier. Figure 6.37(a) shows a CS amplifier with a resistance $R_{s}$ in its source lead. The corresponding small-signal equivalent circuit is shown in Fig. 6.37(b), where we have utilized the T model for the MOSFET. The T model is used in preference to the hybrid- $\pi$ model because it makes the analysis in this case considerably simpler. In general, whenever a resistance is connected in the source lead, the T model is preferred. The source resistance then simply appears in series with the model resistance $1 / g_{m}$ and can be added to it.

From Fig. 6.37(b) we see that as expected, the input resistance $R_{\text {in }}$ is infinite and thus $v_{i}=v_{\text {sig }}$. Unlike the CS amplifier, however, here only a fraction of $v_{i}$ appears between gate and source as $v_{g s}$. The voltage divider composed of $1 / g_{m}$ and $R_{s}$, which appears across the amplifier input, can be used to determine $v_{g s}$, as follows:

$$
\begin{equation*}
v_{g s}=v_{i} \frac{1 / g_{m}}{1 / g_{m}+R_{s}}=\frac{v_{i}}{1+g_{m} R_{s}} \tag{6.98}
\end{equation*}
$$

Thus we can use the value of $R_{s}$ to control the magnitude of the signal $v_{g s}$ and thereby ensure that $v_{g s}$ does not become too large and cause unacceptably high nonlinear distortion. This is the first benefit of including resistor $R_{s}$. Other benefits will be encountered in later sections and chapters. For instance, it will be shown in Chapter 9 that $R_{s}$ causes the useful bandwidth of the amplifier to be extended. The mechanism by which $R_{s}$ causes such improvements in amplifier performance is negative feedback. To see how $R_{s}$ introduces negative feedback, refer to Fig. 6.37(a): If with $v_{\text {sig }}$ and hence $v_{i}$ kept constant, the drain current increases for some reason, the source current also will increase, resulting in an increased voltage drop across $R_{s}$. Thus the source voltage rises, and the gate-to-source voltage decreases. The latter effect causes the drain current to decrease, counteracting the initially assumed change, an indication of the presence of negative feedback. In Chapter 10 we shall study negative feedback formally. There we will learn that the improvements that negative feedback


Figure 6.37 The CS amplifier with a source resistance $R_{s}$ : (a) circuit without bias details; (b) equivalent circuit with the MOSFET represented by its T model.
provides are obtained at the expense of a reduction in gain. We will now show this to be the case in the circuit of Fig. 6.37.

The output voltage $v_{o}$ is obtained by multiplying the controlled-source current $i$ by $R_{D}$,

$$
v_{o}=-i R_{D}
$$

The current $i$ in the source lead can be found by dividing $v_{i}$ by the total resistance in the source,

$$
\begin{equation*}
i=\frac{v_{i}}{1 / g_{m}+R_{s}}=\left(\frac{g_{m}}{1+g_{m} R_{s}}\right) v_{i} \tag{6.99}
\end{equation*}
$$

Thus, the voltage gain $A_{v o}$ can be found as

$$
\begin{equation*}
A_{v o} \equiv \frac{v_{o}}{v_{i}}=-\frac{g_{m} R_{D}}{1+g_{m} R_{s}} \tag{6.100}
\end{equation*}
$$

which can also be expressed as

$$
\begin{equation*}
A_{v o}=-\frac{R_{D}}{1 / g_{m}+R_{s}} \tag{6.101}
\end{equation*}
$$

Equation (6.100) indicates that including the resistance $R_{s}$ reduces the voltage gain by the factor $\left(1+g_{m} R_{s}\right)$. This is the price paid for the improvements that accrue as a result of $R_{s}$. It is interesting to note that in Chapter 10, we will find that the factor $\left(1+g_{m} R_{s}\right)$ is the "amount of negative feedback" introduced by $R_{s}$. It is also the same factor by which linearity, bandwidth, and other performance parameters improve. Because of the negative-feedback action of $R_{s}$ it is known as a source-degeneration resistance.

There is another useful interpretation of the expression for the drain current in Eq. (6.99): The quantity between brackets on the right-hand side can be thought of as the "effective transconductance with $R_{s}$ included." Thus, including $R_{s}$ reduces the transconductance by the factor $\left(1+g_{m} R_{s}\right)$. This, of course, is simply the result of the fact that only a fraction $1 /\left(1+g_{m} R_{s}\right)$ of $v_{i}$ appears as $v_{g s}$ (see Eq. 6.98).

The alternative gain expression in Eq. (6.101) has a powerful and insightful interpretation: The voltage gain between gate and drain is equal to the ratio of the total resistance in the drain $\left(R_{D}\right)$ to the total resistance in the source $\left(1 / g_{m}+R_{s}\right)$,

$$
\begin{equation*}
\text { Voltage gain from gate to drain }=-\frac{\text { Total resistance in drain }}{\text { Total resistance in source }} \tag{6.102}
\end{equation*}
$$

This is a general expression. For instance, setting $R_{s}=0$ in Eq. (6.101) yields $A_{v o}$ of the CS amplifier.
Finally, we consider the situation of a load resistance $R_{L}$ connected at the output. We can obtain the gain $A_{v}$ using the open-circuit voltage gain $A_{v o}$ together with the output resistance $R_{o}$, which can be found by inspection to be

$$
R_{o}=R_{D}
$$

Alternatively, $A_{v}$ can be obtained by simply replacing $R_{D}$ in Eq. (6.101) or (6.100) by ( $R_{D} \| R_{L}$ ); thus,

$$
\begin{equation*}
A_{v}=-\frac{g_{m}\left(R_{D} \| R_{L}\right)}{1+g_{m} R_{s}} \tag{6.103}
\end{equation*}
$$

or

$$
\begin{equation*}
A_{v}=-\frac{R_{D} \| R_{L}}{1 / g_{m}+R_{s}} \tag{6.104}
\end{equation*}
$$

Observe that Eq. (6.104) is a direct application of the ratio of total resistance rule of Eq. (6.102). Finally, note that because $R_{\text {in }}$ is infinite, $v_{i}=v_{\text {sig }}$ and the overall voltage gain $G_{v}$ is equal to $A_{v}$.

## EXERCISE

6.23 In Exercise 6.21 we applied an input signal $v_{\text {sig }}$ of 50 mV peak and obtained an output signal of approximately 1 V peak. Assume that for some reason we now have an input signal $v_{\text {sig }}$ that is 0.2 V peak and that we wish to modify the circuit to keep $v_{g s}$ unchanged, and thus keep the nonlinear distortion from increasing. What value should we use for $R_{s}$ ? What value of $G_{v}$ will result? What will the peak signal at the output become? Assume $r_{o}=\infty$.

Ans. $1.5 \mathrm{k} \Omega ;-5 \mathrm{~V} / \mathrm{V} ; 1 \mathrm{~V}$

We next turn our attention to the BJT case. Figure 6.38(a) shows a CE amplifier with a resistance $R_{e}$ in its emitter. The corresponding equivalent circuit, utilizing the T model, is shown in Fig. 6.38(b). Note that in the BJT case also, as a general rule, the T model results in a simpler analysis and should be employed whenever there is a resistance in series with the emitter.

To determine the amplifier input resistance $R_{\text {in }}$, we note from Fig. 6.38(b) that

$$
R_{\mathrm{in}} \equiv \frac{v_{i}}{i_{b}}
$$

where

$$
\begin{equation*}
i_{b}=(1-\alpha) i_{e}=\frac{i_{e}}{\beta+1} \tag{6.105}
\end{equation*}
$$

and

$$
\begin{equation*}
i_{e}=\frac{v_{i}}{r_{e}+R_{e}} \tag{6.106}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
R_{\mathrm{in}}=(\beta+1)\left(r_{e}+R_{e}\right) \tag{6.107}
\end{equation*}
$$


(b)

Figure 6.38 The CE amplifier with an emitter resistance $R_{e}$; (a) circuit without bias details; (b) equivalent circuit with the BJT replaced with its T model.

This is a very important result. It states that the input resistance looking into the base is $(\beta+1)$ times the total resistance in the emitter, and is known as the resistance-reflection rule. The factor $(\beta+1)$ arises because the base current is $1 /(\beta+1)$ times the emitter current. The expression for $R_{\mathrm{in}}$ in Eq. (6.107) shows clearly that including a resistance $R_{e}$ in the emitter can substantially increase $R_{\mathrm{in}}$, a very desirable result. Indeed, the value of $R_{\mathrm{in}}$ is increased by the ratio

$$
\begin{align*}
\frac{R_{\mathrm{in}}\left(\text { with } R_{e} \text { included }\right)}{R_{\mathrm{in}}\left(\text { without } R_{e}\right)} & =\frac{(\beta+1)\left(r_{e}+R_{e}\right)}{(\beta+1) r_{e}} \\
& =1+\frac{R_{e}}{r_{e}} \simeq 1+g_{m} R_{e} \tag{6.108}
\end{align*}
$$

Thus the circuit designer can use the value of $R_{e}$ to control the value of $R_{\text {in }}$.
To determine the voltage gain $A_{v o}$, we see from Fig. 6.38(b) that

$$
v_{o}=-i_{c} R_{C}=-\alpha i_{e} R_{C}
$$

Substituting for $i_{e}$ from Eq. (6.106) gives

$$
\begin{equation*}
A_{v o}=-\alpha \frac{R_{C}}{r_{e}+R_{e}} \tag{6.109}
\end{equation*}
$$

This is a very useful result: It states that the gain from base to collector is $\alpha$ times the ratio of the total resistance in the collector to the total resistance in the emitter (in this case, $r_{e}+R_{e}$ ),

$$
\begin{equation*}
\text { Voltage gain from base to collector }=-\alpha \frac{\text { Total resistance in collector }}{\text { Total resistance in emitter }} \tag{6.110}
\end{equation*}
$$

This is the BJT version of the MOSFET expression in Eq. (6.102) except that here we have the additional factor $\alpha$. This factor arises because $i_{c}=\alpha i_{e}$, unlike the MOSFET case where $i_{d}=i_{s}$. Usually, $\alpha \simeq 1$ and can be dropped from Eq. (6.110).

The open-circuit voltage gain in Eq. (6.109) can be expressed alternatively as

$$
A_{v o}=-\frac{\alpha}{r_{e}} \frac{R_{C}}{1+R_{e} / r_{e}}
$$

Thus,

$$
\begin{equation*}
A_{v o}=-\frac{g_{m} R_{C}}{1+R_{e} / r_{e}} \simeq-\frac{g_{m} R_{C}}{1+g_{m} R_{e}} \tag{6.111}
\end{equation*}
$$

Thus, including $R_{e}$ reduces the voltage gain by the factor $\left(1+g_{m} R_{e}\right)$, which is the same factor by which $R_{\text {in }}$ is increased. This points out an interesting trade-off between gain and input resistance, a trade-off that the designer can exercise through the choice of an appropriate value for $R_{e}$.

The output resistance $R_{o}$ can be found from the circuit in Fig. 6.38(b) by inspection:

$$
R_{o}=R_{C}
$$

If a load resistance $R_{L}$ is connected at the amplifier output, $A_{v}$ can be found as

$$
\begin{align*}
A_{v} & =A_{v o} \frac{R_{L}}{R_{L}+R_{o}} \\
& =-\alpha \frac{R_{C}}{r_{e}+R_{e}} \frac{R_{L}}{R_{L}+R_{C}}=-\alpha \frac{R_{C} \| R_{L}}{r_{e}+R_{e}} \tag{6.112}
\end{align*}
$$

which could have been written directly using Eq. (6.110). The overall voltage gain $G_{v}$ can now be found:

$$
G_{v}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \times-\alpha \frac{R_{C} \| R_{L}}{r_{e}+R_{e}}
$$

Substituting for $R_{\text {in }}$ from Eq. (6.107) and replacing $\alpha$ with $\beta /(\beta+1)$ results in

$$
\begin{equation*}
G_{v}=-\beta \frac{R_{C} \| R_{L}}{R_{\text {sig }}+(\beta+1)\left(r_{e}+R_{e}\right)} \tag{6.113}
\end{equation*}
$$

Careful examination of this expression reveals that the denominator comprises the total resistance in the base circuit [recall that $(\beta+1)\left(r_{e}+R_{e}\right)$ is the reflection of $\left(r_{e}+R_{e}\right)$ from the emitter side to the base side]. Thus the expression in Eq. (6.113) states that the voltage gain from base to collector is equal to $\beta$ times the ratio of the total resistance in the collector to the total resistance in the base. The factor $\beta$ appears because it is the ratio of the collector current to the base current. This general and useful expression has no counterpart in the MOS case. We observe that the overall voltage gain $G_{v}$ is lower than the value without $R_{e}$, namely,

$$
\begin{equation*}
G_{v}=-\beta \frac{R_{C} \| R_{L}}{R_{\mathrm{sig}}+(\beta+1) r_{e}} \tag{6.114}
\end{equation*}
$$

because of the additional term $(\beta+1) R_{e}$ in the denominator. The gain, however, is now less sensitive to the value of $\beta$, a desirable result because of the typical wide variability in the value of $\beta$.

Another important consequence of including the resistance $R_{e}$ in the emitter is that it enables the amplifier to handle larger input signals without incurring nonlinear distortion. This is because only a fraction of the input signal at the base, $v_{i}$, appears between the base and the emitter. Specifically, from the circuit in Fig. 6.38(b), we see that

$$
\begin{equation*}
\frac{v_{\pi}}{v_{i}}=\frac{r_{e}}{r_{e}+R_{e}} \simeq \frac{1}{1+g_{m} R_{e}} \tag{6.115}
\end{equation*}
$$

Thus, for the same $v_{\pi}$, the signal at the input terminal of the amplifier, $v_{i}$, can be greater than for the CE amplifier by the factor $\left(1+g_{m} R_{e}\right)$.

To summarize, including a resistance $R_{e}$ in the emitter of the CE amplifier results in the following characteristics:

1. The input resistance $R_{\mathrm{in}}$ is increased by the factor $\left(1+g_{m} R_{e}\right)$.
2. The voltage gain from base to collector, $A_{v}$, is reduced by the factor $\left(1+g_{m} R_{e}\right)$.
3. For the same nonlinear distortion, the input signal $v_{i}$ can be increased by the factor $\left(1+g_{m} R_{e}\right)$.
4. The overall voltage gain is less dependent on the value of $\beta$.
5. The high-frequency response is significantly improved (as we shall see in Chapter 9).

With the exception of gain reduction, these characteristics represent performance improvements. Indeed, the reduction in gain is the price paid for obtaining the other performance improvements. In many cases this is a good bargain; it is the underlying philosophy for the use of negative feedback. That the resistance $R_{e}$ introduces negative feedback in the amplifier circuit can be verified by utilizing a procedure similar to that we used above for the MOSFET case. In Chapter 10, where we shall study negative feedback formally, we will find that the factor $\left(1+g_{m} R_{e}\right)$, which appears repeatedly, is the "amount of negative feedback" introduced by $R_{e}$. Finally, we note that the negative-feedback action of $R_{e}$ gives it the name emitter degeneration resistance.

## Example 6.9

For the CE amplifier specified in Example 6.8, what value of $R_{e}$ is needed to raise $R_{\mathrm{in}}$ to a value four times that of $R_{\text {sig }}$ ? With $R_{e}$ included, find $A_{v o}, R_{o}, A_{v}$, and $G_{v}$. Also, if $\hat{v}_{\pi}$ is limited to 5 mV , what are the corresponding values of $\hat{v}_{\text {sig }}$ and $\hat{v}_{o}$ ?

## Solution

To obtain $R_{\text {in }}=4 R_{\text {sig }}=4 \times 5=20 \mathrm{k} \Omega$, the required $R_{e}$ is found from

$$
20=(\beta+1)\left(r_{e}+R_{e}\right)
$$

With $\beta=100$,

$$
r_{e}+R_{e} \simeq 200 \Omega
$$

## Example 6.9 continued

Thus,

$$
\begin{aligned}
R_{e} & =200-25=175 \Omega \\
A_{v o} & =-\alpha \frac{R_{C}}{r_{e}+R_{e}} \simeq-\frac{5000}{25+175}=-25 \mathrm{~V} / \mathrm{V} \\
R_{o} & =R_{C}=5 \mathrm{k} \Omega \text { (unchanged) } \\
A_{v} & =A_{v o} \frac{R_{L}}{R_{L}+R_{o}}=-25 \times \frac{5}{5+5}=-12.5 \mathrm{~V} / \mathrm{V} \\
G_{v} & =\frac{R_{\text {in }}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} A_{v}=-\frac{20}{20+5} \times 12.5=-10 \mathrm{~V} / \mathrm{V}
\end{aligned}
$$

For $\hat{v}_{\pi}=5 \mathrm{mV}$,

$$
\begin{aligned}
& \hat{v}_{i}=\hat{v}_{\pi}\left(\frac{r_{e}+R_{e}}{r_{e}}\right)=5\left(1+\frac{175}{25}\right)=40 \mathrm{mV} \\
& \hat{v}_{\text {sig }}=\hat{v}_{i} \frac{R_{\mathrm{in}}+R_{\text {sig }}}{R_{\mathrm{in}}}=40\left(1+\frac{5}{20}\right)=50 \mathrm{mV} \\
& \hat{v}_{o}=\hat{v}_{\text {sig }} \times\left|G_{v}\right|=50 \times 10=500 \mathrm{mV}=0.5 \mathrm{~V}
\end{aligned}
$$

Thus, while $\left|G_{v}\right|$ has decreased to about a third of its original value, the amplifier is able to produce as large an output signal as before for the same nonlinear distortion.

## EXERCISE

6.24 Show that with $R_{e}$ included, and $v_{\pi}$ limited to a maximum value $\hat{v}_{\pi}$, the maximum allowable input signal, $\hat{v}_{\text {sig }}$, is given by

$$
\hat{v}_{\mathrm{sig}}=\hat{v}_{\pi}\left(1+\frac{R_{e}}{r_{e}}+\frac{R_{\mathrm{sig}}}{r_{\pi}}\right)
$$

If the transistor is biased at $I_{C}=0.5 \mathrm{~mA}$ and has a $\beta$ of 100 , what value of $R_{e}$ is needed to permit an input signal $\hat{v}_{\text {sig }}$ of 100 mV from a source with a resistance $R_{\text {sig }}=10 \mathrm{k} \Omega$ while limiting $\hat{v}_{\pi}$ to 10 mV ? What is $R_{\mathrm{in}}$ for this amplifier? If the total resistance in the collector is $10 \mathrm{k} \Omega$, what $G_{v}$ value results?

Ans. $350 \Omega ; 40.4 \mathrm{k} \Omega ;-19.8 \mathrm{~V} / \mathrm{V}$

### 6.3.5 The Common-Gate (CG) and the Common-Base (CB) Amplifiers

Figure 6.39(a) shows a common-gate amplifier with the biasing circuit omitted. The amplifier is fed with a signal source characterized by $v_{\text {sig }}$ and $R_{\text {sig }}$. Since $R_{\text {sig }}$ appears in series with the source, it is more convenient to
represent the transistor with the T model than with the $\pi$ model. Doing this, we obtain the amplifier equivalent circuit shown in Fig. 6.39(b).

From inspection of the equivalent circuit of Fig. 6.39(b), we see that the input resistance

$$
\begin{equation*}
R_{\mathrm{in}}=\frac{1}{g_{m}} \tag{6.116}
\end{equation*}
$$

This should have been expected, since we are looking into the source and the gate is grounded. Typically $1 / g_{m}$ is a few hundred ohms; thus the CG amplifier has a low input resistance.

To determine the voltage gain $A_{v o}$, we write at the drain node

$$
v_{o}=-i R_{D}
$$

and substitute for the source current $i$ from

$$
i=-\frac{v_{i}}{1 / g_{m}}
$$

to obtain

$$
\begin{equation*}
A_{v o} \equiv \frac{v_{o}}{v_{i}}=g_{m} R_{D} \tag{6.117}
\end{equation*}
$$

which except for the positive sign is identical to the expression for $A_{v o}$ of the CS amplifier.
The output resistance of the CG circuit can be found by inspection of the circuit in Fig. 6.39(b) as

$$
\begin{equation*}
R_{o}=R_{D} \tag{6.118}
\end{equation*}
$$

which is the same as in the case of the CS amplifier.
Although the gain of the CG amplifier proper has the same magnitude as that of the CS amplifier, this is usually not the case as far as the overall voltage gain is concerned. The low input resistance of the CG amplifier can cause the input signal to be severely attenuated. Specifically,

$$
\begin{equation*}
\frac{v_{i}}{v_{\text {sig }}}=\frac{R_{\text {in }}}{R_{\text {in }}+R_{\text {sig }}}=\frac{1 / g_{m}}{1 / g_{m}+R_{\text {sig }}} \tag{6.119}
\end{equation*}
$$


(a)

Figure 6.39 (a) Common-gate (CG) amplifier with bias arrangement omitted. (b) Equivalent circuit of the CG amplifier with the MOSFET replaced with its T model.
from which we see that except for situations in which $R_{\text {sig }}$ is on the order of $1 / g_{m}$, the signal transmission factor $v_{i} / v_{\text {sig }}$ can be very small and the overall voltage gain $G_{v}$ can be correspondingly small. Specifically, with a resistance $R_{L}$ connected at the output

$$
G_{v}=\frac{1 / g_{m}}{R_{\text {sig }}+1 / g_{m}}\left[g_{m}\left(R_{D} \| R_{L}\right)\right]
$$

Thus,

$$
\begin{equation*}
G_{v}=\frac{\left(R_{D} \| R_{L}\right)}{R_{\text {sig }}+1 / g_{m}} \tag{6.120}
\end{equation*}
$$

Observe that the overall voltage gain is simply the ratio of the total resistance in the drain circuit to the total resistance in the source circuit. If $R_{\text {sig }}$ is of the same order as $R_{D}$ and $R_{L}, G_{v}$ will be very small.

Because of its low input resistance, the CG amplifier alone has very limited application. One such application is to amplify high-frequency signals that come from sources with relatively low resistances. These include cables, where it is usually necessary for the input resistance of the amplifier to match the characteristic resistance of the cable. As will be shown in Chapter 9, the CG amplifier has excellent high-frequency response. Thus it can be combined with the CS amplifier in a very beneficial way that takes advantage of the best features of each of the two configurations. A very significant circuit of this kind will be studied in Chapter 7.

## EXERCISE

6.25 A CG amplifier is required to match a signal source with $R_{\text {sig }}=100 \Omega$. At what current $I_{D}$ should the MOSFET be biased if it is operated at an overdrive voltage of 0.20 V ? If the total resistance in the drain circuit is $2 \mathrm{k} \Omega$, what overall voltage gain is realized?

Ans. $1 \mathrm{~mA} ; 10 \mathrm{~V} / \mathrm{V}$

Very similar results can be obtained for the CB amplifier shown in Fig. 6.40(a). Specifically, from the equivalent circuit in Fig. 6.40(b) we can find

$$
\begin{gather*}
R_{\mathrm{in}}=r_{e}=\frac{\alpha}{g_{m}} \simeq 1 / g_{m}  \tag{6.121}\\
A_{v o}=\frac{\alpha}{r_{e}} R_{C}=g_{m} R_{C}  \tag{6.122}\\
R_{o}=R_{C} \tag{6.123}
\end{gather*}
$$

and with a load resistance $R_{L}$ connected to the output, the overall voltage gain is given by

$$
\begin{equation*}
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=\alpha \frac{R_{C} \| R_{L}}{R_{\text {sig }}+r_{e}} \tag{6.124}
\end{equation*}
$$

Since $\alpha \simeq 1$, we see that as in the case of the CG amplifier, the overall voltage gain is simply the ratio of the total resistance in the collector to the total resistance in the emitter. We also note thatthe overall voltage gain


Figure 6.40 (a) CB amplifier with bias details omitted; (b) amplifier equivalent circuit with the BJT represented by its T model.
is almost independent of the value of $\beta$ (except through the small dependence of $\alpha$ on $\beta$ ), a desirable property. Observe that for $R_{\text {sig }}$ of the same order as $R_{C}$ and $R_{L}$, the gain will be very small.

In summary, the CB and CG amplifiers exhibit a very low input resistance ( $1 / g_{m}$ ), an open-circuit voltage gain that is positive and equal in magnitude to that of the CE (CG) amplifier ( $g_{m} R_{C}$ or $g_{m} R_{D}$ ), and, like the $\mathrm{CE}(\mathrm{CS})$ amplifier, a relatively high output resistance ( $R_{C}$ or $R_{D}$ ). Because of its very low input resistance, the CB (CG) circuit alone is not attractive as a voltage amplifier except in specialized applications, such as the cable amplifier mentioned above. The CB (CG) amplifier has excellent high-frequency performance, which as we shall see in Chapters 7 and 9, makes it useful in combination with other circuits in the implementation of high-frequency amplifiers.

## EXERCISES

6.26 Consider a CB amplifier utilizing a BJT biased at $I_{C}=1 \mathrm{~mA}$ and with $R_{C}=5 \mathrm{k} \Omega$. Determine $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$. If the amplifier is loaded in $R_{L}=5 \mathrm{k} \Omega$, what value of $A_{v}$ results? What $G_{v}$ is obtained if $R_{\text {sig }}=5$ $\mathrm{k} \Omega$ ?

Ans. $25 \Omega ; 200 \mathrm{~V} / \mathrm{V} ; 5 \mathrm{k} \Omega ; 100 \mathrm{~V} / \mathrm{V} ; 0.5 \mathrm{~V} / \mathrm{V}$
6.27 A CB amplifier is required to amplify a signal delivered by a coaxial cable having a characteristic resistance of $50 \Omega$. What bias current $I_{C}$ should be utilized to obtain $R_{\mathrm{in}}$ that is matched to the cable resistance? To obtain an overall voltage gain of $G_{v}$ of $40 \mathrm{~V} / \mathrm{V}$, what should the total resistance in the collector (i.e., $R_{C} \| R_{L}$ ) be?

### 6.3.6 The Source and Emitter Followers

The last of the basic transistor amplifier configurations is the common-drain (common-collector) amplifier, an important circuit that finds application in the design of both small-signal amplifiers and amplifiers that are required to handle large signals and deliver substantial amounts of signal power to a load. This latter variety will be studied in Chapter 11. The common-drain amplifier is more commonly known as the source follower, and the common-collector amplifier is more commonly known as the emitter follower. The reason behind these names will become apparent shortly.

The Need for Voltage Buffers Before embarking on the analysis of the source and the emitter followers, it is useful to look at one of their more common applications. Consider the situation depicted in Fig. 6.41(a). A signal source delivering a signal of reasonable strength $(1 \mathrm{~V})$ with an internal resistance of $1 \mathrm{M} \Omega$ is to be connected to a $1-\mathrm{k} \Omega$ load resistance. Connecting the source to the load directly as in Fig. 6.41(b) would result in severe attenuation of the signal; the signal appearing across the load will be only $1 /(1000+1)$ of the input signal, or about 1 mV . An alternative course of action is suggested in Fig. 6.41(c). Here we have interposed an amplifier between the source and the load. Our amplifier, however, is unlike the amplifiers we have been studying in this chapter thus far; it has a voltage gain of only unity. This is because our signal is already of sufficient strength and we do not need to increase its amplitude. Note, however, that our amplifier has a very high input resistance, thus almost all of $v_{\text {sig }}$ (i.e., 1 V ) will appear at the input of the amplifier proper. Since the amplifier has a low output resistance $(100 \Omega), 90 \%$ of this signal $(0.9 \mathrm{~V})$ will appear at the output, obviously a very significant improvement over the situation without the amplifier. As will be seen next, the source follower can easily implement the unity-gain buffer amplifier shown in Fig. 6.41(c).

Characteristic Parameters of the Source Follower Figure 6.42(a) shows a source follower with the bias circuit omitted. The source follower is fed with a signal generator ( $v_{\text {sig }}, R_{\text {sig }}$ ) and has a load resistance $R_{L}$


Figure 6.41 Illustrating the need for a unity-gain voltage buffer amplifier.


Figure 6.42 (a) Common-drain amplifier or source follower with the bias circuit omitted. (b) Equivalent circuit of the source follower obtained by replacing the MOSFET with its T model.
connected between the source terminal and ground. We shall assume that $R_{L}$ includes both the actual load and any other resistance that may be present between the source terminal and ground (e.g., for biasing purposes). Normally, the actual load resistance would be much lower in value than such other resistances and thus would dominate.

Since the MOSFET has a resistance $R_{L}$ connected in its source terminal, it is most convenient to use the T model, as shown in Fig. 6.40(b). From the latter circuit we can write by inspection

$$
R_{\mathrm{in}}=\infty
$$

and obtain $A_{v}$ from the voltage divider formed by $1 / g_{m}$ and $R_{L}$ as

$$
\begin{equation*}
A_{v} \equiv \frac{v_{o}}{v_{i}}=\frac{R_{L}}{R_{L}+1 / g_{m}} \tag{6.125}
\end{equation*}
$$

Setting $R_{L}=\infty$ we obtain

$$
\begin{equation*}
A_{v o}=1 \tag{6.126}
\end{equation*}
$$

The output resistance $R_{o}$ is found by setting $v_{i}=0$ (i.e., by grounding the gate). Now looking back into the output terminal, excluding $R_{L}$, we simply see $1 / g_{m}$, thus

$$
\begin{equation*}
R_{o}=1 / g_{m} \tag{6.127}
\end{equation*}
$$

The unity open-circuit voltage gain together with $R_{o}$ in Eq. (6.127) can be used to find $A_{v}$ when a load resistance $R_{L}$ is connected. The result is simply the expression in Eq. (6.125). Finally, because of the infinite $R_{\mathrm{in}}, v_{i}=v_{\text {sig }}$, and the overall voltage gain is

$$
\begin{equation*}
G_{v}=A_{v}=\frac{R_{L}}{R_{L}+1 / g_{m}} \tag{6.128}
\end{equation*}
$$

Thus $G_{v}$ will be lower than unity. However, because $1 / g_{m}$ is usually low, the voltage gain can be close to unity. The unity open-circuit voltage gain in Eq. (6.126) indicates that the voltage at the source terminal will follow that at the input, hence the name source follower.

In conclusion, the source follower features a very high input resistance (ideally, infinite), a relatively low output resistance $\left(1 / g_{m}\right)$, and an open-circuit voltage gain that is near unity (ideally, unity). Thus the source follower is ideally suited for implementing the unity-gain voltage buffer of Fig. 6.41(c). The source follower is also used as the output (i.e., last) stage in a multistage amplifier, where its function is to equip the overall amplifier with a low output resistance, thus enabling it to supply relatively large load currents without loss of gain (i.e., with little reduction of output signal level). The design of output stages is studied in Chapter 11.

## EXERCISES

D6.28 It is required to design a source follower that implements the buffer amplifier shown in Fig. 6.41(c). If the MOSFET is operated with an overdrive voltage $V_{O V}=0.25 \mathrm{~V}$, at what drain current should it be biased? Find the output signal amplitude and the signal amplitude between gate and source.

Ans. $1.25 \mathrm{~mA} ; 0.91 \mathrm{~V} ; 91 \mathrm{mV}$
D6.29 A MOSFET is connected in the source-follower configuration and employed as the output stage of a cascade amplifier. It is required to provide an output resistance of $200 \Omega$. If the MOSFET has $k_{n}^{\prime}=$ $0.4 \mathrm{~mA} / \mathrm{V}^{2}$ and is operated at $V_{O V}=0.25 \mathrm{~V}$, find the required $W / L$ ratio. Also specify the dc bias current $I_{D}$. If the amplifier load resistance varies over the range $1 \mathrm{k} \Omega$ to $10 \mathrm{k} \Omega$, what is the range of $G_{v}$ of the source follower?

Ans. $50 ; 0.625 \mathrm{~mA} ; 0.83 \mathrm{~V} / \mathrm{V}$ to $0.98 \mathrm{~V} / \mathrm{V}$

Characteristic Parameters of the Emitter Follower Although the emitter follower does not have an infinite input resistance (as in the case of the source follower), it is still widely used as a voltage buffer. In fact, it is a very versatile and popular circuit. We will therefore study it in some detail.

Figure 6.43(a) shows an emitter follower with the equivalent circuit shown in Fig. 6.43(b). The input resistance $R_{\mathrm{in}}$ is found from

$$
R_{\mathrm{in}}=\frac{v_{i}}{i_{b}}
$$

Substituting for $i_{b}=i_{e} /(\beta+1)$ where $i_{e}$ is given by

$$
i_{e}=\frac{v_{i}}{r_{e}+R_{L}}
$$

we obtain

$$
\begin{equation*}
R_{\mathrm{in}}=(\beta+1)\left(r_{e}+R_{L}\right) \tag{6.129}
\end{equation*}
$$

a result that we could have written directly, utilizing the resistance-reflection rule. Note that as expected the emitter follower takes the low load resistance and reflects it to the base side, where the signal source is, after

(a)

(b)

Figure 6.43 (a) Common-collector amplifier or emitter follower with the bias circuit omitted. (b) Equivalent circuit obtained by replacing the BJT with its T model.
increasing its value by a factor $(\beta+1)$. It is this impedance transformation property of the emitter follower that makes it useful in connecting a low-resistance load to a high-resistance source, that is, to implement a buffer amplifier.

The voltage gain $A_{v}$ is given by

$$
\begin{equation*}
A_{v} \equiv \frac{v_{o}}{v_{i}}=\frac{R_{L}}{R_{L}+r_{e}} \tag{6.130}
\end{equation*}
$$

Setting $R_{L}=\infty$ yields $A_{v o}$,

$$
\begin{equation*}
A_{v o}=1 \tag{6.131}
\end{equation*}
$$

Thus, as expected, the open-circuit voltage gain of the emitter follower proper is unity, which means that the signal voltage at the emitter follows that at the base, which is the origin of the name "emitter follower."

To determine $R_{o}$, refer to Fig. 6.43(b) and look back into the emitter (i.e., behind or excluding $R_{L}$ ) while setting $v_{i}=0$ (i.e., grounding the base). You will see $r_{e}$ of the BJT, thus

$$
\begin{equation*}
R_{o}=r_{e} \tag{6.132}
\end{equation*}
$$

This result together with $A_{v o}=1$ yields $A_{v}$ in Eq. (6.130), thus confirming our earlier analysis.
We next determine the overall voltage gain $G_{v}$, as follows:

$$
\begin{aligned}
\frac{v_{i}}{v_{\mathrm{sig}}} & =\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \\
& =\frac{(\beta+1)\left(r_{e}+R_{L}\right)}{(\beta+1)\left(r_{e}+R_{L}\right)+R_{\mathrm{sig}}} \\
G_{v} & \equiv \frac{v_{o}}{v_{\mathrm{sig}}}=\frac{v_{i}}{v_{\mathrm{sig}}} \times A_{v}
\end{aligned}
$$

Substituting for $A_{v}$ from Eq. (6.130) results in

$$
\begin{equation*}
G_{v}=\frac{(\beta+1) R_{L}}{(\beta+1) R_{L}+(\beta+1) r_{e}+R_{\mathrm{sig}}} \tag{6.133}
\end{equation*}
$$

This equation indicates that the overall gain, though lower than one, can be close to one if $(\beta+1) R_{L}$ is larger or comparable in value to $R_{\text {sig }}$. This again confirms the action of the emitter follower in delivering a large proportion of $v_{\text {sig }}$ to a low-valued load resistance $R_{L}$ even though $R_{\text {sig }}$ can be much larger than $R_{L}$. The key point is that $R_{L}$ is multiplied by $(\beta+1)$ before it is "presented to the source." Figure 6.44(a) shows an equivalent circuit of the emitter follower obtained by simply reflecting $r_{e}$ and $R_{L}$ to the base side. The overall voltage gain $G_{v} \equiv v_{o} / v_{\text {sig }}$ can be determined directly and very simply from this circuit by using the voltage divider rule. The result is the expression for $G_{v}$ already given in Eq. (6.133).

Dividing all resistances in the circuit of Fig. 6.44(a) by $\beta+1$ does not change the voltage ratio $v_{o} / v_{\text {sig }}$. Thus we obtain another equivalent circuit, shown in Fig. 6.44(b), that can be used to determine $G_{v} \equiv v_{o} / v_{\text {sig }}$ of the emitter follower. A glance at this circuit reveals that it is simply the equivalent circuit obtained by reflecting $v_{\text {sig }}$ and $R_{\text {sig }}$ from the base side to the emitter side. In this reflection, $v_{\text {sig }}$ does not change, but $R_{\text {sig }}$ is divided by $\beta+1$. Thus, we either reflect to the base side and obtain the circuit in Fig. 6.44(a) or reflect to the emitter side and obtain the circuit in Fig. 6.44(b). From the latter, $G_{v}$ can be found as

$$
\begin{equation*}
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=\frac{R_{L}}{R_{L}+r_{e}+R_{\text {sig }} /(\beta+1)} \tag{6.134}
\end{equation*}
$$

Observe that this expression is the same as that in Eq. (6.133) except for dividing both the numerator and denominator by $\beta+1$.

The expression for $G_{v}$ in Eq. (6.134) has an interesting interpretation: The emitter follower reduces $R_{\text {sig }}$ by the factor $(\beta+1)$ before "presenting it to the load resistance $R_{L}$ ": an impedance transformation that has the same buffering effect.

At this point it is important to note that although the emitter follower does not provide voltage gain it has a current gain of $\beta+1$.

Thévenin Representation of the Emitter-Follower Output A more general representation of the emitter-follower output is shown in Fig. 6.45(a). Here $G_{v o}$ is the overall open-circuit voltage gain that can be obtained by setting $R_{L}=\infty$ in the circuit of Fig. 6.44(b), as illustrated in Fig. 6.45(b). The result is $G_{v o}=1$.


Figure 6.44 Simple equivalent circuits for the emitter follower obtained by (a) reflecting $r_{e}$ and $R_{L}$ to the base side, and (b) reflecting $v_{\text {sig }}$ and $R_{\text {sig }}$ to the emitter side. Note that the circuit in (b) can be obtained from that in (a) by simply dividing all resistances by $(\beta+1)$.


Figure 6.45 (a) Thévenin representation of the output of the emitter follower. (b) Obtaining $G_{v_{o}}$ from the equivalent circuit in Fig. 6.44(b). (c) Obtaining $R_{\text {out }}$ from the equivalent circuit in Fig. 6.44(b) with $v_{\text {sig }}$ set to zero. (d) The emitter follower with $R_{\text {in }}$ and $R_{\text {out }}$ determined simply by looking into the input and output terminals, respectively.

The output resistance $R_{\text {out }}$ is different from $R_{o}$. To determine $R_{\text {out }}$ we set $v_{\text {sig }}$ to zero (rather than setting $v_{i}$ to zero). Again we can use the equivalent circuit in Fig. 6.44(b) to do this, as illustrated in Fig. 6.45(c). We see that

$$
\begin{equation*}
R_{\mathrm{out}}=r_{e}+\frac{R_{\mathrm{sig}}}{\beta+1} \tag{6.135}
\end{equation*}
$$

Finally, we show in Fig. 6.45(d) the emitter-follower circuit together with its $R_{\text {in }}$ and $R_{\text {out }}$. Observe that $R_{\mathrm{in}}$ is determined by reflecting $r_{e}$ and $R_{L}$ to the base side (by multiplying their values by $\beta+1$ ). To determine $R_{\text {out }}$, grab hold of the emitter and walk (or just look!) backward while $v_{\text {sig }}=0$. You will see $r_{e}$ in series with $R_{\text {sig }}$, which because it is in the base must be divided by $(\beta+1)$.

We note that unlike the amplifier circuits we studied earlier, the emitter follower is not unilateral. This is manifested by the fact that $R_{\text {in }}$ depends on $R_{L}$ and $R_{\text {out }}$ depends on $R_{\text {sig }}$.

## Example 6.10

It is required to design an emitter follower to implement the buffer amplifier of Fig. 6.46(a). Specify the required bias current $I_{E}$ and the minimum value the transistor $\beta$ must have. Determine the maximum

## Example 6.10 continued

allowed value of $v_{\text {sig }}$ if $v_{\pi}$ is to be limited to 5 mV in order to obtain reasonably linear operation. With $v_{\text {sig }}=200 \mathrm{mV}$, determine the signal voltage at the output if $R_{L}$ is changed to $2 \mathrm{k} \Omega$, and to $0.5 \mathrm{k} \Omega$.

(a)

(b)

Figure 6.46 Circuit for Example 6.10.

## Solution

The emitter-follower circuit is shown in Fig. 6.46(b). To obtain $R_{o}=10 \Omega$, we bias the transistor to obtain $r_{e}=10 \Omega$. Thus,

$$
\begin{aligned}
10 \Omega & =\frac{V_{T}}{I_{E}} \\
I_{E} & =2.5 \mathrm{~mA}
\end{aligned}
$$

The input resistance $R_{\text {in }}$ will be

$$
\begin{aligned}
R_{\mathrm{in}} & =(\beta+1)\left(r_{e}+R_{L}\right) \\
100 & =(\beta+1)(0.01+1)
\end{aligned}
$$

Thus, the BJT should have a $\beta$ with a minimum value of 98 . A higher $\beta$ would obviously be beneficial.

## Example 6.10 continued

The overall voltage gain can be determined from

$$
G_{v} \equiv \frac{v_{o}}{v_{\mathrm{sig}}}=\frac{R_{L}}{R_{L}+r_{e}+\frac{R_{\mathrm{sig}}}{(\beta+1)}}
$$

Assuming $\beta=100$, the value of $G_{v}$ obtained is

$$
G_{v}=0.5
$$

Thus when $v_{\text {sig }}=200 \mathrm{mV}$, the signal at the output will be 100 mV . Since the 100 mV appears across the $1-\mathrm{k} \Omega$ load, the signal across the base-emitter junction can be found from

$$
v_{\pi}=\frac{v_{o}}{R_{L}} \times r_{e}=\frac{100}{1000} \times 10=1 \mathrm{mV}
$$

If $\hat{v}_{\pi}=5 \mathrm{mV}$ then $v_{\text {sig }}$ can be increased by a factor of 5 , resulting in $\hat{v}_{\text {sig }}=1 \mathrm{~V}$.
To obtain $v_{o}$ as the load is varied, we use the Thévenin equivalent of the emitter follower, shown in Fig. 6.45(a) with $G_{v o}=1$ and

$$
R_{\mathrm{out}}=\frac{R_{\mathrm{sig}}}{\beta+1}+r_{e}=\frac{100}{101}+0.01=1 \mathrm{k} \Omega
$$

to obtain

$$
v_{o}=v_{\text {sig }} \frac{R_{L}}{R_{L}+R_{\text {out }}}
$$

For $R_{L}=2 \mathrm{k} \Omega$,

$$
v_{o}=200 \mathrm{mV} \times \frac{2}{2+1}=133.3 \mathrm{mV}
$$

and for $R_{L}=0.5 \mathrm{k} \Omega$,

$$
v_{o}=200 \mathrm{mV} \times \frac{0.5}{0.5+1}=66.7 \mathrm{mV}
$$

## EXERCISE

6.30 An emitter follower utilizes a transistor with $\beta=100$ and is biased at $I_{C}=5 \mathrm{~mA}$. It operates between a source having a resistance of $10 \mathrm{k} \Omega$ and a load of $1 \mathrm{k} \Omega$. Find $R_{\mathrm{in}}, G_{v o}, R_{\text {out }}$, and $G_{v}$. What is the peak amplitude of $v_{\text {sig }}$ that results in $v_{\pi}$ having a peak amplitude of 5 mV ? Find the resulting peak amplitude at the output.

### 6.3.7 Summary Tables and Comparisons

For easy reference and to enable comparisons, we present in Tables 6.4 and 6.5 the formulas for determining the characteristic parameters for the various configurations of MOSFET and BJT amplifiers, respectively. In addition to the remarks made throughout this section about the characteristics and areas of applicability of the various configurations, we make the following concluding points:

1. MOS amplifiers provide much higher, ideally infinite input resistances (except, of course, for the CG configuration). This is a definite advantage over BJT amplifiers.
2. BJTs exhibit higher $g_{m}$ values than MOSFETs, resulting in higher gains.
3. For discrete-circuit amplifiers-that is, those that are assembled from discrete components on a printed-circuit board (PCB)—the BJT remains the device of choice. This is because discrete BJTs are much easier to handle physically than discrete MOSFETs and, more important, a very wide variety of discrete BJTs is available commercially. The remainder of this chapter is concerned with discrete-circuit amplifiers.
4. Integrated-circuit (IC) amplifiers predominantly use MOSFETs, although BJTs are utilized in certain niche areas. Chapters 7 to 12 are mainly concerned with IC amplifiers.
5. The CS and CE configurations are the best suited for realizing the bulk of the gain required in an amplifier. Depending on the magnitude of the gain required, either a single stage or a cascade of two or three stages can be used.
6. Including a resistance $R_{s}$ in the source of the CS amplifier (a resistance $R_{e}$ in the emitter of the CE amplifier) provides a number of performance improvements at the expense of gain reduction.
7. The low input resistance of the CG and CB amplifiers makes them useful only in specific applications. As we shall see in Chapter 9, these two configurations exhibit a much better high-frequency response than that available from the CS and CE amplifiers. This makes them useful as high-frequency

| Table 6.4 Characteristics of MOSFET Amplifiers |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Amplifier type | Characteristics ${ }^{\text {a }}$ |  |  |  |  |
|  | $R_{\text {in }}$ | $A_{\text {vo }}$ | $R_{o}$ | $A_{v}$ | $G_{v}$ |
| Common source (Fig. 6.35) | $\infty$ | ${ }_{-} g_{m} R_{D}$ | $R_{D}$ | $-g_{m}\left(R_{D} \\| R_{L}\right)$ | $-g_{m}\left(R_{D} \\| R_{L}\right)$ |
| Common source with $R_{s}$ (Fig. 6.37) | $\infty$ | $-\frac{g_{m} R_{D}}{1+g_{m} R_{s}}$ | $R_{D}$ | $\begin{aligned} & \frac{-g_{m}\left(R_{D} \\| R_{L}\right)}{1+g_{m} R_{s}} \\ & -\frac{R_{D} \\| R_{L}}{1 / g_{m}+R_{s}} \end{aligned}$ | $\begin{aligned} & -\frac{g_{m}\left(R_{D} \\| R_{L}\right)}{1+g_{m} R_{s}} \\ & -\frac{R_{D} \\| R_{L}}{1 / g_{m}+R_{s}} \end{aligned}$ |
| Common gate (Fig. 6.39) | $\frac{1}{g_{m}}$ | $g_{m} R_{D}$ | $R_{D}$ | $g_{m}\left(R_{D} \\| R_{L}\right)$ | $\frac{R_{D} \\| R_{L}}{R_{\text {sig }}+1 / g_{m}}$ |
| Source follower (Fig. 6.42) | $\infty$ | 1 | $\frac{1}{g_{m}}$ | $\frac{R_{L}}{R_{L}+1 / g_{m}}$ | $\frac{R_{L}}{R_{L}+1 / g_{m}}$ |
| ${ }^{\text {a }}$ For the interpretation of $R_{\text {in }}, A_{v o}$, and $R_{o}$, refer to Fig. 6.34(b). |  |  |  |  |  |

Table 6.5 Characteristics of BJT Amplifiers ${ }^{\text {a,b }}$

|  | $R_{\text {in }}$ | $A_{\text {vo }}$ | $R_{o}$ | $A_{v}$ | $G_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Common emitter (Fig. 6.36) | $(\beta+1) r_{e}$ | $-g_{m} R_{C}$ | $R_{C}$ | $\begin{aligned} & -g_{m}\left(R_{C} \\| R_{L}\right) \\ & -\alpha \frac{R_{C} \\| R_{L}}{r_{e}} \end{aligned}$ | $-\beta \frac{R_{C} \\| R_{L}}{R_{\text {sig }}+(\beta+1) r_{e}}$ |
| Common emitter with $R_{e}$ (Fig. 6.38) | $(\beta+1)\left(r_{e}+R_{e}\right)$ | $-\frac{g_{m} R_{C}}{1+g_{m} R_{e}}$ | $R_{C}$ | $\begin{aligned} & \frac{-g_{m}\left(R_{C} \\| R_{L}\right)}{1+g_{m} R_{e}} \\ & -\alpha \frac{R_{C} \\| R_{L}}{r_{e}+R_{e}} \end{aligned}$ | $-\beta \frac{R_{C} \\| R_{L}}{R_{\text {sig }}+(\beta+1)\left(r_{e}+R_{e}\right)}$ |
| Common base (Fig. 6.40) | $r_{e}$ | $g_{m} R_{C}$ | $R_{C}$ | $\begin{aligned} & g_{m}\left(R_{C} \\| R_{L}\right) \\ & \alpha \frac{R_{C} \\| R_{L}}{r_{e}} \end{aligned}$ | $\alpha \frac{R_{C} \\| R_{L}}{R_{\mathrm{sig}}+r_{e}}$ |
| Emitter follower (Fig. 6.43) | $(\beta+1)\left(r_{e}+R_{L}\right)$ | 1 | $r_{e}$ | $\frac{R_{L}}{R_{L}+r_{e}}$ | $\begin{aligned} & \frac{R_{L}}{R_{L}+r_{e}+R_{\mathrm{sig}} /(\beta+1)} \\ & G_{\text {vo }}=1 \\ & R_{\text {out }}=r_{e}+\frac{R_{\mathrm{sig}}}{\beta+1} \end{aligned}$ |

${ }^{\text {a }}$ For the interpretation of $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$ refer to Fig. 6.34(b).
${ }^{\mathrm{b}}$ Setting $\beta=\infty(\alpha=1)$ and replacing $r_{e}$ with $1 / g_{m}, R_{C}$ with $R_{D}$, and $R_{e}$ with $R_{s}$ results in the corresponding formulas for MOSFET amplifiers (Table 6.4).
amplifiers, especially when combined with the CS or CE circuit. We shall study one such combination in Chapter 7.
8. The source follower (emitter follower) finds application as a voltage buffer for connecting a high-resistance source to a low-resistance load, and as the output stage in a multistage amplifier, where its purpose is to equip the amplifier with a low output resistance.

### 6.3.8 When and How to Include the Output Resistance $r_{0}$

So far we have been neglecting the output resistance $r_{o}$ of the MOSFET and the BJT. We have done this for two reasons:

1. To keep things simple and focus attention on the significant features of each of the basic configurations, and
2. Because our main interest in this chapter is discrete-circuit design, where the circuit resistances (e.g., $R_{C}, R_{D}$, and $R_{L}$ ) are usually much smaller than $r_{o}$.

Nevertheless, in some instances it is relatively easy to include $r_{o}$ in the analysis. Specifically:

1. In the CS and CE amplifiers, it can be seen that $r_{o}$ of the transistor appears in parallel with $R_{D}$ and $R_{C}$, respectively, and can be simply included in the corresponding formulas in Tables 6.4 and 6.5 by replacing $R_{D}$ with ( $R_{D} \| r_{o}$ ) and $R_{C}$ with $\left(R_{C} \| r_{o}\right)$. The effect will be a reduction in the magnitude of gain, of perhaps $5 \%$ to $10 \%$.

LEE DE FOREST-
A FATHER OF THE ELECTRONICS AGE

In 1906 self-employed inventor Lee de Forest (1873-1961) created a three-terminal vacuum tube; it was the first electronic amplifier of weak signals. The device was known initially as the de Forest valve. The patent filed in 1907, however, used the name Audion, with the "-ion" indicating that the device was not completely evacuated. By 1919, engineers had realized that complete evacuation of internal gases produced a more reliable device.

De Forest's first amplifier became known as the vacuum tube triode.
Through its impact on radio, telephony, motion picture sound, and television, this invention, one of de Forest's 180 patents, is credited with introducing the electronics age. The vacuum tube, in a variety of types, remained the device for implementing amplifiers until the appearance of transistors in the early 1950s.
2. In the source and emitter followers, it can be seen that the transistor $r_{o}$ appears in parallel with $R_{L}$ and can be taken into account by replacing $R_{L}$ in the corresponding formulas with ( $R_{L} \| r_{o}$ ). Thus, here too, the effect of taking $r_{o}$ into account is a small reduction in gain. More significant, however, taking $r_{o}$ into account reduces the open-circuit voltage gain $A_{v o}$ from unity to

$$
\begin{equation*}
A_{v o}=\frac{r_{o}}{r_{o}+\left(1 / g_{m}\right)} \tag{6.136}
\end{equation*}
$$

There are configurations in which taking $r_{o}$ into account complicates the analysis considerably. These are the CS (CE) amplifiers with a source (emitter) resistance, and the CG (CB) amplifier. Fortunately, for discrete implementation of these configurations, the effect of neglecting $r_{o}$ is usually small (which can be verified by computer simulation).

Finally, a very important point: In the analysis and design of IC amplifiers, $r_{o}$ must always be taken into account. This is because, as will be seen in the next chapter, all the circuit resistances are of the same order of magnitude as $r_{o}$; thus, neglecting $r_{o}$ can result in completely erroneous results.

### 6.4 Biasing

As discussed in Section 6.1, an essential step in the design of a transistor amplifier is the establishment of an appropriate dc operating point for the transistor. This is the step known as biasing or bias design. In this section, we study the biasing methods commonly employed in discrete-circuit amplifiers. Biasing of integrated-circuit amplifiers will be studied in Chapter 7 .

Bias design aims to establish in the drain (collector) a dc current that is predictable and insensitive to variations in temperature and to the large variations in parameter values between devices of the same type. For instance, discrete BJTs belonging to the same manufacturer's part number can exhibit $\beta$ values that range, say, from 50 to 150 . Nevertheless, the bias design for an amplifier utilizing this particular transistor type may specify that the dc collector current shall always be within, say, $\pm 10 \%$ of the nominal value of, say, 1 mA . A similar statement can be made about the desired insensitivity of the dc drain current to the wide variations encountered in $V_{t}$ of discrete MOSFETs.

A second consideration in bias design is locating the dc operating point in the active region of operation of the transistor so as to obtain high voltage gain while allowing for the required output signal swing without the transistor leaving the active region at any time (in order to avoid nonlinear distortion). We discussed this point in Section 6.1.7.

Although we shall consider the biasing of MOSFET and BJT amplifiers separately, the resulting circuits are very similar. Also, it will be seen that good bias designs incorporate a feedback mechanism that works to keep the dc bias point as constant as possible.

In order to keep matters simple and thus focus our attention on significant issues, we will neglect the Early effect; that is assume $\lambda=0$ or $V_{A}=\infty$. This is certainly allowed in initial designs of discrete circuits. Of course, the design can be fine-tuned at a later point with the assistance of a circuit-simulation program such as SPICE.

### 6.4.1 The MOSFET Case

Biasing by Fixing $V_{G S}$ The most straightforward approach to biasing a MOSFET is to fix its gate-to-source voltage $V_{G S}$ to the value required ${ }^{6}$ to provide the desired $I_{D}$. This voltage value can be derived from the power-supply voltage $V_{D D}$ through the use of an appropriate voltage divider, as shown in Fig. 6.47(a). Alternatively, it can be derived from another suitable reference voltage that might be available in the system. Independent of how the voltage $V_{G S}$ may be generated, this is not a good approach to biasing a MOSFET. To understand the reason for this statement, recall that

$$
I_{D}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{t}\right)^{2}
$$

and note that the values of the threshold voltage $V_{t}$, the oxide-capacitance $C_{o x}$, and (to a lesser extent) the transistor aspect ratio $W / L$ vary widely among devices of supposedly the same size and type. This is certainly the case for discrete devices, in which large spreads in the values of these parameters occur among devices of the same manufacturer's part number. The spread can also be large in integrated circuits, especially among devices fabricated on different wafers and certainly between different batches of wafers. Furthermore, both $V_{t}$ and $\mu_{n}$ depend on temperature, with the result that if we fix the value of $V_{G S}$, the drain current $I_{D}$ becomes very much temperature dependent.

To emphasize the point that biasing by fixing $V_{G S}$ is not a good technique, we show in Fig. 6.47 two $i_{D}-v_{G S}$ characteristic curves representing extreme values in a batch of MOSFETs of the same type. Observe that for the fixed value of $V_{G S}$, the resultant spread in the values of the drain current can be substantial.

Biasing by Fixing $V_{\mathbf{G}}$ and Connecting a Resistance in the Source An excellent biasing technique for discrete MOSFET circuits consists of fixing the dc voltage at the gate, $V_{G}$, and connecting a resistance in the source lead, as shown in Fig. 6.48(a). For this circuit we can write

$$
\begin{equation*}
V_{G}=V_{G S}+R_{S} I_{D} \tag{6.137}
\end{equation*}
$$

Now, if $V_{G}$ is much greater than $V_{G S}, I_{D}$ will be mostly determined by the values of $V_{G}$ and $R_{S}$. However, even if $V_{G}$ is not much larger than $V_{G S}$, resistor $R_{S}$ provides negative feedback, which acts to stabilize the value of the bias current $I_{D}$. To see how this comes about, consider what happens when $I_{D}$ increases for whatever reason. Equation (6.137) indicates that since $V_{G}$ is constant, $V_{G S}$ will have to decrease. This in turn results in a decrease in $I_{D}$, a change that is opposite to that initially assumed. Thus the action of $R_{S}$ works to keep $I_{D}$ as constant as possible. ${ }^{7}$

Figure 6.48(b) provides a graphical illustration of the effectiveness of this biasing scheme. Here too we show the $i_{D}-v_{G S}$ characteristics for two devices that represent the extremes of a batch of MOSFETs.

[^5]

Figure 6.47 (a) Biasing the MOSFET with a constant $V_{G S}$ generated from $V_{D D}$ using a voltage divider $\left(R_{G 1}, R_{G 2}\right)$; (b) the use of fixed bias (constant $V_{G S}$ ) can result in a large variability in the value of $I_{D}$. Devices 1 and 2 represent extremes among units of the same type.

Superimposed on the device characteristics is a straight line that represents the constraint imposed by the bias circuit-namely, Eq. (6.137). The intersection of this straight line with the $i_{D}-v_{G S}$ characteristic curve provides the coordinates ( $I_{D}$ and $V_{G S}$ ) of the bias point. Observe that compared to the case of fixed $V_{G S}$, here the variability obtained in $I_{D}$ is much smaller. Also, note that the variability decreases as $V_{G}$ and $R_{S}$ are made larger (thus providing a bias line that is less steep).

Two possible practical discrete implementations of this bias scheme are shown in Fig. 6.48(c) and (e). The circuit in Fig. 6.48(c) utilizes one power-supply $V_{D D}$ and derives $V_{G}$ through a voltage divider $\left(R_{G 1}, R_{G 2}\right)$. Since $I_{G}=0, R_{G 1}$ and $R_{G 2}$ can be selected to be very large (in the megohm range), allowing the MOSFET to present a large input resistance to a signal source that may be connected to the gate through a coupling capacitor, as shown in Fig. 6.48(d). Here capacitor $C_{C 1}$ blocks dc and thus allows us to couple the signal $v_{\text {sig }}$ to the amplifier input without disturbing the MOSFET dc bias point. The value of $C_{C 1}$ should be selected large enough to approximate a short circuit at all signal frequencies of interest. We shall study capacitively coupled MOSFET amplifiers, which are suitable only in discrete-circuit design, in Section 6.5. Finally, note that in the circuit of Fig. 6.48(c), resistor $R_{D}$ is selected to be as large as possible to obtain high gain but small enough to allow for the desired signal swing at the drain while keeping the MOSFET in saturation at all times.

When two power supplies are available, as is often the case, the somewhat simpler bias arrangement of Fig. 6.48(e) can be utilized. This circuit is an implementation of Eq. (6.137), with $V_{G}$ replaced by $V_{S S}$. Resistor $R_{G}$ establishes a dc ground at the gate and presents a high input resistance to a signal source that may be connected to the gate through a coupling capacitor.


Figure 6.48 Biasing using a fixed voltage at the gate, $V_{G}$, and a resistance in the source lead, $R_{S}$ : (a) basic arrangement; (b) reduced variability in $I_{D}$; (c) practical implementation using a single supply; (d) coupling of a signal source to the gate using a capacitor $C_{C 1}$; (e) practical implementation using two supplies.

## Example 6.11

It is required to design the circuit of Fig. 6.48(c) to establish a dc drain current $I_{D}=0.5 \mathrm{~mA}$. The MOSFET is specified to have $V_{t}=1 \mathrm{~V}$ and $k_{n}^{\prime} W / L=1 \mathrm{~mA} / \mathrm{V}^{2}$. For simplicity, neglect the channel-length modulation effect (i.e., assume $\lambda=0$ ). Use a power-supply $V_{D D}=15 \mathrm{~V}$. Calculate the percentage change in the value of $I_{D}$ obtained when the MOSFET is replaced with another unit having the same $k_{n}^{\prime} W / L$ but $V_{t}=1.5 \mathrm{~V}$.

## Solution

As a rule of thumb for designing this classical biasing circuit, we choose $R_{D}$ and $R_{S}$ to provide one-third of the power-supply voltage $V_{D D}$ as a drop across each of $R_{D}$, the transistor (i.e., $V_{D S}$ ), and $R_{S}$.

## Example 6.11 continued

For $V_{D D}=15 \mathrm{~V}$, this choice makes $V_{D}=+10 \mathrm{~V}$ and $V_{S}=+5 \mathrm{~V}$. Now, since $I_{D}$ is required to be 0.5 mA , we can find the values of $R_{D}$ and $R_{S}$ as follows:

$$
\begin{aligned}
& R_{D}=\frac{V_{D D}-V_{D}}{I_{D}}=\frac{15-10}{0.5}=10 \mathrm{k} \Omega \\
& R_{S}=\frac{V_{S}}{R_{S}}=\frac{5}{0.5}=10 \mathrm{k} \Omega
\end{aligned}
$$

The required value of $V_{G S}$ can be determined by first calculating the overdrive voltage $V_{O V}$ from

$$
\begin{aligned}
I_{D} & =\frac{1}{2} k_{n}^{\prime}(W / L) V_{O V}^{2} \\
0.5 & =\frac{1}{2} \times 1 \times V_{O V}^{2}
\end{aligned}
$$

which yields $V_{O V}=1 \mathrm{~V}$, and thus,

$$
V_{G S}=V_{t}+V_{O V}=1+1=2 \mathrm{~V}
$$

Now, since $V_{S}=+5 \mathrm{~V}, V_{G}$ must be

$$
V_{G}=V_{S}+V_{G S}=5+2=7 \mathrm{~V}
$$

To establish this voltage at the gate we may select $R_{G 1}=8 \mathrm{M} \Omega$ and $R_{G 2}=7 \mathrm{M} \Omega$. The final circuit is shown in Fig. 6.49. Observe that the dc voltage at the drain $(+10 \mathrm{~V})$ allows for a positive signal swing of +5 V (i.e., up to $V_{D D}$ ) and a negative signal swing of 4 V [i.e., down to $\left(V_{G}-V_{t}\right)$ ].

If the NMOS transistor is replaced with another having $V_{t}=1.5 \mathrm{~V}$, the new value of $I_{D}$ can be found as follows:

$$
\begin{align*}
I_{D} & =\frac{1}{2} \times 1 \times\left(V_{G S}-1.5\right)^{2}  \tag{6.138}\\
V_{G} & =V_{G S}+I_{D} R_{S} \\
7 & =V_{G S}+10 I_{D} \tag{6.139}
\end{align*}
$$

Solving Eqs. (6.138) and (6.139) together yields

$$
I_{D}=0.455 \mathrm{~mA}
$$

Thus the change in $I_{D}$ is

$$
\Delta I_{D}=0.455-0.5=-0.045 \mathrm{~mA}
$$

which is $\frac{-0.045}{0.5} \times 100=-9 \%$ change.

## Example 6.11 continued



Figure 6.49 Circuit for Example 6.11.

## EXERCISES

6.31 Consider the MOSFET in Example 6.11 when fixed- $V_{G S}$ bias is used. Find the required value of $V_{G S}$ to establish a dc bias current $I_{D}=0.5 \mathrm{~mA}$. Recall that the device parameters are $V_{t}=1 \mathrm{~V}, k_{n}^{\prime} W / L=$ $1 \mathrm{~mA} / \mathrm{V}^{2}$, and $\lambda=0$. What is the percentage change in $I_{D}$ obtained when the transistor is replaced with another having $V_{t}=1.5 \mathrm{~V}$ ?

Ans. $V_{G S}=2 \mathrm{~V} ;-75 \%$
D6.32 Design the circuit of Fig. 6.48(e) to operate at a dc drain current of 0.5 mA and $V_{D}=+2 \mathrm{~V}$. Let $V_{t}=1 \mathrm{~V}$, $k_{n}^{\prime} W / L=1 \mathrm{~mA} / \mathrm{V}^{2}, \lambda=0, V_{D D}=V_{S S}=5 \mathrm{~V}$. Use standard $5 \%$ resistor values (see Appendix J), and give the resulting values of $I_{D}, V_{D}$, and $V_{S}$.

Ans. $R_{D}=R_{S}=6.2 \mathrm{k} \Omega ; I_{D}=0.49 \mathrm{~mA}, V_{S}=-1.96 \mathrm{~V}$, and $V_{D}=+1.96 \mathrm{~V} . R_{G}$ can be selected in the range of $1 \mathrm{M} \Omega$ to $10 \mathrm{M} \Omega$.

Biasing Using a Drain-to-Gate Feedback Resistor A simple and effective discrete- circuit biasing arrangement utilizing a feedback resistor connected between the drain and the gate is shown in Fig. 6.50. Here the large feedback resistance $R_{G}$ (usually in the megohm range) forces the dc voltage at the gate to be equal to that at the drain (because $I_{G}=0$ ). Thus we can write


Figure 6.50 Biasing the MOSFET using a large drain-to-gate feedback resistance, $R_{G}$.

$$
V_{G S}=V_{D S}=V_{D D}-R_{D} I_{D}
$$

which can be rewritten in the form

$$
\begin{equation*}
V_{D D}=V_{G S}+R_{D} I_{D} \tag{6.140}
\end{equation*}
$$

which is identical in form to Eq. (6.137), which describes the operation of the bias scheme discussed above (that in Fig. 6.48a). Thus, here too, if $I_{D}$ for some reason changes, say increases, then Eq. (6.140) indicates that $V_{G S}$ must decrease. The decrease in $V_{G S}$ in turn causes a decrease in $I_{D}$, a change that is opposite in direction to the one originally assumed. Thus the negative feedback or degeneration provided by $R_{G}$ works to keep the value of $I_{D}$ as constant as possible.

The circuit of Fig. 6.50 can be utilized as an amplifier by applying the input voltage signal to the gate via a coupling capacitor so as not to disturb the dc bias conditions already established. The amplified output signal at the drain can be coupled to another part of the circuit, again via a capacitor. We considered such an amplifier circuit in Section 6.2 (Example 6.3).

## EXERCISE

D6.33 Design the circuit in Fig. 6.50 to operate at a dc drain current of 0.5 mA . Assume $V_{D D}=+5 \mathrm{~V}, k_{n}^{\prime} W / L=$ $1 \mathrm{~mA} / \mathrm{V}^{2}, V_{t}=1 \mathrm{~V}$, and $\lambda=0$. Use a standard $5 \%$ resistance value for $R_{D}$, and give the actual values obtained for $I_{D}$ and $V_{D}$.

$$
\text { Ans. } R_{D}=6.2 \mathrm{k} \Omega ; I_{D} \simeq 0.49 \mathrm{~mA} ; V_{D} \simeq 1.96 \mathrm{~V}
$$

### 6.4.2 The BJT Case

Before presenting the "good" biasing schemes, we should point out why two obvious arrangements are not good. First, attempting to bias the BJT by fixing the voltage $V_{B E}$ by, for instance, using a voltage divider across the power supply $V_{C C}$, as shown in Fig. 6.51(a), is not a viable approach: The very sharp exponential relationship $i_{C}-v_{B E}$ means that any small and inevitable differences in $V_{B E}$ from the desired value will result in large differences in $I_{C}$ and in $V_{C E}$. Second, biasing the BJT by establishing a constant current in the base, as shown in Fig. $6.51(\mathrm{~b})$, where $I_{B} \simeq\left(V_{C C}-0.7\right) / R_{B}$, is also not a recommended approach. Here the typically large variations in the value of $\beta$ among units of the same device type will result in correspondingly large variations in $I_{C}$ and hence in $V_{C E}$.

The Classical Discrete-Circuit Bias Arrangement Figure 6.52(a) shows the arrangement most commonly used for biasing a discrete-circuit transistor amplifier if only a single power supply is available. The technique consists of supplying the base of the transistor with a fraction of the supply voltage $V_{C C}$ through the voltage divider $R_{1}, R_{2}$. In addition, a resistor $R_{E}$ is connected to the emitter. This circuit is very similar to one we used for the MOSFET (Fig. 6.48c). Here, however, the design must take into account the finite base current.

(a)

(b)

Figure 6.51 Two obvious schemes for biasing the BJT: (a) by fixing $V_{B E}$; (b) by fixing $I_{B}$. Both result in wide variations in $I_{C}$ and hence in $V_{C E}$ and therefore are considered to be "bad." Neither scheme is recommended.

Figure 6.52(b) shows the same circuit with the voltage-divider network replaced by its Thévenin equivalent,

$$
\begin{gather*}
V_{B B}=\frac{R_{2}}{R_{1}+R_{2}} V_{C C}  \tag{6.141}\\
R_{B}=\frac{R_{1} R_{2}}{R_{1}+R_{2}} \tag{6.142}
\end{gather*}
$$

The current $I_{E}$ can be determined by writing a Kirchhoff loop equation for the base-emitterground loop, labeled L , and substituting $I_{B}=I_{E} /(\beta+1)$ :

$$
\begin{equation*}
I_{E}=\frac{V_{B B}-V_{B E}}{R_{E}+R_{B} /(\beta+1)} \tag{6.143}
\end{equation*}
$$

To make $I_{E}$ insensitive to temperature and $\beta$ variation, ${ }^{8}$ we design the circuit to satisfy the following two constraints:

$$
\begin{align*}
V_{B B} & \gg V_{B E}  \tag{6.144}\\
R_{E} & \gg \frac{R_{B}}{\beta+1} \tag{6.145}
\end{align*}
$$

Condition (6.144) ensures that small variations in $V_{B E}(\simeq 0.7 \mathrm{~V})$ will be swamped by the much larger $V_{B B}$. There is a limit, however, on how large $V_{B B}$ can be: For a given value of the supply voltage $V_{C C}$, the higher the value we use for $V_{B B}$, the lower will be the sum of voltages across $R_{C}$ and the collector-base junction $\left(V_{C B}\right)$. On the other hand, we want the voltage across $R_{C}$ to be large in order to obtain high voltage gain and large signal swing (before transistor cutoff). We also want $V_{C B}$ (or $V_{C E}$ ) to be large, to provide a large signal swing

[^6]

Figure 6.52 Classical biasing for BJTs using a single power supply: (a) circuit; (b) circuit with the voltage divider supplying the base replaced with its Thévenin equivalent.
(before transistor saturation). Thus, as is the case in any design, we have a set of conflicting requirements, and the solution must be a trade-off. As a rule of thumb, one designs for $V_{B B}$ about $\frac{1}{3} V_{C C}, V_{C B}$ (or $V_{C E}$ ) about $\frac{1}{3} V_{C C}$, and $I_{C} R_{C}$ about $\frac{1}{3} V_{C C}$.

Condition (6.145) makes $I_{E}$ insensitive to variations in $\beta$ and could be satisfied by selecting $R_{B}$ small. This in turn is achieved by using low values for $R_{1}$ and $R_{2}$. Lower values for $R_{1}$ and $R_{2}$, however, will mean a higher current drain from the power supply, and will result in a lowering of the input resistance of the amplifier (if the input signal is coupled to the base), ${ }^{9}$ which is the trade-off involved in this part of the design. It should be noted that condition (6.145) means that we want to make the base voltage independent of the value of $\beta$ and determined solely by the voltage divider. This will obviously be satisfied if the current in the divider is made much larger than the base current. Typically one selects $R_{1}$ and $R_{2}$ such that their current is in the range of $I_{E}$ to $0.1 I_{E}$.

Further insight regarding the mechanism by which the bias arrangement of Fig. 6.52(a) stabilizes the dc emitter (and hence collector) current is obtained by considering the feedback action provided by $R_{E}$. Consider that for some reason the emitter current increases. The voltage drop across $R_{E}$, and hence $V_{E}$, will increase correspondingly. Now, if the base voltage is determined primarily by the voltage divider $R_{1}, R_{2}$, which is the case if $R_{B}$ is small, it will remain constant, and the increase in $V_{E}$ will result in a corresponding decrease in $V_{B E}$. This in turn reduces the collector (and emitter) current, a change opposite to that originally assumed. Thus $R_{E}$ provides a negative feedback action that stabilizes the bias current. This should remind the reader of the resistance $R_{e}$ that we included in the emitter lead of the CE amplifier in Section 6.3.4. Also, the feedback action of $R_{E}$ in the circuit of Fig. 6.52(a) is similar to the feedback action of $R_{S}$ in the circuit of Fig. 6.48(c). We shall study negative feedback formally in Chapter 10.

## Example 6.12

We wish to design the bias network of the amplifier in Fig. 6.52 to establish a current $I_{E}=1 \mathrm{~mA}$ using a power supply $V_{C C}=+12 \mathrm{~V}$. The transistor is specified to have a nominal $\beta$ value of 100 .

## Solution

We shall follow the rule of thumb mentioned above and allocate one-third of the supply voltage to the voltage drop across $R_{2}$ and another one-third to the voltage drop across $R_{C}$, leaving one-third for possible negative signal swing at the collector. Thus,

$$
\begin{aligned}
& V_{B}=+4 \mathrm{~V} \\
& V_{E}=4-V_{B E} \simeq 3.3 \mathrm{~V}
\end{aligned}
$$

and $R_{E}$ is determined from

[^7]
## Example 6.12 continued

$$
R_{E}=\frac{V_{E}}{I_{E}}=\frac{3.3}{1}=3.3 \mathrm{k} \Omega
$$

From the discussion above we select a voltage-divider current of $0.1 I_{E}=0.1 \times 1=0.1 \mathrm{~mA}$. Neglecting the base current, we find

$$
R_{1}+R_{2}=\frac{12}{0.1}=120 \mathrm{k} \Omega
$$

and

$$
\frac{R_{2}}{R_{1}+R_{2}} V_{C C}=4 \mathrm{~V}
$$

Thus $R_{2}=40 \mathrm{k} \Omega$ and $R_{1}=80 \mathrm{k} \Omega$.
At this point, it is desirable to find a more accurate estimate for $I_{E}$, taking into account the nonzero base current. Using Eq. (6.143),

$$
I_{E}=\frac{4-0.7}{3.3(\mathrm{k} \Omega)+\frac{(80 \| 40)(\mathrm{k} \Omega)}{101}}=0.93 \mathrm{~mA}
$$

This is quite a bit lower than 1 mA , the value we are aiming for. It is easy to see from the above equation that a simple way to restore $I_{E}$ to its nominal value would be to reduce $R_{E}$ from $3.3 \mathrm{k} \Omega$ by the magnitude of the second term in the denominator $(0.267 \mathrm{k} \Omega)$. Thus a more suitable value for $R_{E}$ in this case would be $R_{E}=3 \mathrm{k} \Omega$, which results in $I_{E}=1.01 \mathrm{~mA} \simeq 1 \mathrm{~mA} .{ }^{10}$

It should be noted that if we are willing to draw a higher current from the power supply and to accept a lower input resistance for the amplifier, then we may use a voltage-divider current equal, say, to $I_{E}$ (i.e., 1 mA ), resulting in $R_{1}=8 \mathrm{k} \Omega$ and $R_{2}=4 \mathrm{k} \Omega$. We shall refer to the circuit using these latter values as design 2 , for which the actual value of $I_{E}$ using the initial value of $R_{E}$ of $3.3 \mathrm{k} \Omega$ will be

$$
I_{E}=\frac{4-0.7}{3.3+0.027}=0.99 \simeq 1 \mathrm{~mA}
$$

In this case, design 2, we need not change the value of $R_{E}$.
Finally, the value of $R_{C}$ can be determined from

$$
R_{C}=\frac{12-V_{C}}{I_{C}}
$$

Substituting $I_{C}=\alpha I_{E}=0.99 \times 1=0.99 \mathrm{~mA} \simeq 1 \mathrm{~mA}$ results, for both designs, in

$$
R_{C}=\frac{12-8}{1}=4 \mathrm{k} \Omega
$$

[^8]
## EXERCISE

6.34 For design 1 in Example 6.12, calculate the expected range of $I_{E}$ if the transistor used has $\beta$ in the range of 50 to 150 . Express the range of $I_{E}$ as a percentage of the nominal value ( $I_{E} \simeq 1 \mathrm{~mA}$ ) obtained for $\beta=100$. Repeat for design 2.
Ans. For design 1: 0.94 mA to 1.04 mA , a $10 \%$ range; for design 2: 0.984 mA to 0.995 mA , a $1.1 \%$ range.


Figure 6.53 Biasing the BJT using two power supplies. Resistor $R_{B}$ is needed only if the signal is to be capacitively coupled to the base. Otherwise, the base can be connected directly to ground, or to a grounded signal source, resulting in almost total $\beta$-independence of the bias current.

## A Two-Power-Supply Version of the Classical Bias Arrangement A

 somewhat simpler bias arrangement is possible if two power supplies are available, as shown in Fig. 6.53. Writing a loop equation for the loop labeled L gives$$
\begin{equation*}
I_{E}=\frac{V_{E E}-V_{B E}}{R_{E}+R_{B} /(\beta+1)} \tag{6.146}
\end{equation*}
$$

This equation is identical to Eq. (6.143) except for $V_{E E}$ replacing $V_{B B}$. Thus the two constraints of Eqs. (6.144) and (6.145) apply here as well. Note that if the transistor is to be used with the base grounded (i.e., in the common-base configuration), then $R_{B}$ can be eliminated altogether. On the other hand, if the input signal is to be coupled to the base, then $R_{B}$ is needed. We shall study complete circuits of the various BJT amplifier configurations in Section 6.5. Finally, observe that the circuit in Fig. 6.53 is the counterpart of the MOS circuit in Fig. 6.48(e).

Biasing Using a Collector-to-Base Feedback Resistor In the BJT case, there is a counterpart to the MOSFET circuit of Fig. 6.50. Figure 6.54(a) shows such a simple but effective biasing arrangement that is suitable for common-emitter amplifiers. The circuit employs a resistor $R_{B}$ connected between the collector and the base. Resistor $R_{B}$ provides negative feedback, which helps to stabilize the bias point of the BJT.

Analysis of the circuit is shown in Fig. 6.54(b), from which we can write

$$
\begin{aligned}
V_{C C} & =I_{E} R_{C}+I_{B} R_{B}+V_{B E} \\
& =I_{E} R_{C}+\frac{I_{E}}{\beta+1} R_{B}+V_{B E}
\end{aligned}
$$

Thus the emitter bias current is given by

$$
\begin{equation*}
I_{E}=\frac{V_{C C}-V_{B E}}{R_{C}+R_{B} /(\beta+1)} \tag{6.147}
\end{equation*}
$$

## EXERCISE

D6.35 The bias arrangement of Fig. 6.53 is to be used for a common-base amplifier. Design the circuit to establish a dc emitter current of 1 mA and provide the highest possible voltage gain while allowing for a signal swing at the collector of $\pm 2 \mathrm{~V}$. Use $+10-\mathrm{V}$ and $-5-\mathrm{V}$ power supplies.

$$
\text { Ans. } R_{B}=0 ; R_{E}=4.3 \mathrm{k} \Omega ; R_{C}=8.4 \mathrm{k} \Omega
$$



Figure 6.54 (a) A common-emitter transistor amplifier biased by a feedback resistor $R_{B}$. (b) Analysis of the circuit in (a).

It is interesting to note that this equation is identical to Eq. (6.143), which governs the operation of the traditional bias circuit, except that $V_{C C}$ replaces $V_{B B}$ and $R_{C}$ replaces $R_{E}$. It follows that to obtain a value of $I_{E}$ that is insensitive to variation of $\beta$, we select $R_{B} /(\beta+1) \ll R_{C}$. Note, however, that the value of $R_{B}$ determines the allowable negative signal swing at the collector since

$$
\begin{equation*}
V_{C B}=I_{B} R_{B}=I_{E} \frac{R_{B}}{\beta+1} \tag{6.148}
\end{equation*}
$$

## EXERCISE

D6.36 Design the circuit of Fig. 6.54 to obtain a dc emitter current of 1 mA , maximum gain, and a $\pm 2-\mathrm{V}$ signal swing at the collector; that is, design for $V_{C E}=+2.3 \mathrm{~V}$. Let $V_{C C}=10 \mathrm{~V}$ and $\beta=100$.

Ans. $R_{B}=162 \mathrm{k} \Omega ; R_{C}=7.7 \mathrm{k} \Omega$. Note that if standard $5 \%$ resistor values are used (Appendix J), we select $R_{B}=160 \mathrm{k} \Omega$ and $R_{C}=7.5 \mathrm{k} \Omega$. This results in $I_{E}=1.02 \mathrm{~mA}$ and $V_{C}=+2.3 \mathrm{~V}$.

### 6.5 Discrete-Circuit Amplifiers

With our study of transistor amplifier basics complete, we now put everything together by presenting practical circuits for discrete-circuit amplifiers. These circuits, which utilize the amplifier configurations studied in Section 6.3 and the biasing methods of Section 6.4, can be assembled using off-the-shelf discrete transistors, resistors, and capacitors. Though practical and carefully selected to illustrate some important points, the circuits presented in this section should be regarded as examples of discrete-circuit transistor amplifiers. Indeed, there is a great variety of such circuits, a number of which are explored in the end-of-chapter problems.

As mentioned earlier, the vast majority of discrete-circuit amplifiers utilize BJTs. This is reflected in this section where all the circuits presented except for one utilize BJTs. Of course, if desired, one can utilize MOSFETs in the same amplifier configurations presented here. Also, the MOSFET is the device of choice in the design of integrated-circuit (IC) amplifiers. We begin our study of IC amplifiers in Chapter 7.

As will be seen shortly, the circuits presented in this section utilize large capacitors (in the $\mu \mathrm{F}$ range) to couple the signal source to the input of the amplifier, and to couple the amplifier output signal to a load
resistance or to the input of another amplifier stage. As well, a large capacitor is employed to establish a signal ground at the desired terminal of the transistor (e.g., at the emitter of a CE amplifier). The use of capacitors for these purposes simplifies the design considerably: Since capacitors block dc, one is able to first carry out the dc bias design and then connect the signal source and load to the amplifier without disturbing the dc design. These amplifiers are therefore known as capacitively coupled amplifiers.

### 6.5.1 A Common-Source (CS) Amplifier

As mentioned in Section 6.3, the common-source (CS) configuration is the most widely used of all MOSFET amplifier circuits. A common-source amplifier realized using the bias circuit of Fig. 6.48(c) is shown in Fig. 6.55(a). Observe that to establish a signal ground, or an ac ground as it is sometimes called, at the source, we have connected a large capacitor, $C_{S}$, between the source and ground. This capacitor, usually in the microfarad range, is required to provide a very small impedance (ideally, zero impedance-i.e., in effect, a short circuit) at all signal frequencies of interest. In this way, the signal current passes through $C_{S}$ to ground and thus bypasses the resistance $R_{S}$; hence, $C_{S}$ is called a bypass capacitor. Obviously, the lower the signal frequency, the less effective the bypass capacitor becomes. This issue will be studied in Section 9.1. For our purposes here we shall assume that $C_{S}$ is acting as a perfect short circuit and thus is establishing a zero signal voltage at the MOSFET source.

To prevent disturbances to the dc bias current and voltages, the signal to be amplified, shown as voltage source $v_{\text {sig }}$ with an internal resistance $R_{\text {sig }}$, is connected to the gate through a large capacitor $C_{C 1}$. Capacitor $C_{C 1}$, known as a coupling capacitor, is required to act as a perfect short circuit at all signal frequencies of interest while blocking dc. Here again, we note that as the signal frequency is lowered, the impedance of $C_{C 1}$ (i.e., $1 / j \omega C_{C 1}$ ) will increase and its effectiveness as a coupling capacitor will be correspondingly reduced. This problem, too, will be considered in Section 9.1 in connection with the dependence of the amplifier operation on frequency. For our purposes here we shall assume that $C_{C 1}$ is acting as a perfect short circuit as far as the signal is concerned.

The voltage signal resulting at the drain is coupled to the load resistance $R_{L}$ via another coupling capacitor $C_{C 2}$. We shall assume that $C_{C 2}$ acts as a perfect short circuit at all signal frequencies of interest and thus the output voltage $v_{o}=v_{d}$. Note that $R_{L}$ can be either an actual load resistor, to which the amplifier is required to provide its output voltage signal, or it can be the input resistance of another amplifier stage in cases where more than one stage of amplification is needed. (We will study multistage amplifiers in Chapter 8.)

Since a capacitor behaves as an open circuit at dc, the circuit for performing the dc bias design and analysis is obtained by open-circuiting all capacitors. The resulting circuit is shown in Fig. 6.55(b) and can be designed as discussed in Section 6.4.1.

To determine the terminal characteristics of the CS amplifier of Fig. 6.55(a)—that is, its input resistance, voltage gain, and output resistance-we replace the MOSFET with its hybrid- $\pi$ small-signal model, replace $V_{D D}$ with a signal ground, and replace all coupling and bypass capacitors with short circuits. The result is the circuit in Fig. 6.55(c). Analysis is straightforward and is shown on the figure, thus

$$
\begin{equation*}
R_{\mathrm{in}}=R_{G 1} \| R_{G 2} \tag{6.149}
\end{equation*}
$$

which shows that to keep $R_{\text {in }}$ high, large values should be used for $R_{G 1}$ and $R_{G 2}$, usually in the megohm range. The overall voltage gain $G_{v}$ is

$$
\begin{equation*}
G_{v}=-\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} g_{m}\left(R_{D}\left\|R_{L}\right\| r_{o}\right) \tag{6.150}
\end{equation*}
$$

Observe that we have taken $r_{o}$ into account, simply because it is easy to do so. Its effect, however, is usually small (this is not the case for IC amplifiers, as will be explained in Chapter 7). Finally, to encourage


Figure 6.55 (a) A common-source amplifier using the classical biasing arrangement of Fig. 6.48(c). (b) Circuit for determining the bias point. (c) Equivalent circuit and analysis.
the reader to do the small-signal analysis directly on the original circuit diagram, with the MOSFET model used implicitly, we show some of the analysis on the circuit of Fig. 6.55(a).

## EXERCISES

D6.37 Design the bias circuit in Fig.6.55(b) for the CS amplifier of Fig. 6.55(a). Assume the MOSFET is specified to have $V_{t}=1 \mathrm{~V}, k_{n}=4 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{A}=100 \mathrm{~V}$. Neglecting the Early effect, design for $I_{D}=0.5 \mathrm{~mA}, V_{S}=3.5 \mathrm{~V}$, and $V_{D}=6 \mathrm{~V}$ using a power-supply $V_{D D}=15 \mathrm{~V}$. Specify the values of $R_{S}$ and $R_{D}$. If a current of $2 \mu \mathrm{~A}$ is used in the voltage divider, specify the values of $R_{G 1}$ and $R_{G 2}$. Give the values of the MOSFET parameters $g_{m}$ and $r_{o}$ at the bias point.

$$
\text { Ans. } R_{S}=7 \mathrm{k} \Omega ; R_{D}=18 \mathrm{k} \Omega ; R_{G 1}=5 \mathrm{M} \Omega ; R_{G 2}=2.5 \mathrm{M} \Omega ; g_{m}=2 \mathrm{~mA} / \mathrm{V} ; r_{o}=200 \mathrm{k} \Omega
$$

6.38 For the CS amplifier of Fig. 6.55(a) use the design obtained in Exercise 6.37 to determine $R_{\text {in }}, R_{o}$, and the overall voltage gain $G_{v}$ when $R_{\text {sig }}=100 \mathrm{k} \Omega$ and $R_{L}=20 \mathrm{k} \Omega$.

Ans. $1.67 \mathrm{M} \Omega ; 16.5 \mathrm{k} \Omega ;-17.1 \mathrm{~V} / \mathrm{V}$
D6.39 As discussed in Section 6.3, beneficial effects can be realized by having an unbypassed resistance $R_{s}$ in the source lead of the CS amplifier. This can be implemented in the circuit of Fig. 6.55(a) by splitting the resistance $R_{S}$ into two resistances: $R_{s}$, which is left unbypassed, and ( $R_{S}-R_{s}$ ), across which the bypass capacitor $C_{S}$ is connected. Now, if in order to improve linearity of the amplifier in Exercises 6.37 and $6.38, v_{g s}$ is to be reduced to half its value, what value should $R_{s}$ have? What would the amplifier gain $G_{v}$ become? Recall that when $R_{s}$ is included it becomes difficult to include $r_{o}$ in the analysis, so neglect it.

$$
\text { Ans. } R_{s}=500 \Omega ; G_{v}=-8.9 \mathrm{~V} / \mathrm{V}
$$

### 6.5.2 A Common-Emitter Amplifier

The common-emitter (CE) amplifier is the most widely used of all BJT amplifier configurations. Figure 6.56(a) shows a CE amplifier utilizing the classical biasing arrangement of Fig. 6.48(c), the design of which was considered in Section 6.4. The CE circuit in Fig. 6.54(a) is the BJT counterpart of the CS amplifier of Fig. 6.55(a). It utilizes coupling capacitors $C_{C 1}$ and $C_{C 2}$ and bypass capacitor $C_{E}$. Here we assume that these capacitors, while blocking dc, behave as perfect short circuits at all signal frequencies of interest.

To determine the characteristic parameters of the CE amplifier, we replace the BJT with its hybrid- $\pi$ model, replace $V_{C C}$ with a short circuit to ground, and replace the coupling and bypass capacitor with short circuits. The resulting small-signal equivalent circuit of the CE amplifier is shown in Fig. 6.56(b). The analysis is straightforward and is given in the figure, thus

$$
\begin{equation*}
R_{\mathrm{in}}=R_{B 1}\left\|R_{B 2}\right\| r_{\pi} \tag{6.151}
\end{equation*}
$$

which indicates that to keep $R_{\text {in }}$ relatively high, $R_{B 1}$ and $R_{B 2}$ should be selected large (typically in the range of tens or hundreds of kilohms). This requirement conflicts with the need to keep $R_{B 1}$ and $R_{B 2}$ low so as to minimize the dependence of the dc current $I_{C}$ on the transistor $\beta$. We discussed this design trade-off in Section 6.4.

The voltage gain $G_{v}$ is given by

$$
\begin{equation*}
G_{v}=-\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} g_{m}\left(R_{C}\left\|R_{L}\right\| r_{o}\right) \tag{6.152}
\end{equation*}
$$

Note that we have taken $r_{o}$ into account because it is easy to do so. However, as already mentioned, the effect of this parameter on discrete-circuit amplifier performance is usually small.

## EXERCISES

D6.40 Design the bias circuit of the CE amplifier of Fig. 6.56(a) to obtain $I_{E}=0.5 \mathrm{~mA}$ and $V_{C}=$ +6 V . Design for a dc voltage at the base of 5 V and a current through $R_{B 2}$ of $50 \mu \mathrm{~A}$. Let $V_{C C}=+15 \mathrm{~V}$, $\beta=100$, and $V_{B E} \simeq 0.7 \mathrm{~V}$. Specify the values of $R_{B 1}, R_{B 2}, R_{E}$, and $R_{C}$. Also give the values of the BJT small-signal parameters $g_{m}, r_{\pi}$, and $r_{o}$ at the bias point. (For the calculation of $r_{o}$, let $V_{A}=100 \mathrm{~V}$.)

Ans. $R_{B 1}=182 \mathrm{k} \Omega ; R_{B 2}=100 \mathrm{k} \Omega ; R_{E}=8.6 \mathrm{k} \Omega ; R_{C}=18 \mathrm{k} \Omega ; g_{m}=20 \mathrm{~mA} / \mathrm{V}, r_{\pi}=5 \mathrm{k} \Omega$,
6.41 For the amplifier designed in Exercise 6.40, find $R_{\text {in }}, R_{o}$, and $G_{v}$ when $R_{\text {sig }}=10 \mathrm{k} \Omega$ and $R_{L}=20 \mathrm{k} \Omega$.

$$
\text { Ans. } R_{\mathrm{in}}=4.64 \mathrm{k} \Omega ; R_{o}=16.51 \mathrm{k} \Omega ; G_{v}=-57.3 \mathrm{~V} / \mathrm{V}
$$


(a)

(b)

Figure 6.56 (a) A common-emitter amplifier using the classical biasing arrangement of Fig. 6.52(a). (b) Equivalent circuit and analysis.

### 6.5.3 A Common-Emitter Amplifier with an Emitter Resistance $R_{e}$

As discussed in Section 6.3.4, it is beneficial to include a small resistance in the transistor emitter lead. This can be implemented in the circuit of Fig. 6.56(a) by splitting the emitter bias resistance $R_{E}$ into two components: an unbypassed resistance $R_{e}$, and a resistance ( $R_{E}-R_{e}$ ) across which the bypass capacitor $C_{E}$ is connected. The resulting circuit is shown in Fig. 6.57(a) and its small-signal model is shown in Fig. 6.57(b). In the latter we utilize the T model of the BJT because it results in much simpler analysis (recall that this is always the

(b)

Figure 6.57 (a) A common-emitter amplifier with an unbiased emitter resistance $R_{e} \cdot$ (b) The amplifier small-signal model and analysis.
case when a resistance is connected in series with the emitter). Also note that we have not included $r_{o}$, for doing so would complicate the analysis significantly. This burden would not be justified given that $r_{o}$ has little effect on the performance of discrete-circuit amplifiers.

Analysis of the circuit in Fig. 6.57(b) is straightforward and is shown in the figure. Thus,

$$
\begin{align*}
R_{\mathrm{in}} & =R_{B 1}\left\|R_{B 2}\right\|(\beta+1)\left(r_{e}+R_{e}\right) \\
& =R_{B 1}\left\|R_{B 2}\right\|\left[r_{\pi}+(\beta+1) R_{e}\right] \tag{6.153}
\end{align*}
$$

from which we note that including $R_{e}$ increases $R_{\text {in }}$ because it increases the input resistance looking into the base by adding a component $(\beta+1) R_{e}$ to $r_{\pi}$. The overall voltage gain $G_{v}$ is

$$
\begin{align*}
G_{v} & =-\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \times \alpha \frac{\text { Total resistance in collector }}{\text { Total resistance in emitter }} \\
& =-\alpha \frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \frac{R_{C} \| R_{L}}{r_{e}+R_{e}} \tag{6.154}
\end{align*}
$$

## EXERCISE

6.42 For the amplifier designed in Exercise 6.40 and analyzed in Exercise 6.41, let it be required to raise $R_{\mathrm{in}}$ to $10 \mathrm{k} \Omega$. What is the required value of $R_{e}$, and what does the overall voltage gain $G_{v}$ become?

$$
\text { Ans. } R_{e}=67.7 \Omega ; G_{v}=-39.8 \mathrm{~V} / \mathrm{V}
$$

### 6.5.4 A Common-Base (CB) Amplifier

Figure 6.58(a) shows a CB amplifier designed using the biasing arrangement of Fig. 6.53. Note that the availability of two power supplies, $V_{C C}$ and $-V_{E E}$, enables us to connect the base directly to ground, obviating the need for a large bypass capacitor to establish a signal ground at the base.

The small-signal equivalent circuit of the CB amplifier is shown in Fig 6.58(b). As expected, we have utilized the T model of the BJT and have not included $r_{o}$. Including $r_{o}$ would complicate the analysis significantly without making much difference to the results in the case of discrete-circuit amplifiers. From the circuit in Fig. 6.58(b) we find

$$
R_{\mathrm{in}}=r_{e} \| R_{E} \simeq r_{e} \simeq 1 / g_{m}
$$

which as expected can be very small, causing $v_{i}$ to be a small fraction of $v_{\text {sig }}$,

$$
v_{i}=v_{\mathrm{sig}} \frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}}
$$

Now,

$$
i_{e}=-\frac{v_{i}}{r_{e}}
$$

and

$$
v_{o}=-\alpha i_{e}\left(R_{C} \| R_{L}\right)
$$

Thus, the overall voltage gain is given by

$$
\begin{equation*}
G_{v}=\alpha \frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \frac{R_{C} \| R_{L}}{r_{e}}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} g_{m}\left(R_{C} \| R_{L}\right) \tag{6.155}
\end{equation*}
$$

## EXERCISE

D6.43 Design the CB amplifier of Fig. 6.58(a) to provide an input resistance $R_{\text {in }}$ that matches the source resistance of a cable with a characteristic resistance of $50 \Omega$. Assume that $R_{E} \gg r_{e}$. The available power supplies are $\pm 5 \mathrm{~V}$ and $R_{L}=8 \mathrm{k} \Omega$. Design for a dc collector voltage $V_{C}=+1 \mathrm{~V}$. Specify the values of $R_{C}$ and $R_{E}$. What overall voltage gain is obtained? If $v_{\text {sig }}$ is a sine wave with a peak amplitude of 10 mV , what is the peak amplitude of the output voltage? Let $\alpha \simeq 1$.

Ans. $R_{C}=8 \mathrm{k} \Omega ; R_{E}=8.6 \mathrm{k} \Omega ; 40 \mathrm{~V} / \mathrm{V} ; 0.4 \mathrm{~V}$

### 6.5.5 An Emitter Follower

Figure 6.59(a) shows an emitter follower designed using the bias arrangement of Fig. 6.53 and two power supplies, $V_{C C}$ and $-V_{E E}$. The bias resistance $R_{B}$ affects the input resistance of the follower and should be chosen as large as possible while limiting the dc voltage drop across it to a small fraction of $V_{E E}$; otherwise the dependence of the bias current $I_{C}$ on $\beta$ can become unacceptably large.

Figure 6.59(b) shows the small-signal equivalent circuit of the emitter follower. Here, as expected, we have replaced the BJT with its T model and included $r_{o}$ (since this can be done very simply). The input resistance of the emitter follower can be seen to be

$$
\begin{equation*}
R_{\mathrm{in}}=R_{B} \| R_{i b} \tag{6.156}
\end{equation*}
$$

where $R_{i b}$, the input resistance looking into the base, can be obtained by using the resistance-reflection rule. Toward that end, note that $r_{o}$ appears in parallel with $R_{E}$ and $R_{L}$ (which is why it can be easily taken into account). Thus,

$$
\begin{equation*}
R_{i b}=(\beta+1)\left[r_{e}+\left(R_{E}\left\|r_{o}\right\| R_{L}\right)\right] \tag{6.157}
\end{equation*}
$$

The overall voltage gain can be determined by tracking the signal transmission from source to load,

$$
\begin{equation*}
v_{i}=v_{\mathrm{sig}} \frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} \tag{6.158}
\end{equation*}
$$

and

$$
\begin{equation*}
v_{o}=v_{i} \frac{R_{E}\left\|r_{o}\right\| R_{L}}{r_{e}+\left(R_{E}\left\|r_{o}\right\| R_{L}\right)} \tag{6.159}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=\frac{R_{\text {in }}}{R_{\text {in }}+R_{\text {sig }}} \frac{\left(R_{E}\left\|r_{o}\right\| R_{L}\right)}{r_{e}+\left(R_{E}\left\|r_{o}\right\| R_{L}\right)} \tag{6.160}
\end{equation*}
$$

Finally, the output resistance $R_{\text {out }}$ can be obtained by short-circuiting $v_{\text {sig }}$ and looking back into the output terminal, excluding $R_{L}$, as

$$
\begin{equation*}
R_{\mathrm{out}}=r_{o}\left\|R_{E}\right\|\left[r_{e}+\frac{R_{B} \| R_{\mathrm{sig}}}{\beta+1}\right] \tag{6.161}
\end{equation*}
$$

Note that we have used the inverse resistance-reflection rule, namely, dividing the total resistance in the base, $\left(R_{B} \| R_{\text {sig }}\right)$, by $(\beta+1)$.

(a)

(b)

Figure 6.58 (a) A common-base amplifier using the structure of Fig. 6.53 with $R_{B}$ omitted (since the base is grounded). (b) Equivalent circuit obtained by replacing the transistor with its T model.

## EXERCISE

D6.44 Design the emitter follower of Fig. 6.59(a) to operate at a dc emitter current $I_{E}=1 \mathrm{~mA}$. Allow a dc voltage drop across $R_{B}$ of 1 V . The available power supplies are $\pm 5 \mathrm{~V}, \beta=100, V_{B E}=0.7 \mathrm{~V}$, and $V_{A}=100 \mathrm{~V}$. Specify the values required for $R_{B}$ and $R_{E}$. Now if $R_{\mathrm{sig}}=50 \mathrm{k} \Omega$ and $R_{L}=1 \mathrm{k} \Omega$, find $R_{\mathrm{in}}$, $v_{i} / v_{\text {sig }}, v_{o} / v_{i}, G_{v}$, and $R_{\text {out }}$. (Note: In performing the bias design, neglect the Early effect.)

$$
\text { Ans. } R_{B}=100 \mathrm{k} \Omega ; R_{E}=3.3 \mathrm{k} \Omega ; 44.3 \mathrm{k} \Omega ; 0.469 \mathrm{~V} / \mathrm{V} ; 0.968 \mathrm{~V} / \mathrm{V} ; 0.454 \mathrm{~V} / \mathrm{V} ; 320 \Omega
$$



Figure 6.59 (a) An emitter-follower circuit. (b) Small-signal equivalent circuit of the emitter follower with the transistor replaced by its T model. Note that $r_{o}$ is included because it is easy to do so. Normally, its effect on performance is small.

### 6.5.6 The Amplifier Frequency Response

Thus far, we have assumed that the gain of transistor amplifiers is constant independent of the frequency of the input signal. This would imply that transistor amplifiers have infinite bandwidth, which of course is not true. To illustrate, we show in Fig. 6.60 a sketch of the magnitude of the gain of a common-emitter or a CS amplifier such as those shown in Figs. 6.56 and 6.55, respectively, versus frequency. Observe that there is indeed a wide frequency range over which the gain remains almost constant. This obviously is the useful frequency range of operation for the particular amplifier. Thus far, we have been assuming that our amplifiers are operating in this frequency band, called the midband.

Figure 6.60 indicates that at lower frequencies, the magnitude of amplifier gain falls off. This is because the coupling and bypass capacitors no longer have low impedances. Recall that we assumed that their impedances were small enough to act as short circuits. Although this can be true at midband frequencies, as the frequency
of the input signal is lowered, the reactance $1 / j \omega C$ of each of these capacitors becomes significant, and it can be shown that this results in the overall voltage gain of the amplifier decreasing.

Figure 6.60 indicates also that the gain of the amplifier falls off at the high-frequency end. This is due to the internal capacitive effects in the BJT and the MOSFET. In Chapter 9 we shall study the internal capacitive effects of both transistor types and will augment their hybrid $-\pi$ models with capacitances that model these effects.

We will undertake a detailed study of the frequency response of transistor amplifiers in Chapter 9. For the time being, however, it is important for the reader to realize that for every transistor amplifier, there is a finite band over which the gain is almost constant. The boundaries of this useful frequency band, or midband, are the two frequencies $f_{L}$ and $f_{H}$ at which the gain drops by a certain number of decibels (usually 3 dB ) below its value at midband. As indicated in Fig. 6.60, the amplifier bandwidth, or 3-dB bandwidth, is defined as the difference between the lower $\left(f_{L}\right)$ and upper or higher $\left(f_{H}\right) 3-\mathrm{dB}$ frequencies:

$$
B W=f_{H}-f_{L}
$$

and since usually $f_{L} \ll f_{H}$,

$$
B W \simeq f_{H}
$$

A figure of merit for the amplifier is its gain-bandwidth product, defined as

$$
G B=\left|A_{M}\right| B W
$$

where $\left|A_{M}\right|$ is the magnitude of the amplifier gain in the midband. It will be seen in Chapter 9 that in amplifier design it is usually possible to trade off gain for bandwidth. One way to accomplish this, for instance, is by including resistance $R_{e}$ in the emitter of the CE amplifier.


Figure 6.60 Sketch of the magnitude of the gain of a CE (Fig. 6.56) or CS (Fig. 6.55) amplifier versus frequency. The graph delineates the three frequency bands relevant to frequency-response determination.

## Summary

- The essence of the use of the MOSFET (the BJT) as an amplifier is that when the transistor is operated in the active region, $v_{G S}$ controls $i_{D}\left(v_{B E}\right.$ controls $\left.i_{C}\right)$ in the manner of a voltage-controlled current source. When the device is dc biased in the active region, and the signal $v_{g s}\left(v_{b e}\right)$ is kept small, the operation becomes almost linear, with $i_{d}=g_{m} v_{g s}\left(i_{c}=g_{m} v_{b e}\right)$.
- The most fundamental parameter in characterizing the small-signal linear operation of a transistor is the transconductance $g_{m}$. For a MOSFET, $g_{m}=$ $\mu_{n} C_{o x}(W / L) V_{O V}=\sqrt{2 \mu_{n} C_{o x}(W / L) I_{D}}=2 I_{D} / V_{O V}$; and for the BJT, $g_{m}=I_{C} / V_{T}$.
- A systematic procedure for the analysis of a transistor amplifier circuit is presented in Table 6.1. Tables 6.2 and 6.3 present the small-signal models for the MOSFET and the BJT, respectively.
- When a resistance is connected in series with the source (or emitter), the T model is the most convenient to use.
- The three basic configurations of MOS and BJT amplifiers are presented in Fig. 6.33. Their characteristic parameter values are provided in Table 6.4 (for the MOS case) and in Table 6.5 (for the BJT case).
- The CS amplifier, which has (ideally) infinite input resistance and a reasonably high gain but a rather high output resistance and a limited high-frequency response (more on the latter point in Chapter 9), is used to obtain most of the gain in a cascade amplifier. Similar remarks apply to the CE amplifier, except that it has a relatively low input resistance ( $r_{\pi}=\beta / g_{m}$ ) arising from the finite base current of the BJT (finite $\beta$ ). Its voltage gain, however, can be larger than that of the CS amplifier because of the higher values of $g_{m}$ obtained with BJTs.
- Adding a resistance $R_{s}$ in the source of a CS amplifier (a resistance $R_{e}$ in the emitter of a CE amplifier) can lead to beneficial effects including the following: raising the input resistance of the CE amplifier, increasing linearity, and extending the useful amplifier bandwidth, at the expense of reducing the gain, all by a factor equal to $\left(1+g_{m} R_{s}\right)\left[\left(1+g_{m} R_{e}\right)\right.$ for the BJT case $]$.
- The CG (CB) amplifier has a low input resistance and thus, used alone, it has limited and specialized applications. However, its excellent high-frequency response makes it attractive in combination with the CS (CE) amplifier (Chapters 7 and 9).
- The source follower has (ideally) infinite input resistance, a voltage gain lower than but close to unity, and a low output resistance. It is employed as a voltage buffer and as the output stage of a multistage amplifier. Similar remarks apply to the emitter follower except that its input resistance, though large, is finite. Specifically, the emitter follower multiplies the total resistance in the emitter by $(\beta+1)$ before presenting it to the signal source.
- The resistance-reflection rule is a powerful tool in the analysis of BJT amplifier circuits: All resistances in the emitter circuit including the emitter resistance $r_{e}$ can be reflected to the base side by multiplying them by $(\beta+1)$. Conversely, we can reflect all resistances in the base circuit to the emitter side by dividing them by $(\beta+1)$.
In the analysis and design of discrete-circuit amplifiers, it is rarely necessary to take the transistor output resistance $r_{o}$ into account. In some situations, however, $r_{o}$ can be easily taken into account; specifically in the CS (CE) amplifier and in the source (emitter) follower. In IC amplifiers, $r_{o}$ must always be taken into account.
- A key step in the design of transistor amplifiers is to bias the transistor to operate at an appropriate point in the active region. A good bias design ensures that the parameters of the operating point $\left(I_{D}, V_{O V}\right.$, and $V_{D S}$ for the MOSFET; $I_{C}$ and $V_{C E}$ for the BJT) are predictable and stable and do not vary by large amounts when the transistor is replaced by another of the same type. The bias methods studied in this chapter are suited for discrete-circuit amplifiers only because they utilize large coupling and bypass capacitors.
- Discrete-circuit amplifiers predominantly employ BJTs. The opposite is true for IC amplifiers, where the device of choice is the MOSFET.


## Computer Simulation Problems

sim Problems identified by the Multisim/PSpice icon are intended to demonstrate the value of using SPICE simulation to verify hand analysis and design, and to investigate important issues such as allowable signal swing and amplifier nonlinear distortion. Instructions to assist in setting up PSpice and Multisim simulations for all the indicated problems can be found in the corresponding files on the website. Note that if a particular parameter value is not specified in the problem statement, you are to make a reasonable assumption.

## Section 6.1: Basic Principles

6.1 For the MOS amplifier of Fig. 6.2(a) with $V_{D D}=5 \mathrm{~V}$, $V_{t}=0.5 \mathrm{~V}, k_{n}=10 \mathrm{~mA} / \mathrm{V}^{2}$, and $R_{D}=20 \mathrm{k} \Omega$, determine the coordinates of the active-region segment (AB) of the VTC (Fig. 6.2b).

D 6.2 For the MOS amplifier of Fig. 6.2(a) with $V_{D D}=5 \mathrm{~V}$ and $k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}$, it is required to have the end point of the VTC , point B , at $V_{D S}=0.5 \mathrm{~V}$. What value of $R_{D}$ is required? If the transistor is replaced with another having twice the value of the transconductance parameter $k_{n}$, what new value of $R_{D}$ is needed?

D 6.3 It is required to bias the MOS amplifier of Fig. 6.3 at point Q for which $V_{O V}=0.2 \mathrm{~V}$ and $V_{D S}=1 \mathrm{~V}$. Find the required value of $R_{D}$ when $V_{D D}=5 \mathrm{~V}, V_{t}=0.5 \mathrm{~V}$, and $k_{n}=10 \mathrm{~mA} / \mathrm{V}^{2}$. Also specify the coordinates of the VTC end point $B$. What is the small-signal voltage gain of this amplifier? Assuming linear operation, what is the maximum allowable negative signal swing at the output? What is the corresponding peak input signal?
6.4 The MOS amplifier of Fig. 6.4(a), when operated with $V_{D D}=2 \mathrm{~V}$, is found to have a maximum small-signal voltage gain magnitude of $14 \mathrm{~V} / \mathrm{V}$. Find $V_{O V}$ and $V_{D S}$ for bias point Q at which a voltage gain of $-12 \mathrm{~V} / \mathrm{V}$ is obtained.
6.5 Various measurements are made on an NMOS amplifier for which the drain resistor $R_{D}$ is $20 \mathrm{k} \Omega$. First, dc measurements show the voltage across the drain resistor, $V_{R D}$, to be 1.5 V and the gate-to-source bias voltage to be 0.7 V . Then, ac measurements with small signals show the voltage gain to be $-10 \mathrm{~V} / \mathrm{V}$. What is the value of $V_{t}$ for this
transistor? If the process transconductance parameter $k_{n}^{\prime}$ is $200 \mu \mathrm{~A} / \mathrm{V}^{2}$, what is the MOSFET's W/L?
6.6 Consider the amplifier of Fig. 6.4(a) for the case $V_{D D}=$ $5 \mathrm{~V}, R_{D}=24 \mathrm{k} \Omega, k_{n}^{\prime}(W / L)=1 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{t}=1 \mathrm{~V}$.
(a) Find the coordinates of the two end points of the saturation-region segment of the amplifier transfer characteristic, that is, points A and B on the sketch of Fig. 6.4(b).
(b) If the amplifier is biased to operate with an overdrive voltage $V_{O V}$ of 0.5 V , find the coordinates of the bias point Q on the transfer characteristic. Also, find the value of $I_{D}$ and of the incremental gain $A_{v}$ at the bias point.
(c) For the situation in (b), and disregarding the distortion caused by the MOSFET's square-law characteristic, what is the largest amplitude of a sine-wave voltage signal that can be applied at the input while the transistor remains in saturation? What is the amplitude of the output voltage signal that results? What gain value does the combination of these amplitudes imply? By what percentage is this gain value different from the incremental gain value calculated above? Why is there a difference?
*6.7 The expression for the incremental voltage gain $A_{v}$ given in Eq. (6.16) can be written in as

$$
A_{v}=-\frac{2\left(V_{D D}-V_{D S}\right)}{V_{O V}}
$$

where $V_{D S}$ is the bias voltage at the drain. This expression indicates that for given values of $V_{D D}$ and $V_{O V}$, the gain magnitude can be increased by biasing the transistor at a lower $V_{D S}$. This, however, reduces the allowable output signal swing in the negative direction. Assuming linear operation around the bias point, show that the largest possible negative output signal peak $\hat{v}_{o}$ that is achievable while the transistor remains saturated is

$$
\hat{v}_{o}=\left(V_{D S}-V_{O V}\right) /\left(1+\frac{1}{\left|A_{v}\right|}\right)
$$

For $V_{D D}=5 \mathrm{~V}$ and $V_{O V}=0.5 \mathrm{~V}$, provide a table of values for $A_{v}, \hat{v}_{o}$, and the corresponding $\hat{v}_{i}$ for $V_{D S}=1 \mathrm{~V}, 1.5 \mathrm{~V}$, 2 V , and 2.5 V . If $k_{n}^{\prime}(W / L)=1 \mathrm{~mA} / \mathrm{V}^{2}$, find $I_{D}$ and $R_{D}$ for the design for which $V_{D S}=1 \mathrm{~V}$.

D *6.8 Design the MOS amplifier of Fig. 6.4(a) to obtain maximum gain while allowing for an output voltage swing of at least $\pm 0.5 \mathrm{~V}$. Let $V_{D D}=5 \mathrm{~V}$, and utilize an overdrive voltage of approximately 0.2 V .
(a) Specify $V_{D S}$ at the bias point.
(b) What is the gain achieved? What is the signal amplitude $\hat{v}_{g s}$ that results in the $0.5-\mathrm{V}$ signal amplitude at the output?
(c) If the dc bias current in the drain is to be $100 \mu \mathrm{~A}$, what value of $R_{D}$ is needed?
(d) If $k_{n}^{\prime}=200 \mu \mathrm{~A} / \mathrm{V}^{2}$, what $W / L$ ratio is required for the MOSFET?
*6.9 Figure P6.9 shows an amplifier in which the load resistor $R_{D}$ has been replaced with another NMOS transistor $Q_{2}$ connected as a two-terminal device. Note that because $v_{D G}$ of $Q_{2}$ is zero, it will be operating in saturation at all times, even when $v_{I}=0$ and $i_{D 2}=i_{D 1}=0$. Note also that the two transistors conduct equal drain currents. Using $i_{D 1}=i_{D 2}$, show that for the range of $v_{I}$ over which $Q_{1}$ is operating in saturation, that is, for

$$
V_{t 1} \leq v_{I} \leq v_{o}+V_{t 1}
$$

the output voltage will be given by

$$
v_{O}=V_{D D}-V_{t}+\sqrt{\frac{(W / L)_{1}}{(W / L)_{2}}} V_{t}-\sqrt{\frac{(W / L)_{1}}{(W / L)_{2}}} v_{l}
$$

where we have assumed $V_{t 1}=V_{t 2}=V_{t}$. Thus the circuit functions as a linear amplifier, even for large input signals. For $(W / L)_{1}=(50 \mu \mathrm{~m} / 0.5 \mu \mathrm{~m})$ and $(W / L)_{2}=$ ( $5 \mu \mathrm{~m} / 0.5 \mu \mathrm{~m}$ ), find the voltage gain.


Figure P6.9
6.10 A BJT amplifier circuit such as that in Fig. 6.6 is operated with $V_{C C}=+5 \mathrm{~V}$ and is biased at $V_{C E}=+1 \mathrm{~V}$. Find the voltage gain, the maximum allowed output negative swing without the transistor entering saturation, and the corresponding maximum input signal permitted.
6.11 For the amplifier circuit in Fig. 6.6 with $V_{C C}=+5 \mathrm{~V}$ and $R_{C}=1 \mathrm{k} \Omega$, find $V_{C E}$ and the voltage gain at the following dc collector bias currents: $0.5 \mathrm{~mA}, 1 \mathrm{~mA}, 2.5 \mathrm{~mA}$, 4 mA , and 4.5 mA . For each, give the maximum possible positive- and negative-output signal swing as determined by the need to keep the transistor in the active region. Present your results in a table.

D 6.12 Consider the CE amplifier circuit of Fig. 6.6 when operated with a dc supply $V_{C C}=+5 \mathrm{~V}$. It is required to find the point at which the transistor should be biased; that is, find the value of $V_{C E}$ so that the output sine-wave signal $v_{c e}$ resulting from an input sine-wave signal $v_{b e}$ of $5-\mathrm{mV}$ peak amplitude has the maximum possible magnitude. What is the peak amplitude of the output sine wave and the value of the gain obtained? Assume linear operation around the bias point. (Hint: To obtain the maximum possible output amplitude for a given input, you need to bias the transistor as close to the edge of saturation as possible without entering saturation at any time, that is, without $v_{C E}$ decreasing below 0.3 V .)
6.13 The transistor in the circuit of Fig. P6.13 is biased at a dc collector current of 0.3 mA . What is the voltage gain? (Hint: Use Thévenin's theorem to convert the circuit to the form in Fig. 6.6.)


Figure P6.13

D *6.14 A BJT amplifier such as that in Fig. 6.6 is to be designed to support relatively undistorted sine-wave output signals of peak amplitudes $P$ volt without the BJT entering saturation or cutoff and to have the largest possible voltage gain, denoted $A_{v} \mathrm{~V} / \mathrm{V}$. Show that the minimum supply voltage $V_{C C}$ needed is given by

$$
V_{C C}=V_{C E s a t}+P+\left|A_{v}\right| V_{T}
$$

Also, find $V_{C C}$, specified to the nearest 0.5 V , for the following situations:
(a) $A_{v}=-20 \mathrm{~V} / \mathrm{V}, P=0.2 \mathrm{~V}$
(b) $A_{v}=-50 \mathrm{~V} / \mathrm{V}, P=0.5 \mathrm{~V}$
(c) $A_{v}=-100 \mathrm{~V} / \mathrm{V}, P=0.5 \mathrm{~V}$
(d) $A_{v}=-100 \mathrm{~V} / \mathrm{V}, P=1.0 \mathrm{~V}$
(e) $A_{v}=-200 \mathrm{~V} / \mathrm{V}, P=1.0 \mathrm{~V}$
(f) $A_{v}=-500 \mathrm{~V} / \mathrm{V}, P=1.0 \mathrm{~V}$
(g) $A_{v}=-500 \mathrm{~V} / \mathrm{V}, P=2.0 \mathrm{~V}$
6.15 Sketch and label the voltage-transfer characteristics of the pnp amplifiers shown in Fig. P6.15.


Figure P6.15
*6.16 In deriving the expression for small-signal voltage gain $A_{v}$ in Eq. (6.21) we neglected the Early effect. Derive this expression including the Early effect by substituting

$$
i_{C}=I_{S} e^{v_{B E} / V_{T}}\left(1+\frac{v_{C E}}{V_{A}}\right)
$$

in Eq. (6.4) and including the factor $\left(1+V_{C E} / V_{A}\right)$ in Eq. (6.11). Show that the gain expression changes to

$$
A_{v}=\frac{-I_{C} R_{C} / V_{T}}{\left[1+\frac{I_{C} R_{C}}{V_{A}+V_{C E}}\right]}=-\frac{\left(V_{C C}-V_{C E}\right) / V_{T}}{\left[1+\frac{V_{C C}-V_{C E}}{V_{A}+V_{C E}}\right]}
$$

For the case $V_{C C}=5 \mathrm{~V}$ and $V_{C E}=3 \mathrm{~V}$, what is the gain without and with the Early effect taken into account? Let $V_{A}=100 \mathrm{~V}$.
*6.17 Consider the amplifier circuit of Fig. 6.6 when operated with a supply voltage $V_{C C}=+3 \mathrm{~V}$.
(a) What is the theoretical maximum voltage gain that this amplifier can provide?
(b) What value of $V_{C E}$ must this amplifier be biased at to provide a voltage gain of $-60 \mathrm{~V} / \mathrm{V}$ ?
(c) If the dc collector current $I_{C}$ at the bias point in (b) is to be 0.5 mA , what value of $R_{C}$ should be used?
(d) What is the value of $V_{B E}$ required to provide the bias point mentioned above? Assume that the BJT has $I_{S}=10^{-15} \mathrm{~A}$.
(e) If a sine-wave signal $v_{b e}$ having a $5-\mathrm{mV}$ peak amplitude is superimposed on $V_{B E}$, find the corresponding output voltage signal $v_{c e}$ that will be superimposed on $V_{C E}$ assuming linear operation around the bias point.
(f) Characterize the signal current $i_{c}$ that will be superimposed on the dc bias current $I_{C}$.
(g) What is the value of the dc base current $I_{B}$ at the bias point? Assume $\beta=100$. Characterize the signal current $i_{b}$ that will be superimposed on the base current $I_{B}$.
(h) Dividing the amplitude of $v_{b e}$ by the amplitude of $i_{b}$, evaluate the incremental (or small-signal) input resistance of the amplifier.
(i) Sketch and clearly label correlated graphs for $v_{B E}, v_{C E}$, $i_{C}$, and $i_{B}$ versus time. Note that each graph consists of a dc or average value and a superimposed sine wave. Be careful of the phase relationships of the sine waves.
6.18 The essence of transistor operation is that a change in $v_{B E}, \Delta v_{B E}$, produces a change in $i_{C}, \Delta i_{C}$. By keeping $\Delta v_{B E}$ small, $\Delta i_{C}$ is approximately linearly related to $\Delta v_{B E}, \Delta i_{C}=$ $g_{m} \Delta v_{B E}$, where $g_{m}$ is known as the transistor transconductance. By passing $\Delta i_{C}$ through $R_{C}$, an output voltage signal $\Delta v_{o}$ is obtained. Use the expression for the small-signal voltage gain in Eq. (6.20) to derive an expression for $g_{m}$. Find the value of $g_{m}$ for a transistor biased at $I_{C}=0.5 \mathrm{~mA}$.
6.19 The purpose of this problem is to illustrate the application of graphical analysis to the circuit shown in Fig. P6.19. Sketch $i_{C}-v_{C E}$ characteristic curves for the BJT for $i_{B}=10 \mu \mathrm{~A}, 20 \mu \mathrm{~A}, 30 \mu \mathrm{~A}$, and $40 \mu \mathrm{~A}$. Assume the lines to be horizontal (i.e., neglect the Early effect), and
let $\beta=100$. For $V_{C C}=5 \mathrm{~V}$ and $R_{C}=1 \mathrm{k} \Omega$, sketch the load line. What peak-to-peak collector voltage swing will result for $i_{B}$ varying over the range $10 \mu \mathrm{~A}$ to $40 \mu \mathrm{~A}$ ? If the BJT is biased at $V_{C E}=\frac{1}{2} V_{C C}$, find the value of $I_{C}$ and $I_{B}$. If at this current $V_{B E}=0.7 \mathrm{~V}$ and if $R_{B}=100 \mathrm{k} \Omega$, find the required value of $V_{B B}$.


Figure P6. 19
*6.20 Sketch the $i_{C}-v_{C E}$ characteristics of an $n p n$ transistor having $\beta=100$ and $V_{A}=100 \mathrm{~V}$. Sketch characteristic curves for $i_{B}=20 \mu \mathrm{~A}, 50 \mu \mathrm{~A}, 80 \mu \mathrm{~A}$, and $100 \mu \mathrm{~A}$. For the purpose of this sketch, assume that $i_{C}=\beta i_{B}$ at $v_{C E}=0$. Also, sketch the load line obtained for $V_{C C}=10 \mathrm{~V}$ and $R_{C}=$ $1 \mathrm{k} \Omega$. If the dc bias current into the base is $50 \mu \mathrm{~A}$, write the equation for the corresponding $i_{C}-v_{C E}$ curve. Also, write the equation for the load line, and solve the two equations to obtain $V_{C E}$ and $I_{C}$. If the input signal causes a sinusoidal signal of $30-\mu \mathrm{A}$ peak amplitude to be superimposed on $I_{B}$, find the corresponding signal components of $i_{C}$ and $v_{C E}$.

## Section 6.2: Small-Signal Operation and Models

*6.21 This problem investigates the nonlinear distortion introduced by a MOSFET amplifier. Let the signal $v_{g s}$ be a sine wave with amplitude $V_{g s}$, and substitute $v_{g s}=V_{g s} \sin \omega t$ in Eq. (6.28). Using the trigonometric identity $\sin ^{2} \theta=$ $\frac{1}{2}-\frac{1}{2} \cos 2 \theta$, show that the ratio of the signal at frequency $2 \omega$ to that at frequency $\omega$, expressed as a percentage (known as the second-harmonic distortion) is

$$
\text { Second-harmonic distortion }=\frac{1}{4} \frac{V_{g s}}{V_{O V}} \times 100
$$

If in a particular application $V_{g s}$ is 10 mV , find the minimum overdrive voltage at which the transistor should be operated so that the second-harmonic distortion is kept to less than $1 \%$.
6.22 Consider an NMOS transistor having $k_{n}=10 \mathrm{~mA} / \mathrm{V}^{2}$. Let the transistor be biased at $V_{O V}=0.2 \mathrm{~V}$. For operation in saturation, what dc bias current $I_{D}$ results? If a $0.02-\mathrm{V}$ signal is superimposed on $V_{G S}$, find the corresponding increment in collector current by evaluating the total collector current $i_{D}$ and subtracting the dc bias current $I_{D}$. Repeat for a $-0.02-\mathrm{V}$ signal. Use these results to estimate $g_{m}$ of the FET at this bias point. Compare with the value of $g_{m}$ obtained using Eq. (6.33).
6.23 Consider the FET amplifier of Fig. 6.10 for the case $V_{t}=0.4 \mathrm{~V}, k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}, V_{G S}=0.6 \mathrm{~V}, V_{D D}=1.8 \mathrm{~V}$, and $R_{D}=10 \mathrm{k} \Omega$.
(a) Find the dc quantities $I_{D}$ and $V_{D S}$.
(b) Calculate the value of $g_{m}$ at the bias point.
(c) Calculate the value of the voltage gain.
(d) If the MOSFET has $\lambda=0.1 \mathrm{~V}^{-1}$, find $r_{o}$ at the bias point and calculate the voltage gain.

D *6.24 An NMOS amplifier is to be designed to provide a $0.20-\mathrm{V}$ peak output signal across a $20-\mathrm{k} \Omega$ load that can be used as a drain resistor. If a gain of at least $10 \mathrm{~V} / \mathrm{V}$ is needed, what $g_{m}$ is required? Using a dc supply of 1.8 V , what values of $I_{D}$ and $V_{O V}$ would you choose? What $W / L$ ratio is required if $\mu_{n} C_{o x}=200 \mu \mathrm{~A} / \mathrm{V}^{2}$ ? If $V_{t}=0.4 \mathrm{~V}$, find $V_{G S}$.

D *6.25 In this problem we investigate an optimum design of the CS amplifier circuit of Fig. 6.10. First, use the voltage gain expression $A_{v}=-g_{m} R_{D}$ together with Eq. (6.42) for $g_{m}$ to show that

$$
A_{v}=-\frac{2 I_{D} R_{D}}{V_{O V}}=-\frac{2\left(V_{D D}-V_{D S}\right)}{V_{O V}}
$$

Next, let the maximum positive input signal be $\hat{v}_{i}$. To keep the second-harmonic distortion to an acceptable level, we bias the MOSFET to operate at an overdrive voltage $V_{O V} \gg \hat{v}_{i}$. Let $V_{O V}=m \hat{v}_{i}$. Now, to maximize the voltage gain $\left|A_{v}\right|$, we design for the lowest possible $V_{D S}$. Show that the minimum $V_{D S}$ that is consistent with allowing a negative signal voltage swing at the drain of $\left|A_{v}\right| \hat{v}_{i}$ while maintaining saturation-mode operation is given by

$$
V_{D S}=\frac{V_{O V}+\hat{v}_{i}+2 V_{D D}\left(\hat{v}_{i} / V_{O V}\right)}{1+2\left(\hat{v}_{i} / V_{O V}\right)}
$$

| Voltages (V) Dimensions ( $\mu \mathrm{m}$ ) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | Type | $I_{D}(\mathrm{~mA})$ | $\left\|V_{G S}\right\|$ | $\left\|V_{t}\right\|$ | $V_{\text {ov }}$ | W | $L$ | W/L | $k^{\prime}(W / L)$ | $\mathrm{g}_{\mathrm{m}}(\mathrm{mA} / \mathrm{V})$ |
| a | N | 1 | 3 | 2 |  |  | 1 |  |  |  |
| b | N | 1 |  | 0.7 | 0.5 | 50 |  |  |  |  |
| c | N | 10 |  |  | 2 |  | 1 |  |  |  |
| d | N | 0.5 |  |  | 0.5 |  |  |  |  |  |
| e | N | 0.1 |  |  |  | 10 | 2 |  |  |  |
| f | N |  | 1.8 | 0.8 |  | 40 | 4 |  |  |  |
| g | P | 0.5 |  |  |  |  |  | 25 |  |  |
| h | P |  | 3 | 1 |  |  |  |  | 0.5 |  |
| i | P | 10 |  |  |  | 4000 | 2 |  |  |  |
| j | P | 10 |  |  | 4 |  |  |  |  |  |
| k | P |  |  |  | 1 | 30 | 3 |  |  |  |
| 1 | P |  |  |  | 5 |  |  |  | 0.08 |  |

Now, find $V_{O V}, V_{D S}, A_{v}$, and $\hat{v}_{o}$ for the case $V_{D D}=2.5 \mathrm{~V}, \hat{v}_{i}=$ 20 mV , and $m=15$. If it is desired to operate this transistor at $I_{D}=200 \mu \mathrm{~A}$, find the values of $R_{D}$ and $W / L$, assuming that for this process technology $k_{n}^{\prime}=100 \mu \mathrm{~A} / \mathrm{V}^{2}$.
6.26 In the table above, for MOS transistors operating under a variety of conditions, complete as many entries as possible. Although some data is not available, it is always possible to calculate $g_{m}$ using one of Eqs. (6.40), (6.41), or (6.42). Assume $\mu_{n}=500 \mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{s}, \mu_{p}=250 \mathrm{~cm}^{2} / \mathrm{V} \cdot \mathrm{s}$, and $C_{o x}=0.4 \mathrm{fF} / \mu \mathrm{m}^{2}$.
6.27 For the NMOS amplifier in Fig. P6.27, replace the transistor with its T equivalent circuit, assuming $\lambda=0$. Derive expressions for the voltage gains $v_{s} / v_{i}$ and $v_{d} / v_{i}$.


Figure P6.27
sim 6.28 In the circuit of Fig. P6.28, the NMOS transistor has $\left|V_{t}\right|=0.5 \mathrm{~V}$ and $V_{A}=50 \mathrm{~V}$ and operates with $V_{D}=1 \mathrm{~V}$. What is the voltage gain $v_{o} / v_{i}$ ? What do $V_{D}$ and the gain become for $I$ increased to 1 mA ?


Figure P6. 28
6.29 For a $0.18-\mu \mathrm{m}$ CMOS fabrication process: $V_{t n}=$ $0.5 \mathrm{~V}, V_{t p}=-0.5 \mathrm{~V}, \mu_{n} C_{o x}=400 \mu \mathrm{~A} / \mathrm{V}^{2}, \mu_{p} C_{o x}=$ $100 \mu \mathrm{~A} / \mathrm{V}^{2}, C_{o x}=8.6 \mathrm{fF} / \mu \mathrm{m}^{2}, V_{A}$ ( $n$-channel devices) $=5 L(\mu \mathrm{~m})$, and $\left|V_{A}\right|(p$-channel devices $)=6 L(\mu \mathrm{~m})$. Find the small-signal model parameters ( $g_{m}$ and $r_{o}$ ) for both an NMOS and a PMOS transistor having $W / L=$ $10 \mu \mathrm{~m} / 0.5 \mu \mathrm{~m}$ and operating at $I_{D}=100 \mu \mathrm{~A}$. Also, find the overdrive voltage at which each device must be operating.
*6.30 Figure P6.30 shows a discrete-circuit amplifier. The input signal $v_{\text {sig }}$ is coupled to the gate through a very


Figure P6.30
large capacitor (shown as infinite). The transistor source is connected to ground at signal frequencies via a very large capacitor (shown as infinite). The output voltage signal that develops at the drain is coupled to a load resistance via a very large capacitor (shown as infinite). All capacitors behave as short circuits for signals and as open circuits for dc.
(a) If the transistor has $V_{t}=1 \mathrm{~V}$, and $k_{n}=4 \mathrm{~mA} / \mathrm{V}^{2}$, verify that the bias circuit establishes $V_{G S}=1.5 \mathrm{~V}, I_{D}=$ 0.5 mA , and $V_{D}=+7.0 \mathrm{~V}$. That is, assume these values, and verify that they are consistent with the values of the circuit components and the device parameters.
(b) Find $g_{m}$ and $r_{o}$ if $V_{A}=100 \mathrm{~V}$.
(c) Draw a complete small-signal equivalent circuit for the amplifier, assuming all capacitors behave as short circuits at signal frequencies.
(d) Find $R_{\text {in }}, v_{g s} / v_{\text {sig }}, v_{o} / v_{g s}$, and $v_{o} / v_{\text {sig }}$.
6.31 Consider a transistor biased to operate in the active mode at a dc collector current $I_{C}$. Calculate the collector signal current as a fraction of $I_{C}$ (i.e., $i_{c} I_{C}$ ) for input signals $v_{b e}$ of $+1 \mathrm{mV},-1 \mathrm{mV},+2 \mathrm{mV},-2 \mathrm{mV},+5 \mathrm{mV},-5 \mathrm{mV}$, $+8 \mathrm{mV},-8 \mathrm{mV},+10 \mathrm{mV},-10 \mathrm{mV},+12 \mathrm{mV}$, and -12 mV . In each case do the calculation two ways:
(a) using the exponential characteristic, and
(b) using the small-signal approximation.

Present your results in the form of a table that includes a column for the error introduced by the small-signal
approximation. Comment on the range of validity of the small-signal approximation.
6.32 An npn BJT with grounded emitter is operated with $V_{B E}=0.700 \mathrm{~V}$, at which the collector current is 0.5 mA . A $5-\mathrm{k} \Omega$ resistor connects the collector to a $+5-\mathrm{V}$ supply. What is the resulting collector voltage $V_{C}$ ? Now, if a signal applied to the base raises $v_{B E}$ to 705 mV , find the resulting total collector current $i_{C}$ and total collector voltage $v_{C}$ using the exponential $i_{C}-v_{B E}$ relationship. For this situation, what are $v_{b e}$ and $v_{c}$ ? Calculate the voltage gain $v_{c} / v_{b e}$. Compare with the value obtained using the small-signal approximation, that is, $-g_{m} R_{C}$.
6.33 A transistor with $\beta=100$ is biased to operate at a dc collector current of 0.5 mA . Find the values of $g_{m}, r_{\pi}$, and $r_{e}$. Repeat for a bias current of $50 \mu \mathrm{~A}$.

D 6.34 A designer wishes to create a BJT amplifier with a $g_{m}$ of $30 \mathrm{~mA} / \mathrm{V}$ and a base input resistance of $3000 \Omega$ or more. What collector-bias current should he choose? What is the minimum $\beta$ he can tolerate for the transistor used?
6.35 A pnp BJT is biased to operate at $I_{C}=1.0 \mathrm{~mA}$. What is the associated value of $g_{m}$ ? If $\beta=100$, what is the value of the small-signal resistance seen looking into the emitter $\left(r_{e}\right)$ ? Into the base $\left(r_{\pi}\right)$ ? If the collector is connected to a $5-\mathrm{k} \Omega$ load, with a signal of $5-\mathrm{mV}$ peak applied between base and emitter, what output signal voltage results?
6.36 In the circuit of Fig. 6.20, $V_{B E}$ is adjusted so that $V_{C}=$ 1 V . If $V_{C C}=3 \mathrm{~V}, R_{C}=2 \mathrm{k} \Omega$, and a signal $v_{b e}=0.005 \sin \omega t$ volts is applied, find expressions for the total instantaneous quantities $i_{C}(t), v_{C}(t)$, and $i_{B}(t)$. The transistor has $\beta=100$. What is the voltage gain?

D *6.37 We wish to design the amplifier circuit of Fig. 6.20 under the constraint that $V_{C C}$ is fixed. Let the input signal $v_{b e}=\hat{V}_{b e} \sin \omega t$, where $\hat{V}_{b e}$ is the maximum value for acceptable linearity. For the design that results in the largest signal at the collector, without the BJT leaving the active region, show that

$$
R_{C} I_{C}=\left(V_{C C}-0.3\right) /\left(1+\frac{\hat{V}_{b e}}{V_{T}}\right)
$$

and find an expression for the voltage gain obtained. For $V_{C C}=3 \mathrm{~V}$ and $\hat{V}_{b e}=5 \mathrm{mV}$, find the dc voltage at the collector, the amplitude of the output voltage signal, and the voltage gain.
6.38 The table below summarizes some of the basic attributes of a number of BJTs of different types, operating as amplifiers under various conditions. Provide the missing entries. (Note: Isn't it remarkable how much two parameters can reveal?)
6.39 A BJT is biased to operate in the active mode at a dc collector current of 1 mA . It has a $\beta$ of 100 and $V_{A}$ of 100 V . Give the four small-signal models (Figs. 6.25 and 6.27) of the BJT complete with the values of their parameters.
6.40 Using the T model of Fig. 6.26(a), show that the input resistance between base and emitter, looking into the base, is equal to $r_{\pi}$.
6.41 Show that the collector current provided by the model of Fig. 6.26(b) is equal to that provided by the model in Fig. 6.26(a).
6.42 Show that the hybrid- $\pi$ model of Fig. 6.24(b) is the incremental version of the large-signal model of Fig. 4.5(d).
6.43 Show that the T model of Fig. 6.26(b) is the incremental version of the large-signal model of Fig. 4.5(b).
6.44 The transistor amplifier in Fig. P6.44 is biased with a current source $I$ and has a very high $\beta$. Find the dc voltage at the collector, $V_{C}$. Also, find the value of $r_{e}$. Replace the transistor with the T model of Fig. 6.26(b) (note that the dc current source $I$ should be replaced with an open circuit). Hence find the voltage gain $v_{c} / v_{i}$.


Figure P6.44
6.45 Figure P6.45 shows the circuit of an amplifier fed with a signal source $v_{\text {sig }}$ with a source resistance $R_{\text {sig }}$. The bias circuitry is not shown. Replace the BJT with its hybrid- $\pi$ equivalent circuit of Fig. 6.24(a). Find the input resistance $R_{\text {in }} \equiv v_{\pi} / i_{b}$, the voltage transmission from source to amplifier input, $v_{\pi} / v_{\text {sig }}$, and the voltage gain from base to collector, $v_{0} / v_{\pi}$.

| Transistor | a | b | c | d | e | f | g |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | 1.000 |  |  |  |  | 0.90 |  |
| $\beta$ |  | 100 |  | $\infty$ |  |  |  |
| $I_{C}(\mathrm{~mA})$ | 1.00 |  | 1.00 |  |  |  |  |
| $I_{E}(\mathrm{~mA})$ |  | 1.00 |  |  |  | 5 |  |
| $I_{B}(\mathrm{~mA})$ |  |  | 0.020 |  |  |  |  |
| $g_{m}(\mathrm{~mA} / \mathrm{V})$ |  |  |  |  |  |  | 700 |
| $r_{e}(\Omega)$ |  |  |  | 25 |  |  |  |
| $r_{\pi}(\Omega)$ |  |  |  |  | $10.1 \mathrm{k} \Omega$ |  |  |

Use these to show that the overall voltage gain $v_{o} / v_{\text {sig }}$ is given by

$$
\frac{v_{o}}{v_{\mathrm{sig}}}=-\frac{\beta R_{C}}{r_{\pi}+R_{\mathrm{sig}}}
$$



Figure P6.45
6.46 For the conceptual circuit shown in Fig. 6.23, $R_{C}=2 \mathrm{k} \Omega, g_{m}=50 \mathrm{~mA} / \mathrm{V}$, and $\beta=100$. If a peak-to-peak output voltage of 1 V is measured at the collector, what are the peak-to-peak values of $v_{b e}$ and $i_{b}$ ?
6.47 Figure P6.47 shows a transistor with the collector connected to the base. The bias arrangement is not shown. Since a zero $v_{B C}$ implies operation in the active mode, the BJT can be replaced by one of the small-signal models of Figs. 6.24 and 6.26. Use the model of Fig. 6.26(b) and show that the resulting two-terminal device, known as a diode-connected transistor, has a small-signal resistance $r$ equal to $r_{e}$.


Figure P6.47
6.48 Figure P6.48 shows a particular configuration of BJT amplifiers known as "emitter follower." The bias
arrangement is not shown. Replace the BJT with its T equivalent-circuit model of Fig. 6.26(b). Show that

$$
\begin{aligned}
R_{\mathrm{in}} \equiv \frac{v_{i}}{i_{b}} & =(\beta+1)\left(r_{e}+R_{e}\right) \\
\frac{v_{o}}{v_{i}} & =\frac{R_{e}}{R_{e}+r_{e}}
\end{aligned}
$$



Figure P6.48
6.49 In the circuit shown in Fig. P6.49, the transistor has a $\beta$ of 200. What is the dc voltage at the collector? Replacing the BJT with one of the hybrid- $\pi$ models (neglecting $r_{o}$ ), draw the equivalent circuit of the amplifier. Find the input resistances $R_{i b}$ and $R_{\mathrm{in}}$ and the overall voltage gain $\left(v_{o} / v_{\text {sig }}\right)$. For an output signal of $\pm 0.4 \mathrm{~V}$, what values of $v_{\text {sig }}$ and $v_{b}$ are required?


Figure P6.49
6.50 For the circuit shown in Fig. P6.50, draw a complete small-signal equivalent circuit utilizing an appropriate T model for the BJT (use $\alpha=0.99$ ). Your circuit should show the values of all components, including the model parameters. What is the input resistance $R_{\text {in }}$ ? Calculate the overall voltage gain $\left(v_{o} / v_{\text {sig }}\right)$.


Figure P6.50
6.51 Consider the augmented hybrid- $\pi$ model shown in Fig. 6.25(a). Disregarding how biasing is to be done, what is the largest possible voltage gain available for a signal source connected directly to the base and a very-high-resistance load? Calculate the value of the maximum possible gain for $V_{A}=25 \mathrm{~V}$ and $V_{A}=125 \mathrm{~V}$.

D *6.52 Design an amplifier using the configuration of Fig. 6.30(a). The power supplies available are $\pm 5 \mathrm{~V}$. The input signal source has a resistance of $50 \Omega$, and it is required that the amplifier input resistance match this value. (Note that $R_{\mathrm{in}}=r_{e} \| R_{E} \simeq r_{e}$.) The amplifier is to have the greatest possible voltage gain and the largest possible output signal but retain small-signal linear operation (i.e., the signal component across the base-emitter junction should be limited to no more than 10 mV ). Find appropriate values for $R_{E}$ and $R_{C}$. What is the value of voltage gain realized from signal source to output?
*6.53 The transistor in the circuit shown in Fig. P6.53 is biased to operate in the active mode. Assuming that $\beta$ is very large, find the collector bias current $I_{C}$. Replace the transistor with the small-signal equivalent-circuit model of Fig. 6.26(b) (remember to replace the dc power supply with
a short circuit). Analyze the resulting amplifier equivalent circuit to show that

$$
\begin{aligned}
& \frac{v_{o 1}}{v_{i}}=\frac{R_{E}}{R_{E}+r_{e}} \\
& \frac{v_{o 2}}{v_{i}}=\frac{-\alpha R_{C}}{R_{E}+r_{e}}
\end{aligned}
$$



Figure P6.53
Find the values of these voltage gains (for $\alpha \simeq 1$ ). Now, if the terminal labeled $v_{o 1}$ is connected to ground, what does the voltage gain $v_{o 2} / v_{i}$ become?

## Section 6.3: Basic Configurations

6.54 An amplifier with an input resistance of $100 \mathrm{k} \Omega$, an open-circuit voltage gain of $100 \mathrm{~V} / \mathrm{V}$, and an output resistance of $100 \Omega$ is connected between a $20-\mathrm{k} \Omega$ signal source and a $2-\mathrm{k} \Omega$ load. Find the overall voltage gain $G_{v}$. Also find the current gain, defined as the ratio of the load current to the current drawn from the signal source.

D 6.55 Specify the parameters $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$ of an amplifier that is to be connected between a $100-\mathrm{k} \Omega$ source and a $2-\mathrm{k} \Omega$ load and is required to meet the following specifications:
(a) No more than $5 \%$ of the signal strength is lost in the connection to the amplifier input;
(b) If the load resistance changes from the nominal value of $2 \mathrm{k} \Omega$ to a low value of $1 \mathrm{k} \Omega$, the change in output voltage is limited to $5 \%$ of nominal value; and
(c) The nominal overall voltage gain is $10 \mathrm{~V} / \mathrm{V}$.
6.56 Figure P6.56 shows an alternative equivalent-circuit representation of an amplifier. If this circuit is to be
equivalent to that in Fig. 6.34(b) show that $G_{m}=A_{v o} / R_{o}$. Also convince yourself that the transconductance $G_{m}$ is defined as

$$
G_{m}=\left.\frac{i_{o}}{v_{i}}\right|_{R_{L}=0}
$$

and hence is known as the short-circuit transconductance. Now, if the amplifier is fed with a signal source ( $v_{\text {sig }}, R_{\text {sig }}$ ) and is connected to a load resistance $R_{L}$ show that the gain of the amplifier proper $A_{v}$ is given by $A_{v}=G_{m}\left(R_{o} \| R_{L}\right)$ and the overall voltage gain $G_{v}$ is given by

$$
G_{v}=\frac{R_{\mathrm{in}}}{R_{\mathrm{in}}+R_{\mathrm{sig}}} G_{m}\left(R_{o} \| R_{L}\right)
$$



Figure P6.56
6.57 An alternative equivalent circuit of an amplifier fed with a signal source ( $v_{\text {sig }}, R_{\text {sig }}$ ) and connected to a load $R_{L}$ is shown in Fig. P6.57. Here $G_{v o}$ is the open-circuit overall voltage gain,

$$
G_{v o}=\left.\frac{v_{o}}{v_{\text {sig }}}\right|_{R_{L}=\infty}
$$

and $R_{\text {out }}$ is the output resistance with $v_{\text {sig }}$ set to zero. This is different than $R_{o}$. Show that

$$
G_{v o}=\frac{R_{i}}{R_{i}+R_{\mathrm{sig}}} A_{v o}
$$

where $R_{i}=\left.R_{\text {in }}\right|_{R_{L}=\infty}$.
Also show that the overall voltage gain is

$$
G_{v}=G_{v o} \frac{R_{L}}{R_{L}+R_{\text {out }}}
$$

**6.58 Most practical amplifiers have internal feedback that make them non-unilateral. In such a case, $R_{\text {in }}$ depends on $R_{L}$. To illustrate this point we show in Fig. P6.58 the equivalent circuit of an amplifier where a feedback resistance $R_{f}$ models the internal feedback mechanism that is present in


Figure P6.57


Figure P6.58
this amplifier. It is $R_{f}$ that makes the amplifier non-unilateral. Show that

$$
\begin{aligned}
R_{\mathrm{in}} & =R_{1} \|\left[\frac{R_{f}+\left(R_{2} \| R_{L}\right)}{1+g_{m}\left(R_{2} \| R_{L}\right)}\right] \\
A_{v o} & =-g_{m} R_{2} \frac{1-1 /\left(g_{m} R_{f}\right)}{1+\left(R_{2} / R_{f}\right)} \\
R_{o} & =R_{2} \| R_{f}
\end{aligned}
$$

Evaluate $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$ for the case $R_{1}=100 \mathrm{k} \Omega, R_{f}=$ $1 \mathrm{M} \Omega, g_{m}=100 \mathrm{~mA} / \mathrm{V}, R_{2}=100 \Omega$, and $R_{L}=1 \mathrm{k} \Omega$. Which of the amplifier characteristic parameters is most affected by $R_{f}$ (that is, relative to the case with $R_{f}=\infty$ )? For $R_{\text {sig }}=100 \mathrm{k} \Omega$ determine the overall voltage gain, $G_{v}$, with and without $R_{f}$ present.
6.59 Calculate the overall voltage gain of a CS amplifier fed with a $1-\mathrm{M} \Omega$ source and connected to a $10-\mathrm{k} \Omega$ load. The MOSFET has $g_{m}=2 \mathrm{~mA} / \mathrm{V}$, and a drain resistance $R_{D}=10 \mathrm{k} \Omega$ is utilized.
6.60 A common-source amplifier utilizes a MOSFET operated at $V_{O V}=0.25 \mathrm{~V}$. The amplifier feeds a load resistance $R_{L}=15 \mathrm{k} \Omega$. The designer selects $R_{D}=2 R_{L}$. If it is required to realize an overall voltage gain $G_{v}$ of $-10 \mathrm{~V} / \mathrm{V}$ what $g_{m}$ is needed? Also specify the bias current $I_{D}$. If, to increase the output signal swing, $R_{D}$ is reduced to $R_{D}=R_{L}$, what does $G_{v}$ become?
6.61 A CS amplifier utilizes a MOSFET with $\mu_{n} C_{o x}=$ $400 \mu \mathrm{~A} / \mathrm{V}^{2}$ and $W / L=10$. It is biased at $I_{D}=320 \mu \mathrm{~A}$ and uses $R_{D}=10 \mathrm{k} \Omega$. Find $R_{\mathrm{in}}, A_{v o}$, and $R_{o}$. Also, if a load resistance of $10 \mathrm{k} \Omega$ is connected to the output, what overall voltage gain $G_{v}$ is realized? Now, if a $0.2-\mathrm{V}$ peak sine-wave signal is required at the output, what must the peak amplitude of $v_{\text {sig }}$ be?
6.62 Two identical CS amplifiers are connected in cascade. The first stage is fed with a source $v_{\text {sig }}$ having a resistance $R_{\text {sig }}=200 \mathrm{k} \Omega$. A load resistance $R_{L}=10 \mathrm{k} \Omega$ is connected to the drain of the second stage. Each MOSFET is biased at $I_{D}=0.3 \mathrm{~mA}$ and operates with $V_{O V}=0.2 \mathrm{~V}$. Each stage utilizes a drain resistance $R_{D}=10 \mathrm{k} \Omega$.
(a) Sketch the equivalent circuit of the two-stage amplifier.
(b) Calculate the overall voltage gain $G_{v}$.
6.63 A CE amplifier utilizes a BJT with $\beta=100$ biased at $I_{C}=0.5 \mathrm{~mA}$; it has a collector resistance $R_{C}=10 \mathrm{k} \Omega$. Find
$R_{\text {in }}, R_{o}$, and $A_{v o}$. If the amplifier is fed with a signal source having a resistance of $10 \mathrm{k} \Omega$, and a load resistance $R_{L}=10$ $\mathrm{k} \Omega$ is connected to the output terminal, find the resulting $A_{v}$ and $G_{v}$. If the peak voltage of the sine wave appearing between base and emitter is to be limited to 5 mV , what $\hat{\mathrm{s}}_{\text {sig }}$ is allowed, and what output voltage signal appears across the load?

D *6.64 In this problem we investigate the effect of the inevitable variability of $\beta$ on the realized gain of the CE amplifier. For this purpose, use the overall gain expression in Eq. (6.114).

$$
\left|G_{v}\right|=\frac{R_{L}^{\prime}}{\left(R_{\mathrm{sig}} / \beta\right)+\left(1 / g_{m}\right)}
$$

where $R_{L}^{\prime}=R_{L} \| R_{C}$.
Consider the case $R_{L}^{\prime}=10 \mathrm{k} \Omega$ and $R_{\text {sig }}=10 \mathrm{k} \Omega$, and let the BJT be biased at $I_{C}=1 \mathrm{~mA}$. The BJT has a nominal $\beta$ of 100 .
(a) What is the nominal value of $\left|G_{v}\right|$ ?
(b) If $\beta$ can be anywhere between 50 and 150 , what is the corresponding range of $\left|G_{v}\right|$ ?
(c) If in a particular design, it is required to maintain $\left|G_{v}\right|$ within $\pm 20 \%$ of its nominal value, what is the maximum allowable range of $\beta$ ?
(d) If it is not possible to restrict $\beta$ to the range found in (c), and the designer has to contend with $\beta$ in the range 50 to 150 , what value of bias current $I_{C}$ would result in $\left|G_{v}\right|$ falling in a range of $\pm 20 \%$ of a new nominal value? What is the nominal value of $\left|G_{v}\right|$ in this case?
6.65 Two identical CE amplifiers are connected in cascade. The first stage is fed with a source $v_{\text {sig }}$ having a resistance $R_{\text {sig }}=10 \mathrm{k} \Omega$. A load resistance $R_{L}=10 \mathrm{k} \Omega$ is connected to the collector of the second stage. Each BJT is biased at $I_{C}=0.25 \mathrm{~mA}$ and has $\beta=100$. Each stage utilizes a collector resistance $R_{C}=10 \mathrm{k} \Omega$.
(a) Sketch the equivalent circuit of the two-stage amplifier.
(b) Find the overall voltage gain, $v_{o 2} / v_{\text {sig }}$.
6.66 A MOSFET connected in the CS configuration has a transconductance $g_{m}=5 \mathrm{~mA} / \mathrm{V}$. When a resistance $R_{s}$ is connected in the source lead, the effective transconductance is reduced to $2 \mathrm{~mA} / \mathrm{V}$. What do you estimate the value of $R_{s}$ to be?
6.67 The overall voltage gain of a CS amplifier with a resistance $R_{s}=0.5 \mathrm{k} \Omega$ in the source lead was measured and
found to be $-10 \mathrm{~V} / \mathrm{V}$. When $R_{s}$ was shorted, but the circuit operation remained linear, the gain doubled. What must $g_{m}$ be? What value of $R_{s}$ is needed to obtain an overall voltage gain of $-16 \mathrm{~V} / \mathrm{V}$ ?
6.68 A CE amplifier utilizes a BJT with $\beta=100$ biased at $I_{C}=0.5 \mathrm{~mA}$ and has a collector resistance $R_{C}=12 \mathrm{k} \Omega$ and a resistance $R_{e}=250 \Omega$ connected in the emitter. Find $R_{\mathrm{in}}$, $A_{v o}$, and $R_{o}$. If the amplifier is fed with a signal source having a resistance of $10 \mathrm{k} \Omega$, and a load resistance $R_{L}=12 \mathrm{k} \Omega$ is connected to the output terminal, find the resulting $A_{v}$ and $G_{v}$. If the peak voltage of the sine wave appearing between base and emitter is to be limited to 5 mV , what $\hat{\mathrm{v}}_{\text {sig }}$ is allowed, and what output voltage signal appears across the load?

D 6.69 Design a CE amplifier with a resistance $R_{e}$ in the emitter to meet the following specifications:
(i) Input resistance $R_{\text {in }}=15 \mathrm{k} \Omega$.
(ii) When fed from a signal source with a peak amplitude of 0.15 V and a source resistance of $30 \mathrm{k} \Omega$, the peak amplitude of $v_{\pi}$ is 5 mV .

Specify $R_{e}$ and the bias current $I_{C}$. The BJT has $\beta=74$. If the total resistance in the collector is $6 \mathrm{k} \Omega$, find the overall voltage gain $G_{v}$ and the peak amplitude of the output signal $v_{o}$.
sim D 6.70 Inclusion of an emitter resistance $R_{e}$ reduces the variability of the gain $G_{v}$ due to the inevitable wide variance in the value of $\beta$. Consider a CE amplifier operating between a signal source with $R_{\text {sig }}=10 \mathrm{k} \Omega$ and a total collector resistance $R_{C} \| R_{L}$ of $10 \mathrm{k} \Omega$. The BJT is biased at $I_{C}=1 \mathrm{~mA}$ and its $\beta$ is specified to be nominally 100 but can lie in the range of 50 to 150 . First determine the nominal value and the range of $\left|G_{v}\right|$ without resistance $R_{e}$. Then select a value for $R_{e}$ that will ensure that $\left|G_{v}\right|$ be within $\pm 20 \%$ of its new nominal value. Specify the value of $R_{e}$, the new nominal value of $\left|G_{v}\right|$, and the expected range of $\left|G_{v}\right|$.
6.71 A CG amplifier using an NMOS transistor for which $g_{m}=2 \mathrm{~mA} / \mathrm{V}$ has a $5-\mathrm{k} \Omega$ drain resistance $R_{D}$ and a $5-\mathrm{k} \Omega$ load resistance $R_{L}$. The amplifier is driven by a voltage source having a $750-\Omega$ resistance. What is the input resistance of the amplifier? What is the overall voltage gain $G_{v}$ ? By what factor must the bias current $I_{D}$ of the MOSFET be changed so that $R_{\mathrm{in}}$ matches $R_{\text {sig }}$ ?
6.72 For the circuit in Fig. P6.72, let $R_{\text {sig }} \gg r_{e}$ and $\alpha \simeq 1$. Find $v_{o}$.


Figure P6.72
D 6.73 A CB amplifier is operating with $R_{L}=10 \mathrm{k} \Omega$, $R_{C}=10 \mathrm{k} \Omega$, and $R_{\text {sig }}=50 \Omega$. At what current $I_{C}$ should the transistor be biased for the input resistance $R_{\text {in }}$ to equal that of the signal source? What is the resulting overall voltage gain? Assume $\alpha \simeq 1$.
6.74 A CB amplifier is biased at $I_{E}=0.2 \mathrm{~mA}$ with $R_{C}=$ $R_{L}=10 \mathrm{k} \Omega$ and is driven by a signal source with $R_{\text {sig }}=$ $0.5 \mathrm{k} \Omega$. Find the overall voltage gain $G_{v}$. If the maximum signal amplitude of the voltage between base and emitter is limited to 10 mV , what are the corresponding amplitudes of $v_{\text {sig }}$ and $v_{o}$ ? Assume $\alpha \simeq 1$.

D 6.75 A source follower is required to deliver a $0.5-\mathrm{V}$ peak sinusoid to a $2-\mathrm{k} \Omega$ load. If the peak amplitude of $v_{g s}$ is to be limited to 50 mV , and the MOSFET transconductance parameter $k_{n}$ is $5 \mathrm{~mA} / \mathrm{V}^{2}$, what is the lowest value of $I_{D}$ at which the MOSFET can be biased? At this bias current, what are the maximum and minimum currents that the MOSFET will be conducting (at the positive and negative peaks of the output sine wave)? What must the peak amplitude of $v_{\text {sig }}$ be?
6.76 A source follower is required to connect a high-resistance source to a load whose resistance is nominally $2 \mathrm{k} \Omega$ but can be as low as $1.5 \mathrm{k} \Omega$ and as high as $5 \mathrm{k} \Omega$. What is the maximum output resistance that the source follower must have if the output voltage is to remain within $\pm 10 \%$ of nominal value? If the MOSFET has $k_{n}=2.5 \mathrm{~mA} / \mathrm{V}^{2}$, at what current $I_{D}$ must it be biased? At what overdrive voltage is the MOSFET operating?

D 6.77 An emitter follower is required to deliver a $0.5-\mathrm{V}$ peak sinusoid to a $2-\mathrm{k} \Omega$ load. If the peak amplitude of $v_{b e}$
is to be limited to 5 mV , what is the lowest value of $I_{E}$ at which the BJT can be biased? At this bias current, what are the maximum and minimum currents that the BJT will be conducting (at the positive and negative peaks of the output sine wave)? If the resistance of the signal source is $200 \mathrm{k} \Omega$, what value of $G_{v}$ is obtained? Thus determine the required amplitude of $v_{\text {sig }}$. Assume $\beta=100$.
6.78 An emitter follower with a BJT biased at $I_{C}=2 \mathrm{~mA}$ and having $\beta=100$ is connected between a source with $R_{\text {sig }}=10 \mathrm{k} \Omega$ and a load $R_{L}=0.5 \mathrm{k} \Omega$.
(a) Find $R_{\text {in }}, v_{b} / v_{\text {sig }}$, and $v_{o} / v_{\text {sig }}$.
(b) If the signal amplitude across the base-emitter junction is to be limited to 10 mV , what is the corresponding amplitude of $v_{\text {sig }}$ and $v_{o}$ ?
(c) Find the open-circuit voltage gain $G_{v o}$ and the output resistance $R_{\text {out }}$. Use these values first to verify the value of $G_{v}$ obtained in (a), then to find the value of $G_{v}$ obtained with $R_{L}$ reduced to $250 \Omega$.
6.79 An emitter follower, when driven from a $5-\mathrm{k} \Omega$ source, was found to have an output resistance $R_{\text {out }}$ of $150 \Omega$. The output resistance increased to $250 \Omega$ when the source resistance was increased to $10 \mathrm{k} \Omega$. Find the overall voltage gain when the follower is driven by a $10-\mathrm{k} \Omega$ source and loaded by a $1-\mathrm{k} \Omega$ resistor.
6.80 For the general amplifier circuit shown in Fig. P6.80 neglect the Early effect.
(a) Find expressions for $v_{c} / v_{\text {sig }}$ and $v_{e} / v_{\text {sig }}$.
(b) If $v_{\text {sig }}$ is disconnected from node X , node X is grounded, and node Y is disconnected from ground and connected to $v_{\text {sig }}$, find the new expression for $v_{c} / v_{\text {sig }}$.


Figure P6.80
6.81 When the Early effect is neglected, the overall voltage gain of a CE amplifier with a collector resistance $R_{C}=$ $10 \mathrm{k} \Omega$ is calculated to be $-100 \mathrm{~V} / \mathrm{V}$. If the BJT is biased at $I_{C}=1 \mathrm{~mA}$ and the Early voltage is 100 V , provide a better estimate of the voltage gain $G_{v}$.

D 6.82 In this problem, we investigate the effect of changing the bias current $I_{C}$ on the overall voltage gain $G_{v}$ of a CE amplifier. Consider the situation of a CE amplifier operating with a signal source having $R_{\text {sig }}=10 \mathrm{k} \Omega$ and having $R_{C} \| R_{L}=10 \mathrm{k} \Omega$. The BJT is specified to have $\beta=100$ and $V_{A}=25 \mathrm{~V}$. Use Eq. (6.114) (with $r_{o}$ included in parallel with $R_{C}$ and $R_{L}$ in the numerator) to find $\left|G_{v}\right|$ at $I_{C}=0.1 \mathrm{~mA}, 0.2 \mathrm{~mA}, 0.5 \mathrm{~mA}, 1.0 \mathrm{~mA}$, and 1.25 mA . Observe the effect of $r_{o}$ on limiting $\left|G_{v}\right|$ as $I_{C}$ is increased. Find the value of $I_{C}$ that results in $\left|G_{v}\right|=50 \mathrm{~V} / \mathrm{V}$.
*6.83 Show that when $r_{o}$ is taken into account, the voltage gain of the source follower becomes

$$
G_{v} \equiv \frac{v_{o}}{v_{\text {sig }}}=\frac{R_{L} \| r_{o}}{\left(R_{L} \| r_{o}\right)+\frac{1}{g_{m}}}
$$

Now, with $R_{L}$ removed, the voltage gain is carefully measured and found to be 0.98 . Then, when $R_{L}$ is connected and its value is varied, it is found that the gain is halved at $R_{L}=500 \Omega$. If the amplifier remained linear throughout this measurement, what must the values of $g_{m}$ and $r_{o}$ be?

## Section 6.4: Biasing

D 6.84 Consider the classical biasing scheme shown in Fig. 6.48(c), using a 9-V supply. For the MOSFET, $V_{t}=1 \mathrm{~V}$, $\lambda=0$, and $k_{n}=2 \mathrm{~mA} / \mathrm{V}^{2}$. Arrange that the drain current is 1 mA , with about one-third of the supply voltage across each of $R_{S}$ and $R_{D}$. Use $22 \mathrm{M} \Omega$ for the larger of $R_{G 1}$ and $R_{G 2}$. What are the values of $R_{G 1}, R_{G 2}, R_{S}$, and $R_{D}$ that you have chosen? Specify them to two significant digits. For your design, how far is the drain voltage from the edge of saturation?

D 6.85 Using the circuit topology displayed in Fig. 6.48(e), arrange to bias the NMOS transistor at $I_{D}=$ 0.5 mA with $V_{D}$ midway between cutoff and the beginning of triode operation. The available supplies are $\pm 5 \mathrm{~V}$. For the NMOS transistor, $V_{t}=1.0 \mathrm{~V}, \lambda=0$, and $k_{n}=1 \mathrm{~mA} / \mathrm{V}^{2}$. Use a gate-bias resistor of $10 \mathrm{M} \Omega$. Specify $R_{S}$ and $R_{D}$ to two significant digits.
6.86 An NMOS transistor is connected in the bias circuit of Fig. 6.48(c), with $V_{G}=5 \mathrm{~V}$ and $R_{S}=3 \mathrm{k} \Omega$. The transistor
has $V_{t}=1 \mathrm{~V}$ and $k_{n}=2 \mathrm{~mA} / \mathrm{V}^{2}$. What bias current results? If a transistor for which $k_{n}$ is $50 \%$ higher is used, what is the resulting percentage increase in $I_{D}$ ?

D 6.87 Design the circuit of Fig. 6.48(e) for a MOSFET having $V_{t}=1 \mathrm{~V}$ and $k_{n}=4 \mathrm{~mA} / \mathrm{V}^{2}$. Let $V_{D D}=V_{S S}=5 \mathrm{~V}$. Design for a dc bias current of 0.5 mA and for the largest possible voltage gain (and thus the largest possible $R_{D}$ ) consistent with allowing a 2-V peak-to-peak voltage swing at the drain. Assume that the signal voltage on the source terminal of the FET is zero.
sim 6.88 The bias circuit of Fig. 6.48(c) is used in a design with $V_{G}=5 \mathrm{~V}$ and $R_{S}=2 \mathrm{k} \Omega$. For a MOSFET with $k_{n}=2 \mathrm{~mA} / \mathrm{V}^{2}$, the source voltage was measured and found to be 2 V . What must $V_{t}$ be for this device? If a device for which $V_{t}$ is 0.5 V less is used, what does $V_{S}$ become? What bias current results?
sim D 6.89 Design the circuit in Fig. P6.89 so that the transistor operates in saturation with $V_{D}$ biased 1 V from the edge of the triode region, with $I_{D}=1 \mathrm{~mA}$ and $V_{D}=3 \mathrm{~V}$, for each of the following two devices (use a $10-\mu \mathrm{A}$ current in the voltage divider):
(a) $\left|V_{t}\right|=1 \mathrm{~V}$ and $k_{p}^{\prime} W / L=0.5 \mathrm{~mA} / \mathrm{V}^{2}$
(b) $\left|V_{t}\right|=2 \mathrm{~V}$ and $k_{p}^{\prime} W / L=1.25 \mathrm{~mA} / \mathrm{V}^{2}$

For each case, specify the values of $V_{G}, V_{D}, V_{S}, R_{1}, R_{2}, R_{S}$, and $R_{D}$.


Figure P6.89
D **6.90 A very useful way to characterize the stability of the bias current $I_{D}$ is to evaluate the sensitivity of $I_{D}$ relative
to a particular transistor parameter whose variability might be large. The sensitivity of $I_{D}$ relative to the MOSFET parameter $K \equiv \frac{1}{2} k^{\prime}(W / L)$ is defined as

$$
S_{K}^{I_{D}} \equiv \frac{\partial I_{D} / I_{D}}{\partial K / K}=\frac{\partial I_{D}}{\partial K} \frac{K}{I_{D}}
$$

and its value, when multiplied by the variability (or tolerance) of $K$, provides the corresponding expected variability of $I_{D}$,

$$
\frac{\Delta I_{D}}{I_{D}}=S_{K}^{I_{D}}\left(\frac{\Delta K}{K}\right)
$$

The purpose of this problem is to investigate the use of the sensitivity function in the design of the bias circuit of Fig. 6.48(e).
(a) Show that for $V_{t}$ constant,

$$
S_{K}^{I_{D}}=1 /\left(1+2 \sqrt{K I_{D}} R_{S}\right)
$$

(b) For a MOSFET having $K=100 \mu \mathrm{~A} / \mathrm{V}^{2}$ with a variability of $\pm 10 \%$ and $V_{t}=1 \mathrm{~V}$, find the value of $R_{S}$ that would result in $I_{D}=100 \mu \mathrm{~A}$ with a variability of $\pm 1 \%$. Also, find $V_{G S}$ and the required value of $V_{S S}$.
(c) If the available supply $V_{S S}=5 \mathrm{~V}$, find the value of $R_{S}$ for $I_{D}=100 \mu \mathrm{~A}$. Evaluate the sensitivity function, and give the expected variability of $I_{D}$ in this case.

D **6.91 The variability $\left(\triangle I_{D} / I_{D}\right)$ in the bias current $I_{D}$ due to the variability $\left(\Delta V_{t} / V_{t}\right)$ in the threshold voltage $V_{t}$ can be evaluated from

$$
\frac{\Delta I_{D}}{I_{D}}=S_{V_{t}}^{I_{D}}\left(\frac{\Delta V_{t}}{V_{t}}\right)
$$

where $S_{V_{t}}^{L_{D}}$, the sensitivity of $I_{D}$ relative to $V_{t}$, is defined as

$$
S_{V_{t}}^{I_{D}}=\frac{\partial I_{D}}{\partial V_{t}} \frac{V_{t}}{I_{D}}
$$

(a) For the case of a MOSFET biased with a fixed $V_{G S}$, show that

$$
S_{V_{t}}^{I_{D}}=-\frac{2 V_{t}}{V_{O V}}
$$

and find the variability in $I_{D}$ for $V_{t}=0.5 \mathrm{~V}$ and $\Delta V_{t} / V_{t}=$ $\pm 5 \%$. Let the MOSFET be biased at $V_{O V}=0.25 \mathrm{~V}$.
(b) For the case of a MOSFET biased with a fixed gate voltage $V_{G}$ and a resistance $R_{S}$ included in the source lead, show that

$$
S_{V_{t}}^{I_{D}}=-\frac{2 V_{t}}{V_{O V}+2 I_{D} R_{S}}
$$

For the same parameters given in (a), find the required value of $\left(I_{D} R_{S}\right)$ and $V_{G}$ to limit $\Delta I_{D} / I_{D}$ to $\pm 5 \%$. What value of $R_{S}$ is needed if $I_{D}$ is $100 \mu \mathrm{~A}$ ?

D 6.92 Using the feedback bias arrangement shown in Fig. 6.50 with a $5-\mathrm{V}$ supply and an NMOS device for which $V_{t}=1 \mathrm{~V}$ and $k_{n}=10 \mathrm{~mA} / \mathrm{V}^{2}$, find $R_{D}$ to establish a drain current of 0.2 mA .
sim 6.93 In the circuit of Fig. 6.50, let $R_{G}=10 \mathrm{M} \Omega$, $R_{D}=10 \mathrm{k} \Omega$, and $V_{D D}=10 \mathrm{~V}$. For each of the following two transistors, find the voltages $V_{D}$ and $V_{G}$.
(a) $V_{t}=1 \mathrm{~V}$ and $k_{n}=0.5 \mathrm{~mA} / \mathrm{V}^{2}$
(b) $V_{t}=2 \mathrm{~V}$ and $k_{n}=1.25 \mathrm{~mA} / \mathrm{V}^{2}$

D 6.94 Figure P6.94 shows a variation of the feedbackbias circuit of Fig. 6.50. Using a $5-\mathrm{V}$ supply with an NMOS transistor for which $V_{t}=0.8 \mathrm{~V}, k_{n}=8 \mathrm{~mA} / \mathrm{V}^{2}$, and $\lambda=0$, provide a design that biases the transistor at $I_{D}=1 \mathrm{~mA}$, with $V_{D S}$ large enough to allow saturation operation for a $2-\mathrm{V}$ negative signal swing at the drain. Use $22 \mathrm{M} \Omega$ as the largest resistor in the feedback-bias network. What values of $R_{D}, R_{G 1}$, and $R_{G 2}$ have you chosen? Specify all resistors to two significant digits.


## Figure P6.94

D 6.95 For the circuit in Fig. 6.51(a), neglect the base current $I_{B}$ in comparison with the current in the voltage
divider. It is required to bias the transistor at $I_{C}=1 \mathrm{~mA}$, which requires selecting $R_{B 1}$ and $R_{B 2}$ so that $V_{B E}=0.710 \mathrm{~V}$. If $V_{C C}=3 \mathrm{~V}$, what must the ratio $R_{B 1} / R_{B 2}$ be? Now, if $R_{B 1}$ and $R_{B 2}$ are $1 \%$ resistors, that is, each can be in the range of 0.99 to 1.01 of its nominal value, what is the range obtained for $V_{B E}$ ? What is the corresponding range of $I_{C}$ ? If $R_{C}=2 \mathrm{k} \Omega$, what is the range obtained for $V_{C E}$ ? Comment on the efficacy of this biasing arrangement.

D 6.96 It is required to bias the transistor in the circuit of Fig. 6.51(b) at $I_{C}=1 \mathrm{~mA}$. The transistor $\beta$ is specified to be nominally 100 , but it can fall in the range of 50 to 150 . For $V_{C C}=+3 \mathrm{~V}$ and $R_{C}=2 \mathrm{k} \Omega$, find the required value of $R_{B}$ to achieve $I_{C}=1 \mathrm{~mA}$ for the "nominal" transistor. What is the expected range for $I_{C}$ and $V_{C E}$ ? Comment on the efficacy of this bias design.

D 6.97 Consider the single-supply bias network shown in Fig. 6.52(a). Provide a design using a $9-\mathrm{V}$ supply in which the supply voltage is equally split between $R_{C}, V_{C E}$, and $R_{E}$ with a collector current of 0.6 mA . The transistor $\beta$ is specified to have a minimum value of 90 . Use a voltage-divider current of $I_{E} / 10$, or slightly higher. Since a reasonable design should operate for the best transistors for which $\beta$ is very high, do your initial design with $\beta=\infty$. Then choose suitable 5\% resistors (see Appendix J), making the choice in a way that will result in a $V_{B B}$ that is slightly higher than the ideal value. Specify the values you have chosen for $R_{E}, R_{C}, R_{1}$, and $R_{2}$. Now, find $V_{B}, V_{E}, V_{C}$, and $I_{C}$ for your final design using $\beta=90$.

D *6.98 It is required to design the bias circuit of Fig. 6.52 for a BJT whose nominal $\beta=100$.
(a) Find the largest ratio $\left(R_{B} / R_{E}\right)$ that will guarantee $I_{E}$ remains within $\pm 5 \%$ of its nominal value for $\beta$ as low as 50 and as high as 150 .
(b) If the resistance ratio found in (a) is used, find an expression for the voltage $V_{B B} \equiv V_{C C} R_{2} /\left(R_{1}+R_{2}\right)$ that will result in a voltage drop of $V_{C C} / 3$ across $R_{E}$.
(c) For $V_{C C}=5 \mathrm{~V}$, find the required values of $R_{1}, R_{2}$, and $R_{E}$ to obtain $I_{E}=0.5 \mathrm{~mA}$ and to satisfy the requirement for stability of $I_{E}$ in (a).
(d) Find $R_{C}$ so that $V_{C E}=1.0 \mathrm{~V}$ for $\beta$ equal to its nominal value.

Check your design by evaluating the resulting range of $I_{E}$.

D *6.99 Utilizing $\pm 3-\mathrm{V}$ power supplies, it is required to design a version of the circuit in Fig. 6.53 in which the signal will be coupled to the emitter and thus $R_{B}$ can be set to zero. Find values for $R_{E}$ and $R_{C}$ so that a dc emitter current of 0.4 mA is obtained and so that the gain is maximized while allowing $\pm 1 \mathrm{~V}$ of signal swing at the collector. If temperature increases from the nominal value of $25^{\circ} \mathrm{C}$ to $125^{\circ} \mathrm{C}$, estimate the percentage change in collector bias current. In addition to the $-2 \mathrm{mV} /{ }^{\circ} \mathrm{C}$ change in $V_{B E}$, assume that the transistor $\beta$ changes over this temperature range from 50 to 150 .

D *6.100 Consider the two-supply bias arrangement shown in Fig. 6.53 using $\pm 5-\mathrm{V}$ supplies. It is required to design the circuit so that $I_{C}=0.5 \mathrm{~mA}$ and $V_{C}$ is placed 2 V above $V_{E}$.
(a) For $\beta=\infty$, what values of $R_{E}$ and $R_{C}$ are required?
(b) If the BJT is specified to have a minimum $\beta$ of 50 , find the largest value for $R_{B}$ consistent with the need to limit the voltage drop across it to one-tenth the voltage drop across $R_{E}$.
(c) What standard 5\% resistor values (see Appendix J) would you use for $R_{B}, R_{E}$, and $R_{C}$ ? In making your selection, use somewhat lower values in order to compensate for the low- $\beta$ effects.
(d) For the values you selected in (c), find $I_{C}, V_{B}, V_{E}$, and $V_{C}$ for $\beta=\infty$ and for $\beta=50$.
sim D 6.101 Using a 3-V power supply, design a version of the circuit of Fig. 6.54 to provide a dc emitter current of 0.5 mA and to allow a $\pm 1-\mathrm{V}$ signal swing at the collector. The BJT has a nominal $\beta=100$. Use standard 5\% resistor values (see Appendix J). If the actual BJT used has $\beta=50$, what emitter current is obtained? Also, what is the allowable signal swing at the collector? Repeat for $\beta=150$.

D *6.102 (a) Using a 3-V power supply, design the feedback bias circuit of Fig. 6.54 to provide $I_{C}=1 \mathrm{~mA}$ and $V_{C}=V_{C C} / 2$ for $\beta=100$.
(b) Select standard 5\% resistor values, and reevaluate $V_{C}$ and $I_{C}$ for $\beta=100$.
(c) Find $V_{C}$ and $I_{C}$ for $\beta=\infty$.
(d) To improve the situation that obtains when high $-\beta$ transistors are used, we have to arrange for an additional current to flow through $R_{B}$. This can be achieved by connecting a resistor between base and emitter, as shown in Fig. P6.102.

Design this circuit for $\beta=100$. Use a current through $R_{B 2}$ equal to the base current. Now, what values of $V_{C}$ and $I_{C}$ result with $\beta=\infty$ ?


Figure P6. 102
D 6.103 A circuit that can provide a very large voltage gain for a high-resistance load is shown in Fig. P6.103. Find the values of $I$ and $R_{B}$ to bias the BJT at $I_{C}=1 \mathrm{~mA}$ and $V_{C}=$ 1.5 V. Let $\beta=100$.


Figure P6. 103
6.104 The circuit in Fig. P6.104 provides a constant current $I_{O}$ as long as the circuit to which the collector is connected maintains the BJT in the active mode. Show that

$$
I_{O}=\alpha \frac{V_{C C}\left[R_{2} /\left(R_{1}+R_{2}\right)\right]-V_{B E}}{R_{E}+\left(R_{1} \| R_{2}\right) /(\beta+1)}
$$



Figure P6. 104
sim D *6.105 For the circuit in Fig. P6.105, assuming all transistors to be identical with $\beta$ infinite, derive an expression for the output current $I_{o}$, and show that by selecting

$$
R_{1}=R_{2}
$$

and keeping the current in each junction the same, the current $I_{O}$ will be

$$
I_{O}=\frac{V_{C C}}{2 R_{E}}
$$



Figure P6. 105
which is independent of $V_{B E}$. What must the relationship of $R_{E}$ to $R_{1}$ and $R_{2}$ be? For $V_{C C}=10 \mathrm{~V}$ and $V_{B E}=0.7 \mathrm{~V}$, design
the circuit to obtain an output current of 0.5 mA . What is the lowest voltage that can be applied to the collector of $Q_{3}$ ?

D 6.106 For the circuit in Fig. P6. 106 find the value of $R$ that will result in $I_{O} \simeq 0.5 \mathrm{~mA}$. What is the largest voltage that can be applied to the collector? Assume $\left|V_{B E}\right|=0.7 \mathrm{~V}$.


Figure P6.106

## Section 6.5: Discrete-Circuit Amplifiers

6.107 Calculate the overall voltage gain $G_{v}$ of a common-source amplifier for which $g_{m}=3 \mathrm{~mA} / \mathrm{V}, r_{o}=$ $100 \mathrm{k} \Omega, R_{D}=10 \mathrm{k} \Omega$, and $R_{G}=10 \mathrm{M} \Omega$. The amplifier is fed from a signal source with a Thévenin resistance of 1 $\mathrm{M} \Omega$, and the amplifier output is coupled to a load resistance of $20 \mathrm{k} \Omega$.

SIM 6.108 The NMOS transistor in the CS amplifier shown in Fig. P6. 108 has $V_{t}=0.7 \mathrm{~V}$ and $V_{A}=50 \mathrm{~V}$.
(a) Neglecting the Early effect, verify that the MOSFET is operating in saturation with $I_{D}=0.5 \mathrm{~mA}$ and $V_{O V}=$ 0.3 V . What must the MOSFET's $k_{n}$ be? What is the dc voltage at the drain?
(b) Find $R_{\text {in }}$ and $G_{v}$.
(c) If $v_{\text {sig }}$ is a sinusoid with a peak amplitude $\hat{v}_{\text {sig }}$, find the maximum allowable value of $\hat{v}_{\text {sig }}$ for which the transistor remains in saturation. What is the corresponding amplitude of the output voltage?
(d) What is the value of resistance $R_{s}$ that needs to be inserted in series with capacitor $C_{S}$ in order to allow us to double the input signal $\hat{v}_{\text {sig }}$ ? What output voltage now results?


Figure P6.108
sim $\mathbf{D}$ *6.109 The PMOS transistor in the CS amplifier of Fig. P6.109 has $V_{t p}=-0.7 \mathrm{~V}$ and a very large $\left|V_{A}\right|$.


Figure P6. 109
(a) Select a value for $R_{S}$ to bias the transistor at $I_{D}=$ 0.3 mA and $\left|V_{O V}\right|=0.3 \mathrm{~V}$. Assume $v_{\text {sig }}$ to have a zero dc component.
(b) Select a value for $R_{D}$ that results in $G_{v}=-10 \mathrm{~V} / \mathrm{V}$.
(c) Find the largest sinusoid $\hat{v}_{\text {sig }}$ that the amplifier can handle while remaining in the saturation region. What is the corresponding signal at the output?
(d) If to obtain reasonably linear operation, $\hat{v}_{\text {sig }}$ is limited to 50 mV , what value can $R_{D}$ be increased to while
maintaining saturation-region operation? What is the new value of $G_{v}$ ?
6.110 Figure P6.110 shows a scheme for coupling and amplifying a high-frequency pulse signal. The circuit utilizes two MOSFETs whose bias details are not shown and a $50-\Omega$ coaxial cable. Transistor $Q_{1}$ operates as a CS amplifier and $Q_{2}$ as a CG amplifier. For proper operation, transistor $Q_{2}$ is required to present a $50-\Omega$ resistance to the cable. This situation is known as "proper termination" of the cable and


Figure P6. 110
ensures that there will be no signal reflection coming back on the cable. When the cable is properly terminated, its input resistance is $50 \Omega$. What must $g_{m 2}$ be? If $Q_{1}$ is biased at the same point as $Q_{2}$, what is the amplitude of the current pulses in the drain of $Q_{1}$ ? What is the amplitude of the voltage pulses at the drain of $Q_{1}$ ? What value of $R_{D}$ is required to provide 1-V pulses at the drain of $Q_{2}$ ?

## *6.111

(a) The NMOS transistor in the source-follower circuit of Fig. P6.111(a) has $g_{m}=10 \mathrm{~mA} / \mathrm{V}$ and a large $r_{o}$. Find the open-circuit voltage gain and the output resistance.
(b) The NMOS transistor in the common-gate amplifier of Fig. P6.111(b) has $g_{m}=10 \mathrm{~mA} / \mathrm{V}$ and a large $r_{o}$. Find the input resistance and the voltage gain.

(c) If the output of the source follower in (a) is connected to the input of the common-gate amplifier in (b), use the results of (a) and (b) to obtain the overall voltage gain $v_{o} / v_{i}$.

D *6.112 The MOSFET in the circuit of Fig. P6.112 has $V_{t}=0.8 \mathrm{~V}, k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{A}=40 \mathrm{~V}$.
(a) Find the values of $R_{S}, R_{D}$, and $R_{G}$ so that $I_{D}=$ 0.4 mA , the largest possible value for $R_{D}$ is used while a maximum signal swing at the drain of $\pm 0.8 \mathrm{~V}$ is possible, and the input resistance at the gate is $10 \mathrm{M} \Omega$. Neglect the Early effect.
(b) Find the values of $g_{m}$ and $r_{o}$ at the bias point.
(c) If terminal Z is grounded, terminal X is connected to a signal source having a resistance of $1 \mathrm{M} \Omega$, and terminal Y is connected to a load resistance of $10 \mathrm{k} \Omega$, find the voltage gain from signal source to load.
(d) If terminal Y is grounded, find the voltage gain from X to Z with Z open-circuited. What is the output resistance of the source follower?
(e) If terminal X is grounded and terminal Z is connected to a current source delivering a signal current of $50 \mu \mathrm{~A}$ and having a resistance of $100 \mathrm{k} \Omega$, find the voltage signal that can be measured at Y. For simplicity, neglect the effect of $r_{o}$.


Figure P6. 112
D **6.113 The MOSFET in the amplifier circuit of Fig. P6.113 has $V_{t}=0.6 \mathrm{~V}$ and $k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}$. We shall
assume that $V_{A}$ is sufficiently large so that we can ignore the Early effect. The input signal $v_{\text {sig }}$ has a zero average.
(a) It is required to bias the transistor to operate at an overdrive voltage $V_{O V}=0.2 \mathrm{~V}$. What must the dc voltage at the drain be? Calculate the dc drain current $I_{D}$. What value must $R_{D}$ have?
(b) Calculate the value of $g_{m}$ at the bias point.
(c) Use the small-signal equivalent circuit of the amplifier to show that

$$
\frac{v_{o}}{v_{\text {sig }}}=\frac{1+\left(R_{2} / R_{1}\right)}{1+\frac{\left(1+R_{2} / R_{1}\right)}{g_{m} R_{D}^{\prime}}}
$$

and

$$
R_{\mathrm{in}}=\frac{1}{g_{m}}\left(1+g_{m} R_{D}^{\prime} \frac{R_{1}}{R_{1}+R_{2}}\right)
$$

where

$$
R_{D}^{\prime}=R_{D} \|\left(R_{1}+R_{2}\right)
$$

(d) Evaluate $v_{o} / v_{\text {sig }}$ and $R_{\text {in }}$.


Figure P6.113
P.S. This feedback amplifier circuit and the gain formula should remind you of an op amp connected in the noninverting configuration. We shall study feedback formally in Chapter 10.

D **6.114 The MOSFET in the amplifier circuit of Fig. P6.114 has $V_{t}=0.6 \mathrm{~V}, k_{n}=5 \mathrm{~mA} / \mathrm{V}^{2}$, and $V_{A}=60 \mathrm{~V}$. The signal $v_{\text {sig }}$ has a zero average.
(a) It is required to bias the transistor to operate at an overdrive voltage $V_{O V}=0.2 \mathrm{~V}$. What must the dc voltage at the drain be? Calculate the dc drain current $I_{D}$ taking into account $V_{A}$. Now, what value must the drain resistance $R_{D}$ have?
(b) Calculate the values of $g_{m}$ and $r_{o}$ at the bias point established in (a).
(c) Using the small-signal equivalent circuit of the amplifier, show that the voltage gain is given by

$$
\frac{v_{o}}{v_{\text {sig }}}=-\frac{R_{2} / R_{1}}{1+\frac{1+R_{2} / R_{1}}{g_{m}\left(R_{D}\left\|r_{o}\right\| R_{2}\right)\left(1-1 / g_{m} R_{2}\right)}}
$$

and find the value of the gain.


Figure P6.114
P.S. This feedback amplifier and the gain expression should remind you of an op amp utilized in the inverting configuration. We shall study feedback formally in Chapter 10.
6.115 For the common-emitter amplifier shown in Fig. P6.115, let $V_{C C}=15 \mathrm{~V}, R_{1}=27 \mathrm{k} \Omega, R_{2}=15 \mathrm{k} \Omega$, $R_{E}=2.4 \mathrm{k} \Omega$, and $R_{C}=3.9 \mathrm{k} \Omega$. The transistor has $\beta=100$. Calculate the dc bias current $I_{C}$. If the amplifier operates between a source for which $R_{\text {sig }}=2 \mathrm{k} \Omega$ and a load of $2 \mathrm{k} \Omega$, replace the transistor with its hybrid- $\pi$ model, and find the values of $R_{\text {in }}$, and the overall voltage gain $v_{o} / v_{\text {sig }}$.


Figure P6. 115
D 6.116 Using the topology of Fig. P6.115, design an amplifier to operate between a $2-\mathrm{k} \Omega$ source and a $2-\mathrm{k} \Omega$ load with a gain $v_{o} / v_{\text {sig }}$ of $-40 \mathrm{~V} / \mathrm{V}$. The power supply available is 15 V . Use an emitter current of approximately 2 mA and a current of about one-tenth of that in the voltage divider that feeds the base, with the dc voltage at the base about one-third of the supply. The transistor available has $\beta=100$. Use standard 5\% resistors (see Appendix J).

D 6.117 The CE amplifier circuit of Fig. P6.117 is biased with a constant-current source $I$. It is required to design the circuit (i.e., find values for $I, R_{B}$, and $R_{C}$ ) to meet the following specifications:
(a) $R_{\mathrm{in}} \simeq 10 \mathrm{k} \Omega$.
(b) The dc voltage drop across $R_{B}$ is approximately 0.2 V .


Figure P6. 117
(c) The open-circuit voltage gain from base to collector is the maximum possible, consistent with the requirement that the collector voltage never fall by more than approximately 0.4 V below the base voltage with the signal between base and emitter being as high as 5 mV .

Assume that $v_{\text {sig }}$ is a sinusoidal source, the available supply $V_{C C}=5 \mathrm{~V}$, and the transistor has $\beta=100$. Use standard $5 \%$ resistance values, and specify the value of $I$ to one significant digit. What base-to-collector open-circuit voltage gain does your design provide? If $R_{\text {sig }}=R_{L}=20 \mathrm{k} \Omega$, what is the overall voltage gain?

D 6.118 In the circuit of Fig. P6.118, $v_{\text {sig }}$ is a small sine-wave signal with zero average. The transistor $\beta$ is 100 .
(a) Find the value of $R_{E}$ to establish a dc emitter current of about 0.5 mA .
(b) Find $R_{C}$ to establish a dc collector voltage of about +0.5 V .
(c) For $R_{L}=10 \mathrm{k} \Omega$, draw the small-signal equivalent circuit of the amplifier and determine its overall voltage gain.


Figure P6.118
*6.119 The amplifier of Fig. P6.119 consists of two identical common-emitter amplifiers connected in cascade. Observe that the input resistance of the second stage, $R_{\mathrm{in} 2}$, constitutes the load resistance of the first stage.


Figure P6. 119
(a) For $V_{C C}=15 \mathrm{~V}, \quad R_{1}=100 \mathrm{k} \Omega, \quad R_{2}=47 \mathrm{k} \Omega$, $R_{E}=3.9 \mathrm{k} \Omega, R_{C}=6.8 \mathrm{k} \Omega$, and $\beta=100$, determine the dc collector current and dc collector voltage of each transistor.
(b) Draw the small-signal equivalent circuit of the entire amplifier and give the values of all its components.
(c) Find $R_{\text {in } 1}$ and $v_{b 1} / v_{\text {sig }}$ for $R_{\text {sig }}=5 \mathrm{k} \Omega$.
(d) Find $R_{\mathrm{in} 2}$ and $v_{b 2} / v_{b 1}$.
(e) For $R_{L}=2 \mathrm{k} \Omega$, find $v_{o} / v_{b 2}$.
(f) Find the overall voltage gain $v_{o} / v_{\text {sig }}$.
6.120 In the circuit of Fig. P6.120, the BJT is biased with a constant-current source, and $v_{\text {sig }}$ is a small sine-wave signal.

Find $R_{\text {in }}$ and the gain $v_{o} / v_{\text {sig }}$. Assume $\beta=100$. If the amplitude of the signal $v_{b e}$ is to be limited to 5 mV , what is the largest signal at the input? What is the corresponding signal at the output?
*6.121 The BJT in the circuit of Fig. P6.121 has $\beta=100$.
(a) Find the dc collector current and the dc voltage at the collector.
(b) Replacing the transistor by its T model, draw the small-signal equivalent circuit of the amplifier. Analyze the resulting circuit to determine the voltage gain $v_{o} / v_{i}$.


Figure P6.121
6.122 For the emitter-follower circuit shown in Fig. P6.122, the BJT used is specified to have $\beta$ values in the range of 50 to 200 (a distressing situation for the circuit designer).
For the two extreme values of $\beta$ ( $\beta=50$ and $\beta=200$ ), find:
(a) $I_{E}, V_{E}$, and $V_{B}$
(b) the input resistance $R_{\text {in }}$
(c) the voltage gain $v_{o} / v_{\text {sig }}$


Figure P6. 122
6.123 For the circuit in Fig. P6.123, find the input resistance $R_{\text {in }}$ and the voltage gain $v_{o} / v_{\text {sig }}$. Assume that the source provides a small signal $v_{\text {sig }}$ and that $\beta=100$.


Figure P6.123
**6.124 For the circuit in Fig. P6.124, called a bootstrapped follower:
(a) Find the dc emitter current and $g_{m}, r_{e}$, and $r_{\pi}$. Use $\beta=100$.
(b) Replace the BJT with its T model (neglecting $r_{o}$ ), and analyze the circuit to determine the input resistance $R_{\text {in }}$ and the voltage gain $v_{o} / v_{\text {sig }}$.
(c) Repeat (b) for the case when capacitor $C_{B}$ is open-circuited. Compare the results with those obtained in (b) to find the advantages of bootstrapping.


Figure P6. 124
**6.125 For the follower circuit in Fig. P6.125, let transistor $Q_{1}$ have $\beta=50$ and transistor $Q_{2}$ have $\beta=100$, and neglect the effect of $r_{o}$. Use $V_{B E}=0.7 \mathrm{~V}$.


Figure P6. 125
（a）Find the dc emitter currents of $Q_{1}$ and $Q_{2}$ ．Also，find the dc voltages $V_{B 1}$ and $V_{B 2}$ ．
（b）If a load resistance $R_{L}=1 \mathrm{k} \Omega$ is connected to the output terminal，find the voltage gain from the base to the emitter of $Q_{2}, v_{o} / v_{b 2}$ ，and find the input resistance $R_{i b 2}$ looking into the base of $Q_{2}$ ．（Hint：Consider $Q_{2}$ as an emitter follower fed by a voltage $v_{b 2}$ at its base．）
（c）Replacing $Q_{2}$ with its input resistance $R_{i b 2}$ found in（b）， analyze the circuit of emitter follower $Q_{1}$ to determine its input resistance $R_{\mathrm{in}}$ ，and the gain from its base to its emitter，$v_{e 1} / v_{b 1}$ ．
（d）If the circuit is fed with a source having a $100-\mathrm{k} \Omega$ resistance，find the transmission to the base of $Q_{1}$ ， $v_{b 1} / v_{\text {sig }}$ ．
（e）Find the overall voltage gain $v_{o} / v_{\text {sig }}$ ．


## PART II

## Integrated-Circuit Amplifiers

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Having studied the MOSFET and the BJT and become familiar with their basic circuit applications, we are now ready to consider their use in the design of practical amplifier circuits that can be fabricated in inte-grated-circuit (IC) form. Part II is devoted to this rich subject. Its four chapters constitute a coherent treatment of IC amplifier design and can thus serve as a second course in electronic circuits.

Beginning with a brief introduction to the philosophy of IC design, Chapter 7 presents the basic circuit building blocks that are utilized in the design of IC amplifiers. However, the most important building block of all, the differential-pair configuration, is deferred to Chapter 8, where it is the main topic. Chapter 8 also considers the design of amplifiers that require a number of cascaded stages.

As mentioned at various points in Part I, amplifiers have finite bandwidths. Chapter 9 is devoted to the fre-quency-response analysis of amplifiers; it provides a comprehensive study of the mechanisms that limit the bandwidth and the tools and methods that are utilized to estimate it for a wide variety of amplifier circuit configurations. While the study of the first half or so of Chapter 9 is essential, some of its later sections can be postponed until a later point in the course or even to subsequent courses.

An essential tool in amplifier design is the judicious use of feedback. Chapter 10 deals with this exceedingly important subject. A thorough understanding of feedback concepts, insight into feedback configurations, and proficiency in the use of the feedback-analysis method are invaluable to the serious circuit designer.

Throughout Part II, MOSFET and BJT circuits are treated side by side. Because over $90 \%$ of ICs today employ the MOSFET, its circuits are presented first. Nevertheless, BJT circuits are presented with equal depth, although sometimes somewhat more briefly. In this regard, we draw the reader's attention to Appendix G (on the website with Table G. 3 in the printed book), which presents a valuable compilation of the properties of transistors of both types, allowing interesting comparisons to be made. As well, typical device parameter values are provided in Appendix K for a number of CMOS and bipolar fabrication process technologies.


[^0]:    ${ }^{1}$ To focus on essentials, we shall neglect the Early effect until a later point.

[^1]:    ${ }^{2}$ The only exception is a nonsaturating form of BJT logic circuits known as emitter-coupled logic (ECL).

[^2]:    ${ }^{3}$ Practical biasing arrangements will be studied in Section 6.4.

[^3]:    ${ }^{4}$ This assumes that the circuit designer is also designing the device, as is typically the case in IC design. On the other hand, a circuit designer working with a discrete-circuit MOSFET obviously does not have the freedom to change its $W / L$ ratio. Thus, in this case there are only two design parameters- $V_{O V}$ and $I_{D}$, and only one can be specified by the designer.

[^4]:    ${ }^{5}$ At high frequencies, the input capacitance at the MOSFET gate makes the input current finite (see Chapter 9).

[^5]:    ${ }^{6}$ That is indeed what we were doing in Section 6.1. However, the amplifier circuits studied there were conceptual ones, not actual practical circuits. Our purpose in this section is to study the latter.
    ${ }^{7}$ The action of $R_{S}$ in stabilizing the value of the bias current $I_{D}$ is not unlike that of the resistance $R_{s}$, which we included in the source lead of a CS amplifier in Section 6.3.4. In the latter case also, $R_{s}$ works to reduce the change in $i_{D}$ with the result that the amplifier gain is reduced.

[^6]:    ${ }^{8}$ Bias design seeks to stabilize either $I_{E}$ or $I_{C}$ since $I_{C}=\alpha I_{E}$ and $\alpha$ varies very little. That is, a stable $I_{E}$ will result in an equally stable $I_{C}$, and vice versa.

[^7]:    ${ }^{9}$ If the input signal is coupled to the transistor base, the two bias resistances $R_{1}$ and $R_{2}$ effectively appear in parallel between the base and ground. Thus, low values for $R_{1}$ and $R_{2}$ will result in lowering $R_{\mathrm{in}}$.

[^8]:    ${ }^{10}$ Although reducing $R_{E}$ restores $I_{E}$ to the design value of 1 mA , it does not solve the problem of the dependence of the value of $I_{E}$ on $\beta$. See Exercise 6.34.

