Engineering PHYSICS

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Preface

Science and engineering form the backbone of any technological innovation. Physics is a fundamental aspect of science. Engineering focuses on the conversion of scientific ideas into viable products and technologies. A sound knowledge of physics relevant to engineering is critical for converting ideas to new designs and products. An understanding of physics also helps engineers understand the working and limitations of existing devices and techniques, which eventually leads to new innovations and improvements.

It is interesting to note that in spite of the complexities of modern technology, the underlying principle behind it still remains simple. In fact, it would not be wrong to say that unless the basic physics behind a technology is fully understood, it would be impossible to implement the full potential of the technology. The fundamental concepts of physics have laid the foundation for advances in engineering technology.

ABOUT THE BOOK

Engineering Physics is designed as a textbook for first year undergraduate engineering students. The book thoroughly explains all relevant and important topics in a student-friendly manner. The language and approach towards understanding the fundamental topics of physics is clear. The mathematics has been kept simple and understandable, enabling readers to easily comprehend the idea behind a concept. The book lays emphasis on explaining the principles as well as the applications of a given topic using numerous solved examples and self-explanatory figures and diagrams. It includes plenty of chapter-end practice questions, such as multiple-choice questions, review questions, and numerical problems, provided under the self-assessment section. Answers to multiple-choice questions and numerical problems are also provided at the end of the book.

KEY FEATURES OF THE BOOK AND THEIR BENEFITS

Features	Benefits
Solved examples: 166 solved examples are provided	This will help readers learn how to apply concepts in a given problem.
Figures: 176 well-labelled figures are given	This will help readers visualize the concepts and principles of physics.

Features	Benefits
List of symbols: A list of symbols is given at the beginning of each chapter	This facilitates easy referencing of the symbols used in equations and figures across the text.
Summary of concepts, applications, and key formulae: These are given at the end of each chapter	This helps in quick revision of the important formulae, concepts, and their applications.
Chapter-end self-assessment section: Contains **217** multiple-choice questions, **273** review exercises, and **169** numerical problems. *Answers to MCQs and numerical problems are given at the end of the book*	This will help students practice and apply the concepts learnt and also self-check their understanding while preparing for examinations.
Interactive animations: Links for interactive animations, provided as online resources, are indicated by a 'mouse icon ' within the text	These animations will help readers understand the practical implementation of a concept or the occurrence of a phenomenon.

ORGANIZATION OF THE BOOK

The book consists of 11 chapters. A chapter-wise scheme of the book is presented here.

Chapter 1 on interference discusses the principle of superposition and the generation of coherent sources. It covers Young's double-slit interference. It also explains the phenomenon of interference with due emphasis on the division of amplitude including interference in thin films and Newton's rings. The chapter details the construction and working principle of various interferometers that are used for observing the phenomenon of interference.

Chapter 2 presents basics of diffraction, Huygen's principle, and Fraunhoffer's diffraction in detail. It elucidates the cases of single slit, double slit, circular aperture and N-slits. The chapter explains the resolving power of important optical instruments such as plane diffraction grating, telescope, and microscope.

Chapter 3 describes the phenomenon of polarization, types of polarization, and methods of producing polarization. It discusses in detail topics such as Malus law, Nicol prism as a polarizer and analyzer, quarter- and half-wave plates, Fresnel's theory of optical rotation, and polarimeter.

Chapter 4 discusses the ordered excited state—lasers. It covers the various properties, types, components, and applications of lasers in detail. The Einstein's transition probabilities have been mathematically derived giving the difference between the three different phenomenon of spontaneous, stimulated emission and absorption. The chapter also covers Ruby laser and He-Ne lasers.

Chapter 5 covers architectural acoustics which has become an important feature of building design. This chapter details the classification of sound, characteristics of musical sounds, intensity of sound, reverberation, Sabine's formula, and absorption coefficient.

Ultrasonic waves are used in non-destructive testing techniques and are produced by the magnetostriction method and piezoelectric effect. *Chapter 6* deals with the properties and detection of ultrasonic waves, cavitation, acoustic grating, SONAR, and the industrial and medical applications.

Chapter 7 on crystal physics introduces lattices, miller indices, atomic radius, coordination number, and crystal structures.

X-rays, to date, have made useful contributions towards material analysis and medical applications. *Chapter 8* presents a discussion on diffraction of X-rays, X-ray spectrum, the different methods of production of X-rays, and its important applications.

Chapter 9 on nuclear physics and radioactivity has been included in the book, in view of its importance in energy generation, in addition to the use of fossil fuels. The chapter covers nuclear forces, conservation laws, and radioactive laws. The theory of nuclear fusion and fission has been explained with a mention of nuclear reactors.

Chapter 10 on dielectric properties of materials includes important topics such as electric dipole, dipole moment, dielectric constant, and polarizability. The different types of polarizations in dielectrics, their frequency and temperature dependence, and Clausius–Mossotti equation are presented in detail. Dielectric losses, their breakdown, and the applications of dielectric materials are also covered.

Chapter 11 discusses the magnetic properties of materials such as dia, para, and ferromagnetism in detail. The phenomenon of hysteresis, ferrites, and important applications of magnetic materials are also included in this chapter.

ONLINE RESOURCES

For the benefit of faculty and students reading this book, additional resources are available online at india.oup.com/orcs/9780199487127.

For Faculty

- Solutions manual
- Chapter-wise PPTs

For Students

- Test generator
- Model question papers
- Links to interactive animations (indicated with in text)

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CHAPTER 1

Interference

Learning objectives

After studying this chapter, students will be able to

- understand the concept of superposition of waves
- comprehend the meaning of coherence
- understand the physics of interference from two-point sources
- elucidate Young's double-slit interference experiment in detail
- explain different types of interference
- explain the formation of interference pattern in thin films
- understand the formation of Newton's rings
- elucidate the construction and working principle of interferometers

List of Symbols

A = Amplitude	c = Velocity of light	β = Fringe width
ω = Angular velocity	ΔL = Coherence length	μ = Refractive index
I = Intensity	ψ = Wave function	t = Thickness
λ = Wavelength	R = Radius	δ = Phase difference

1.1 INTRODUCTION

When two or more waves travel simultaneously through a medium, the resultant displacement at any point of the medium is given by the vector sum of the displacements of the individual waves. This is called the principle of superposition of waves. In sound waves, this results in two interesting consequences: stationary waves and beats. In the case of light, one such interesting consequence of the principle of superposition is *interference*. Application of the principle of interference can easily be observed in nature. Waves in water get superimposed and result in an interference pattern. Such interference patterns can be observed when small pebbles are thrown into lakes or ponds.

A single stone will create waves in water, but if a second stone is dropped at a small distance from the first one, the waves generated by the two stones will interact and create interference patterns due to superposition. At regions of constructive interference, water waves can reach great heights, and create hazardous conditions and extreme damage. Superposition has been discussed in detail in this chapter. Concepts of superimposition have then been used for understanding the phenomenon of interference. Different types of interference patterns observed in thin films have also been discussed in detail. Some applications of interference have been presented towards the end of this chapter.

1.2 PRINCIPLE OF SUPERPOSITION OF WAVES

A wave represents a travelling disturbance. It is represented mathematically in the following form:

$$\psi = A\sin(\omega t + \delta) \tag{1.1}$$

where A represents the amplitude, w the angular frequency, and δ the phase of a wave with respect to some reference. As the wave travels through a medium, the particles of the medium get acted upon by the wave.

According to the principle of superposition, the resultant displacement of a particle of the medium acted upon by two or more waves simultaneously is given by the algebraic sum of the displacements produced by the individual waves.

Thus, if two waves are represented by

$$\psi_1 = A_1 \sin \omega t \tag{1.2}$$

and $$\psi_2 = A_2 \sin(\omega t + \delta) \tag{1.3}$$

where δ represents the phase difference between the two waves, then the resultant displacement, y, is given by the following equation:

$$\psi = \psi_1 + \psi_2 = A_1 \sin \omega t + A_2 \sin(\omega t + \delta) \tag{1.4}$$

When more than two waves are involved, the general expression for resultant displacement becomes as follows:

$$\psi = \psi_1 + \psi_2 + \psi_3 + \ldots + \psi_n \tag{1.5}$$

$$= A_1 \sin(\omega t + \delta_1) + A_2 \sin(\omega t + \delta_2) + \ldots + A_n \sin(\omega t + \delta_n) \tag{1.6}$$

where A_1, A_2, …, A_n represent the amplitudes and δ_1, δ_2, …, δ_n represent the phases of the waves. Equation (1.6) is not easy to solve for general situations. For some special situations, however, a solution can be visualized. The easiest situation to visualize is the one in which all the individual waves have the same amplitude, say A, and also all the waves are in phase. The amplitude of the resultant of n waves in this case will be nA, and the corresponding intensity will be n^2A^2. For a random distribution of phases, a graphical method has to be used for evaluating the resultant amplitude. A typical figure is shown in Fig. 1.1, where A_1 represents the amplitude of the resultant.

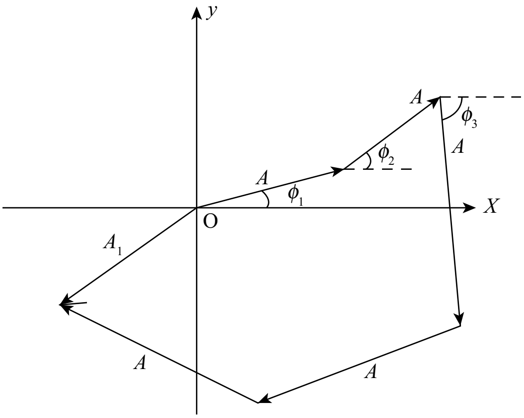

Fig. 1.1 Evaluation of resultant amplitude

Summation of projections of amplitudes along the x-direction results in the following equation:

$$A_{1x} = A(\cos\delta_1 + \cos\delta_2 + \ldots + \cos\delta_n) \tag{1.7}$$

To calculate the intensity, square of Eq. (1.7) should be estimated. This would result in terms such as $\cos^2\delta_1$, $2\cos(\delta_1)\cos(\delta_2)$, and $\cos^2(\delta_2)$. For the superimposition of a large number of waves, the terms having products such as $\cos\delta_1 \cos\delta_2$ will average out to zero. Thus, only terms such as $\cos^2\delta_1$ and $\cos^2\delta_2$ can be assumed to survive. Similarly, the y-components would lead to the following relation:

$$A_{1y} = A(\sin\delta_1 + \sin\delta_2 + \ldots + \sin\delta_n) \tag{1.8}$$

For evaluating intensity, square of Eq. (1.8) would also be needed. Once again, this would involve terms such as $\sin^2\delta_1$, $\sin^2\delta_2$, and $2\sin\delta_1\sin\delta_2$. For a large number of waves, the cross-terms like $\sin\delta_1\sin\delta_2$ can be assumed to result in zero average value, leaving out terms such as $\sin^2\delta_1$ and $\sin^2\delta_2$. The total resultant intensity can be obtained by adding contribution from Eqs (1.7) and (1.8). Thus, we have the following relation:

$$\begin{aligned} I = A_1^2 &= A^2\left(\cos^2\delta_1 + \cos^2\delta_2 + \cos^2\delta_3 + \ldots + \cos^2\delta_n\right) \\ &\quad + A^2\left(\sin^2\delta_1 + \sin^2\delta_2 + \sin^2\delta_3 + \ldots + \sin^2\delta_n\right) \end{aligned} \tag{1.9}$$

Since $(\cos^2\delta + \sin^2\delta) = 1$, we get the following expression:

$$I = A_1^2 = nA^2 \tag{1.10}$$

The average resultant intensity is, therefore, n times the average intensity of a single wave. From Eq. (1.10) it can also be concluded that the resultant average amplitude is proportional to $\sqrt{n}$, where n represents the number of waves.

Example 1.1 Two coherent sources whose intensity ratio is 64:1 produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity.

Solution Suppose A_1 and A_2 represent the amplitudes of waves emitted by the two sources. Then we have

$$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \tag{1.11}$$

which yields the following equation:

$$\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \frac{8}{1} \tag{1.12}$$

giving $A_1 = 8A_2$ (1.13)

We also know that

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} \tag{1.14}$$

Using Eq. (1.13) in Eq. (1.14), we get the following relation:

$$\frac{I_{max}}{I_{min}} = \frac{(8A_2 + A_2)^2}{(8A_2 - A_2)^2} = \frac{(9A_2)^2}{(7A_2)^2} = \frac{81}{49}$$

Thus, $I_{max} : I_{min} = 81:49$

Example 1.2 Two coherent sources whose intensity ratio is 4:1 produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity.

Solution Let us consider the following equation: $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2}$

which yields $\frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \frac{2}{1}$

giving, $A_1 = 2A_2$

We also know that $\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(2A_2 + A_2)^2}{(2A_1 - A_2)^2} = \frac{3^2}{1^2} = 9$

Therefore, the required ratio is 9:1.

1.3 COHERENCE

Coherent sources are sources that have the same wavelength. The waves emitted by these sources at any point exhibit a correlation between the amplitudes and the phases. In other words, two waves are coherent if they have a constant phase difference between them; coherent waves also have the same frequency and amplitude. Coherent sources are obtained by splitting a light source into parts. Some of the common methods used for the generation of coherent waves are as follows: 1. Young's double-slit experiment 2. Fresnel's biprism 3. Llyod's mirror. The different types of coherence are discussed as follows.

Temporal coherence A wave travels along its direction of propagation. Different points along the direction of propagation have a phase associated with them. If the phase difference between any two points along the direction of propagation is independent of time, then the wave is said to be temporarily coherent. *Temporal coherence* is also called *longitudinal coherence*. A time-independent phase difference also implies that the wave is monochromatic or of one wavelength. Suppose δ_1 and δ_2 represent the phases at points 1 and 2 for a wave at a particular instant t_1. In addition, δ_1' and δ_2' represent the phases at the same two points 1 and 2 at a different time t_2. For a wave having temporal coherence, the following relation must hold:

$$\delta_2 - \delta_1 = \delta_2{'} - \delta_1{'} \tag{1.15}$$

Spatial coherence Spatial coherence is related to the phase of a wave at different points that are transverse to the direction of propagation. If the phase difference between any points located transverse to the direction of propagation is independent of time, then the wave is said to be spatially coherent.

Coherence time and coherence length The time interval over which the phase of a wave remains constant is called the *coherence time*. For a perfectly monochromatic sinusoidal wave, the coherence time is infinity. In reality, no wave is perfectly monochromatic, and therefore a finite coherence time exists. The coherence time is generally represented by a symbol Δt. The distance travelled by light during one coherence time is called the *coherence length* and is represented by the symbol ΔL for light waves:

$$\Delta L = c\Delta t \tag{1.16}$$

where c represents the velocity of light.

1.4 INTERFERENCE OF LIGHT FROM TWO POINT SOURCES

Sustainable interference patterns will occur only when overlapping waves satisfy the following conditions:

1. The waves must be of similar types (e.g., both of them are either light or sound waves).
2. Wave sources must be coherent.
3. The waves must have comparable (but not necessarily equal) amplitudes.

If two coherent, monochromatic point sources are set up, then an interference pattern is formed in the region where their waves overlap. Assume that the two sources are in phase with one another. Maxima will be formed where the waves from both sources arrive exactly half a cycle (π) out of phase. Phase differences are caused by the different distances travelled by the waves from the source to the point concerned. An extra path of one half-wavelength from one source will introduce a phase difference of π radians, resulting in cancellation (a minimum), whereas a path difference of any whole-number multiple of wavelength results in the waves arriving in phase and adding (a maximum). The situation is depicted schematically in Fig. 1.2. Figure 1.2(a) depicts the situation where the path difference between the waves originating

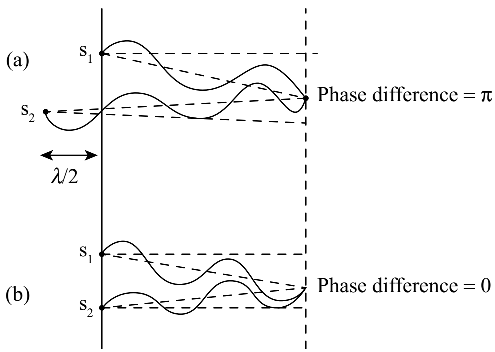

Fig. 1.2 Interference from two point sources (a) Path difference is $\lambda/2$ (b) Path difference and the phase difference are zero

from S_1 and S_2 is $\lambda/2$, resulting in a phase difference of $\pi/2$. The situation in which the path difference and the phase difference are zero is depicted in Fig. 1.2(b).

Path difference x is the difference in distance from each source to a particular point and δ represents the difference in phase of the waves at a point. In general,

$$\delta = \frac{2\pi x}{\lambda} \tag{1.17}$$

Equation (1.17) implies that $x = \lambda$; a path difference of a whole wavelength leads to a phase difference of $\delta = 2\pi$.

Maxima are not regions with a permanent large disturbance—they oscillate like any other part of the wave, passing through zero to negative values every cycle. They represent the positions where this oscillation has the maximum amplitude.

Constructive interference occurs due to the superposition of two waves at a point such that the crest of one wave falls on the crest of the other, that is, *the path difference between two waves is an integral multiple of the wavelength ($n\lambda$). Intensity is maximum at these points (n is an integer or zero).*

Destructive interference occurs due to the superposition of two waves at a point such that the crest of one wave falls on the trough of the other, that is, the path difference between the two waves is $\left(n + \frac{1}{2}\right)\lambda$, where λ is the wavelength and n is an integer or zero. *Intensity is minimum at these points.*

Monochromatic sources are sources of light waves having the same wavelength or frequency. In Section 1.3, we have learnt that *coherent sources* are sources of light waves having the same wavelength or frequency and a constant phase difference.

1.4.1 Mathematical Treatment of Interference

Let the waves from two coherence sources be represented as follows:

$$\psi_1 = \psi_{10} \sin(\omega t - kx) \tag{1.18}$$

$$\psi_2 = \psi_{20} \sin(\omega t - kx + \delta) \tag{1.19}$$

where δ is the constant phase difference between them, ψ_{10} and ψ_{20} are the amplitudes, and w represents the angular frequency of the two waves.

The resultant of their superposition is given by [from Eqs (1.18) and (1.19)] the following relation:

$$\begin{aligned} \psi = \psi_1 + \psi_2 = \psi_{10} \sin(\omega t - kx) + \psi_{20} \sin(\omega t - kx)\cos\delta \\ + \psi_{20} \cos(\omega t - kx)\sin\delta \end{aligned} \tag{1.20}$$

or
$$\psi = (\psi_{10} + \psi_{20}\cos\delta)\sin(\omega t - kx) + \psi_{20}\sin\delta\cos(\omega t - kx) \tag{1.21}$$

Let
$$A\cos\theta = \psi_{10} + \psi_{20}\cos\delta \tag{1.22}$$

and $$A\sin\theta = \psi_{20}\sin\delta \tag{1.23}$$

Therefore, $$A^2 = \psi_{10}{}^2 + \psi_{20}{}^2 + 2\psi_{10}\psi_{20}\cos\delta \tag{1.24}$$

and $$\tan\theta = \frac{\psi_{20}\sin\delta}{\psi_{10} + \psi_{20}\cos\delta} \tag{1.25}$$

Using Eqs (1.21)–(1.23) in Eq. (1.24), the following relation can be obtained:

$$\psi = A\sin(\omega t - kx)\cos\theta + A\cos(\omega t - kx)\sin\theta \tag{1.26}$$

which gives the following expression:

$$\psi = A\sin(\omega t - kx + \theta) \tag{1.27}$$

We see that the resultant wave has an amplitude A, given by Eq. (1.24), and a phase angle of θ, given by Eq. (1.25), with respect to the wave of source [Eq. (1.18)].

1.4.2 Constructive Interference

From Eq. (1.24), A^2 is maximum when

$$\cos\delta = 1 \text{ or } \delta = 0, \ldots, 2n\pi \tag{1.28}$$

$$A_{\max}^2 = \psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}$$

or $$A_{\max}^2 = (\psi_{10} + \psi_{20})^2$$

or $$A_{\max} = \psi_{10} + \psi_{20} \tag{1.29}$$

Since intensity $I \sim A^2$, the maximum intensity is expressed as follows:

$$I_{\max} = k_1\left(\psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}\right), \tag{1.30}$$

where k_1 is a constant of proportionality.

1.4.3 Destructive Interference

Again from Eq. (1.24), A^2 is minimum when $\cos\delta = -1$ or

$$\delta = (2n+1)\pi \tag{1.31}$$

$$\therefore A_{\min}^2 = \psi_{10}^2 + \psi_{20}^2 - 2\psi_{10}\psi_{20} = (\psi_{10} - \psi_{20})^2$$

or $$A_{\min} = \psi_{10} - \psi_{20} \tag{1.32}$$

$$\therefore \text{Minimum intensity } I_{\min} = k\left(\psi_{10}^2 + \psi_{20}^2 - 2\psi_{10}\psi_{20}\right) \tag{1.33}$$

where k is a constant of proportionality.

Thus, for constructive interference, the following relations hold:

$$\text{Phase difference} = \delta = 0, \ldots, 2n\pi$$

$$\text{Path difference} = \frac{\lambda}{2\pi}\delta = 0, \ldots, n\lambda$$

Whereas for destructive interference, the following relations hold:

$$\text{Phase difference} = \delta = (2n+1)\pi$$

$$\text{Path difference} = \frac{\lambda}{2\pi}\delta = \left(n+\frac{1}{2}\right)\lambda$$

From Eqs (1.29) and (1.32) we can conclude that A_{max} will be the greatest and A_{min} the least when $\psi_{10} = \psi_{20} = \psi_0$, that is, the two superposing waves have equal amplitude, because in that case the following relations are true:

$$A_{max} = 2\psi_0$$

$$A_{min} = 0$$

$$I = k_1\left(\psi_0^2 + \psi_0^2 + 2\psi_0^2\cos\delta\right) = 2\psi_0^2\left(1+\cos\delta\right)k_1$$

Example 1.3 Determine the ratio of intensity at the centre of a bright fringe to the intensity found at a point one-quarter of the distance between two fringes from the centre.

Solution From Eq. (1.24), wet get the following relation:

$$I = A^2 = \psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}\cos\delta \tag{1.34}$$

When $\psi_{10} = \psi_{20}$, Eq. (1.34) can be rewritten in the following form:

$$I = \psi_{10}^2 + \psi_{10}^2 + 2\psi_{10}^2\cos\delta \tag{1.35}$$

$$= 2\psi_{10}^2\left(1+\cos\delta\right) \tag{1.36}$$

At the centre, $\delta = 0$. Using Eq. (1.36), we get the following expression:

$$I_0 = 2\psi_{10}^2\left(1+1\right) = 4\psi_{10}^2 \tag{1.37}$$

The phase difference between two consecutive fringes is 2π. Thus, the phase difference at a distance that is one-quarter of the distance between two fringes will be $\frac{2\pi}{4} = \frac{\pi}{2}$.

Suppose I_1 represents the intensity at a distance that is one-quarter of the distance between two fringes, then using Eq. (1.36) we get the following relation:

$$I_1 = 2\psi_{10}^2\left(1+\cos\frac{\pi}{2}\right) = 2\psi_{10}^2 \tag{1.38}$$

Using Eqs (1.37) and (1.38), we obtain the following relation: $\frac{I_0}{I_1} = \frac{4\psi_{10}^2}{2\psi_{10}^2} = 2$.

1.5 YOUNG'S DOUBLE-SLIT INTERFERENCE

The phenomenon of interference was first demonstrated experimentally by Thomas Young in the year 1801. A schematic of the phenomenon is shown in Fig. 1.3.

Sunlight is made to pass through the pin hole S. Two closely spaced pin holes S_1 and S_2 are placed on the way, and the interference pattern was observed on screen XY. Young observed a few coloured bright and dark bands on the screen. Some modern modifications in the original set-up use narrow slits in place of pin holes S_1 and S_2, and sunlight is replaced by monochromatic light. As a result of these modifications, the interference pattern consists of equally spaced bright and dark bands.

As sunlight passes through the pin hole S, spherical waves originating from the pin holes start spreading out, as shown in Fig. 1.3. These spherical waves are incident on pin holes S_1 and S_2. According to the Huygen's principle, each point on the wavefront is a centre of secondary wavelets. Thus, secondary waves start spreading out from pin holes S_1 and S_2, as shown in Fig. 1.3. As the secondary waves spread out, their radii increase and they superimpose on each other. In Fig. 1.3, crests and troughs are represented, respectively, by continuous and dotted circular arcs. There are points at which the crest of one of the secondary waves falls on that of another wave or the trough of one of the waves coincides with that of another wave. One such point is point A (shown in the inset of Fig. 1.3). The resultant amplitude is the sum of the amplitudes of the two individual waves, and therefore at such points, the resultant amplitude increases. This phenomenon, as mentioned earlier, is known as *constructive interference*. Since intensity is proportional to the square of the amplitude, the resultant intensity at points undergoing constructive interference also increases. On the contrary, there are points where the crest of one wave falls on the trough of another wave or vice versa. One such point is point B (shown in the inset of Fig. 1.3). The resultant amplitude at these points is the difference between the two individual amplitudes and is therefore minimum. Such points are called regions of destructive interference.

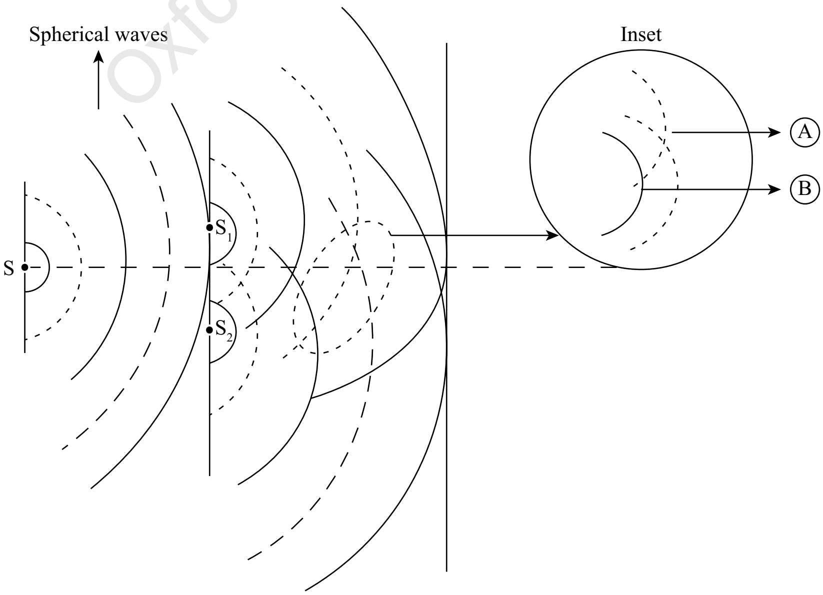

Fig. 1.3 Schematic of Young's double-slit experiment

Since intensity is proportional to the square of amplitude, intensity gets minimized at points having destructive interference.

Theory of fringe formation Figure 1.4 shows the source slit S and the two slits S_1 and S_2 that are equidistant from S.

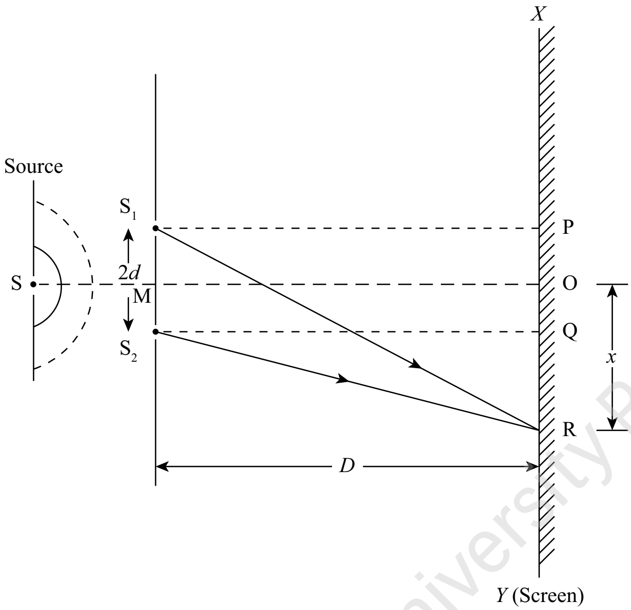

Fig. 1.4 Condition for fringe formation on screen

The distance between slits S_1 and S_2 is $2d$, and the screen is assumed to be at distance D from the plane containing slits S_1 and S_2. Point O on the screen is equidistant from S_1 and S_2. Therefore, the path difference between the waves reaching point O from S_1 and S_2 is zero. This also means that there is no phase difference between the waves reaching point O from S_1 and S_2. The intensity at point O is, therefore, maximum. Let us now consider a point R at a distance x from O, as shown in Fig. 1.4. From the right-angle triangle S_1PR, we get the following relation:

$$(S_1R)^2 = (S_1P)^2 + (PR)^2$$

yielding $(S_1R)^2 = D^2 + (x + d)^2$ (1.39)

In addition, from the right-angle triangle S_2QR we get the following relation:

$$(S_2R)^2 = (S_2Q)^2 + (OR)^2$$

yielding $(S_2R)^2 = D^2 + (x - d)^2$ (1.40)

Using Eqs (1.39) and (1.40), we get the following relation:

$$(S_1R)^2 - (S_2R)^2 = (x + d)^2 - (x - d)^2 = 4xd$$

leading to $(S_1R - S_2R)(S_1R + S_2R) = 4xd$ (1.41)

In Young's set-up, the distance between the screen and the plane containing slits S_1 and S_2, D, is much greater than the distance between the slits, $2d$ or x; therefore,

$(S_1R + S_2R)$ can be replaced by $2D$ without introducing an appreciable error. Thus, Eq. (1.41) can be rewritten as follows:

$$(S_1R - S_2R)\,2D = 4xd$$

or
$$(S_1R - S_2R) = \frac{4xd}{2D} = \frac{2xd}{D} \tag{1.42}$$

Let us now determine the location of bright and dark fringes.

Bright fringes Point P is bright if the path difference is a whole-number multiple of wavelength λ. Thus,

$$S_2P - S_1P = n\lambda \quad \text{where} \quad n = 0, 1, 2, \ldots \tag{1.43}$$

Substitution of Eq. (1.43) into Eq. (1.42) leads to the following expression:

$$\frac{2xd}{D} = n\lambda$$

or
$$x = \frac{n\lambda D}{2d} \tag{1.44}$$

Equation (1.44) gives the distance of the bright fringe from point O on the screen. The central bright fringe is at point O, since the path difference is zero. Other bright fringes are found for $n = 1, 2, 3, \ldots$ From Eq. (1.44), we obtain the following relations:

$$n = 1,\ x_1 = \frac{\lambda D}{2d} \tag{1.45}$$

$$n = 2,\ x_2 = \frac{2\lambda D}{2d} \tag{1.46}$$

$$n = 3,\ x_3 = \frac{3\lambda D}{2d}$$

$\vdots$
$$n = n,\ x_n = \frac{n\lambda D}{2d} \tag{1.47}$$

The linear distance between any two consecutive fringes is given as follows:

$$x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d} = \frac{\lambda D}{2d} \tag{1.48}$$

Dark fringes Point P is dark if the path difference is an odd number multiple of a half-wavelength. In this case,

$$(S_2P - S_1P) = (2n+1)\frac{\lambda}{2} \tag{1.49}$$

where $n = 0, 1, 2, 3, \ldots$

Using Eqs (1.42) and (1.49), we get the following relation: $\dfrac{2xd}{D} = \dfrac{(2n+1)}{2}\lambda$ which implies that

$$x = \frac{(2n+1)\lambda D}{4d} \tag{1.50}$$

From Eq. (1.50), we get the following relations for the dark fringes:

$$n = 0 \text{ gives } x_0 = \frac{\lambda D}{4d} \tag{1.51}$$

$$n = 1 \text{ gives } x_1 = \frac{3\lambda D}{4d} \tag{1.52}$$

$$n = 2 \text{ gives } x_2 = \frac{5\lambda D}{4d} \tag{1.53}$$

$$n = n \text{ gives } x_n = \frac{(2n+1)\lambda D}{4d} \tag{1.54}$$

The distance between two consecutive dark fringes is given as follows:

$$x_2 - x_1 = \frac{5\lambda D}{4d} - \frac{3\lambda D}{4d} = \frac{2\lambda D}{4d} = \frac{\lambda D}{2d} \tag{1.55}$$

From Eqs (1.48) and (1.55), it is clear that the spacing between two consecutive bright fringes (maxima) is the same as the distance between two consecutive dark fringes (minima). This expression is also called the *fringe width*, and a symbol β is often used to represent it. From Eqs (1.48) and (1.55), one can conclude that the fringe width is directly proportional to D and λ, and inversely proportional to the distance between the two slits, $2d$.

Example 1.4 Two straight and narrow parallel slits 0.9 mm apart are illuminated using a monochromatic light source. A screen placed at a distance of 90 cm is used to obtain fringes. It is found that the distance between consecutive fringes is 0.4 mm. Determine the wavelength of light.

Solution Using Eq. (1.48), we can write the following expression for fringe width β:

$$\beta = \frac{\lambda \mathrm{D}}{2d} \tag{1.56}$$

We can rewrite Eq. (1.56) in the following form:

$$\lambda = \frac{\beta \times 2d}{\mathrm{D}} \tag{1.57}$$

Using the given values in Eq. (1.57), we get the following relation:

$$\lambda = \frac{0.04 \times 0.09}{90} = 4 \times 10^{-5} \text{ cm} \quad \text{or } \lambda = 4000\text{Å}.$$

Example 1.5 In Young's experiment, let a light of wavelengths 5.4×10^{-7} m and 6.85×10^{-8} m be used in turn, keeping the geometry same. Compare the fringe widths in the two cases.

Solution From Eq. (1.55), we have the following relation:

$$\beta = \frac{\lambda \mathrm{D}}{2d} \tag{1.58}$$

For the two wavelengths λ_1 and λ_2, we can write the following equations:

$$\beta_1 = \frac{\lambda_1 \mathrm{D}}{2d} \quad \text{and} \quad \beta_2 = \frac{\lambda_2 \mathrm{D}}{2d} \tag{1.59}$$

Using Eq. (1.59), we can write the following expression: $\dfrac{\beta_1}{\beta_2} = \dfrac{\dfrac{\mathrm{D}}{2d} \times 5.4 \times 10^{-7}}{\dfrac{\mathrm{D}}{2d} \times 6.85 \times 10^{-8}} = 8$

Thus, $\beta_1 = 8\beta_2$

Example 1.6 In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe has been measured to be 1.2 cm. Determine the wavelength of light.

Solution From Eq. (1.55), we get the following relation: $\beta = \dfrac{\lambda D}{2d}$ which yields the following equation:

$$\lambda = \frac{\beta(2d)}{D} \tag{1.60}$$

Use of these values in Eq. (1.60) leads to the following expression:

$$\lambda = \left(\frac{1 \cdot 2}{4}\right) \times \frac{0.28 \times 10^{-5}}{1.4} = 600 \times 10^{-9}\ \text{m}$$

Example 1.7 Two coherent sources are placed 0.9 mm apart and generate interference fringes on a screen 1 m away. The second dark fringe is formed at a distance of 0.08 cm from the central fringe. Determine the wavelength of the monochromatic light used.

Solution Wavelength of light, $\lambda = \dfrac{4dx_n}{D(2n+1)} = \dfrac{0.09 \times 2 \times 0.08}{100(5)} = 2.9 \times 10^{-5}\ \text{cm}$

Example 1.8 In the Young's experiment, let light of wavelengths 6.2×10^{-7} m and 7.1×10^{-8} m be used in turn, keeping the geometry same. Compare the fringe widths in these two cases.

Solution On comparing fringe widths, $\dfrac{\beta_1}{\beta_2} = \dfrac{\lambda_1}{\lambda_2} = \dfrac{6.2 \times 10^{-7}}{7.1 \times 10^{-8}} = \dfrac{62}{7.1} = 8.73$

Example 1.9 In a Young's double-slit experiment, the slits are separated by 0.32 mm and the screen is placed 1.5 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.3 cm. Determine the wavelength of light.

Solution The wavelength is calculated as follows:

$$\lambda = \frac{\beta(2d)}{D} = \left(\frac{1.3}{4}\right) \times 10^{-2} \times \frac{0.32 \times 10^{-3}}{1.5} = \left(\frac{1.3}{4}\right) \times \frac{6.32 \times 10^{-5}}{1.5} = 693.3 \times 10^{-9}\ \text{m.}$$

1.6 TYPES OF INTERFERENCE

The phenomenon of interference requires two wavefronts to interact. These wavefronts can be obtained in two different ways, resulting in two different types of interference: (a) division of wavefront and (b) division of amplitude.

1.6.1 Division of Wavefront

In this type of interference, the incident wavefront is divided into two parts using the phenomenon of reflection, refraction, or diffraction. The two parts of the wavefronts are then made to travel unequal distances before reuniting at some angle. This process leads to the production of an interference pattern. Fresnel's biprism and Lloyd's mirror are examples of this type of interference.

1.6.2 Division of Amplitude

In this type of interference, the amplitude of the incoming beam is divided into two parts through the process of reflection or refraction. The two parts then travel along different optical paths and finally superimpose to produce an interference pattern. To produce this type of an interference pattern, point or narrow line sources are not essential. Broad light sources can be employed to yield bright interference bands.

1.7 INTERFERENCE IN THIN FILMS

Interference in thin films occurs due to division of amplitude. When light is incident on thin films, it gets reflected from the top as well as the bottom surfaces of the films. There are multiple reflections within the films and therefore, the light also gets transmitted multiple times as it is incident on the bottom surface of the films. Since, the light rays have phase difference between them, interference fringes are created both in the reflected light as well as the transmitted light.

In Fig. 1.5, GH and G_1H_1 represent the two surfaces of a transparent film of uniform thickness. The path difference between reflected and refracted rays is given as follows:

$$\text{Path difference, PD} = \mu(\text{BC} + \text{CD}) - \text{BE} \tag{1.61}$$

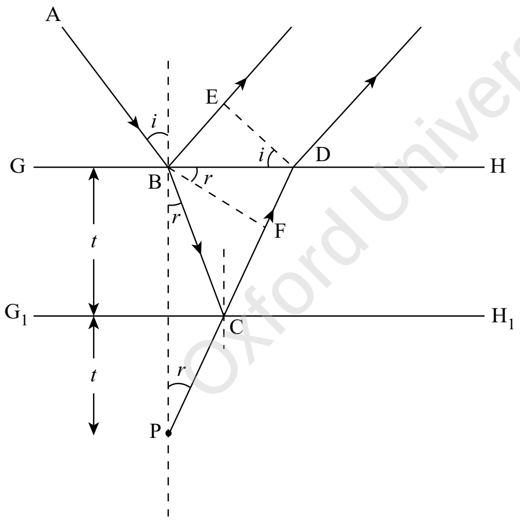

Fig. 1.5 Interference pattern in reflected light

Using Snell's law at the interface, we get

$$\mu = \frac{\sin i}{\sin r} = \frac{\text{BE/BD}}{\text{FD/BD}} = \frac{\text{BE}}{\text{FD}}$$

giving the following relation:

$$\text{BE} = \mu(\text{FD}) \tag{1.62}$$

Using Eq. (1.62) in Eq. (1.61), we obtain the following expression:

$$\text{PD} = \mu(\text{BC+CD}) - \mu(\text{FD})$$

which results in the following relation: $\text{PD} = \mu(\text{BC+CF+FD}) - \mu(\text{FD})$

yielding $PD = \mu(BC+CF)$ (1.63)

Since $BC = PC$, Eq. (1.63) leads to the following form:

$$PD = \mu(PF) \tag{1.64}$$

From triangle BPF, we get the following relation: $\cos r = \dfrac{PF}{BP}$ resulting in

$$PF = BP\cos r = 2t\cos r \tag{1.65}$$

Substitution of the expression for PF [Eq. (1.65)] into Eq. (1.64) yields the following expression:

$$PD = \mu \times 2t\cos r = 2\mu t\cos r \tag{1.66}$$

It is known that a ray reflected from a denser medium suffers a phase change of π, which corresponds to a path difference of $\dfrac{\lambda}{2}$.

The effective path differences $(PD)_{eff}$ thus becomes as follows:

$$(PD)_{eff} = (2\mu t\cos r \pm \lambda/2) \tag{1.67}$$

For maxima, the following relation must hold: $2\mu t\cos r \pm \dfrac{\lambda}{2} = n\lambda$ which gives the following expression:

$$2\mu t\cos r = (2n \pm 1)\lambda/2 \tag{1.68}$$

When Eq. (1.68) is fulfilled, the thin film would appear bright in the reflected pattern.

For minima, we must satisfy the following relation: $2\mu t\cos r \pm \dfrac{\lambda}{2} = (2n \pm 1)\dfrac{\lambda}{2}$

yielding $2\mu t\cos r = (2n \pm 1)\dfrac{\lambda}{2} \pm \dfrac{\lambda}{2}$ (1.69)

Since $(n+1)$ or $(n-1)$ can also be taken as an integer, we can rewrite Eq. (1.69) in the following form:

$$2\mu t\cos r = n\lambda \tag{1.70}$$

where $n = 0, 1, 2, 3, \dots$

When Eq. (1.70) is fulfilled, the films appear dark in the reflected pattern.

***Note*:** If the thickness of the soap bubble is such that the condition for minima, as given in Eq. (1.70) is fulfilled by the reflected light, the soap bubble will appear dark for that particular wavelength.

An interference pattern can also be observed in the transmitted light. Figure 1.6 is a schematic representation of this situation.

Figure 1.6 shows two transmitted rays, BT and DS, which are obtained after reflection and refraction of the corresponding incident rays. BF and DE are normals drawn on DC and BT, respectively. When DC is extended in the backward direction, it meets the extended BH at I.

The effective path difference $(PD)_{eff}$ is given as follows:

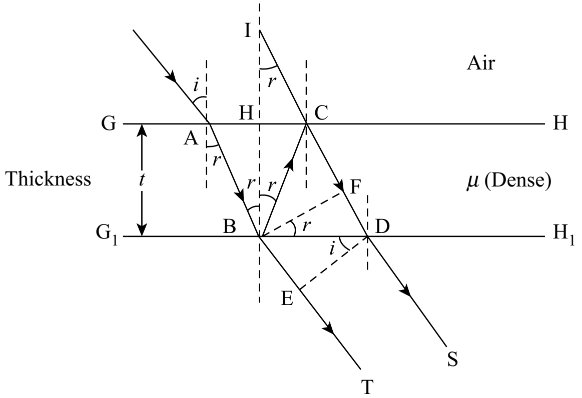

Fig. 1.6 Interference pattern in transmitted light

$$(PD)_{eff} = \mu(BC + CD) - BE \tag{1.71}$$

We know that $\mu = \sin i/\sin r = [BE/BD]/[FD/BD]$

Thus, $\quad BE = \mu FD \qquad (1.72)$

Using Eq. (1.72) in Eq. (1.71), we get the following relation:

$$(PD)_{eff} = \mu\,(BC + CF + FD) - \mu FD$$

leading to $(PD)_{eff} = \mu(BC + CF) = \mu(BI)$

implying that

$$(PD)_{eff} = 2\mu t \cos r \tag{1.73}$$

In this case, reflections have taken place inside the film, and therefore the ray travels from a denser medium to a rarer medium, that is, air. Thus, no additional phase change of π is involved.

For maxima,

$$2\mu t \cos r = n\lambda \tag{1.74}$$

The condition indicated by Eq. (1.74) results in the film appearing bright in the transmitted light.

For minima,

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \tag{1.75}$$

where n can take values 1, 2, 3, …

The condition indicated by Eq. (1.75) results in the film appearing dark in the transmitted light.

A comparison of Eqs (1.68), (1.70), (1.74), and (1.75) reveals that the conditions of maxima and minima get reversed as we change from reflected light to transmitted light.

Let us now see what happens if we replace monochromatic light with white light. The effective path difference is dependent upon μ, which in turn depends upon the wavelength of the incident light. For any particular region on the film and for a particular viewing position, the condition for maxima would be satisfied only for some wavelengths. Bright-coloured fringes would appear in this position. Neighbouring wavelengths would result in reduced intensity. Wavelengths for which the condition for minima is satisfied would be absent in the observed pattern. The basic pattern would remain the same as we change the position of the eye or the region of the film that we are looking at. We also see that conditions for maxima and minima get reversed as we go from the reflected to the transmitted light. This is the reason why thin films (or soap bubbles) appear coloured when viewed under sunlight/white light. The colours absent in the reflected light are visible in the transmitted light and vice versa. Thus, the colours of the reflected and transmitted light are complementary to each other.

Example 1.10 A soap film has a refractive index of 4/3 and is 1×10^{-4} cm thick. It is illuminated by white light incident at an angle of 45°. On examining the reflected light using a spectroscope, a dark band is found corresponding to a wavelength of 5×10^{-5} cm. Determine the order of interference band.

Solution For a dark band,

$$2\mu t \cos r = n\lambda \tag{1.76}$$

Further, sin r = $\sin r = \frac{\sin i}{\mu} = \frac{\sin 45}{4/3} = \frac{1}{\sqrt{2}} \times \frac{3}{4}$

$$\therefore \cos r = \sqrt{1 - \frac{9}{32}} = \frac{1}{4}\sqrt{\frac{23}{2}} \tag{1.77}$$

From Eq. 1.77, we get

$$n = \frac{2\mu t \cos r}{\lambda} = \frac{2 \times 4 \times 1 \times 10^{-4} \times \sqrt{23}}{3 \times 4\sqrt{2} \times 5.1 \times 10^{-5}} = 4.5$$

Thus, 5th order interference fringe is involved.

Example 1.11 White light is incident normally on a soapy water film of thickness 4×10^{-5} cm and $\mu = 1.33$. Which wavelength is reflected strongly in zeroth order of the resulting interference pattern?

Solution For maxima,

$$2\mu t \cos r = (2n + 1)\,\lambda/2 \tag{1.78}$$

Where, $n = 0, 1, 2, 3, \ldots$

Further, $\cos r = 1$ for normal incidence

Using Eq. (1.78), we get

$$\lambda = \frac{4\mu t \cos r}{(2n+1)} \tag{1.79}$$

Putting $n = 0$ and other given values in Eq. (1.79), we get

$$\lambda = \frac{4 \times 1.33 \times 4 \times 10^{-5}}{1} = 21.28 \times 10^{-5}$$

Thus, it can be noted here that this wavelength is not in the visible region.

Example 1.12 In a thin film, between points A and B, six fringes are seen with a light of wavelength 6000 Å. If the light used is of wavelength 4500 Å, what are the number of fringes observed between A and B?

Solution If t represents the thickness of the film between points A and B, then from Eq. (1.70), we get the following relation:

$$2\mu t\cos r = n\lambda \tag{1.80}$$

Since two wavelengths are involved, we can write the following formula:

$$2\mu t\cos r = n_1\lambda_1 = n_2\lambda_2 \tag{1.81}$$

or $6\times 6000 = n_2 \times 4500$, that is, $n_2 = 8$

Example 1.13 In a thin film, between points A and B, five fringes are seen with a light of wavelength 5000 Å. What are the number of fringes observed between A and B?

Solution Let us consider the following expression: $50\times 5800 = n_2 \times 5000$

This yields the following value: $n_2 = \dfrac{5\times 5800}{5000} \cong 6$

1.8 NEWTON'S RINGS

Figure 1.7 shows a schematic representation of a plano-convex lens L kept on a plane glass plate GP. An air film of variable thickness is then formed between the bottom surface of the lens and the top surface of the glass plate. From Fig. 1.7, it is clear that the thickness of the film increases as we move away from the point of contact. Thickness of the air film is zero at the point of contact, while it is constant along the circles drawn using the point of contact as the centre. The resultant interference pattern thus consists of alternate dark and bright rings that are concentric around the point of contact. These rings are also known as *Newton's rings*, as they were first analysed by Newton. The rings are formed because the air has a circular symmetry. These rings can be viewed using a travelling microscope.

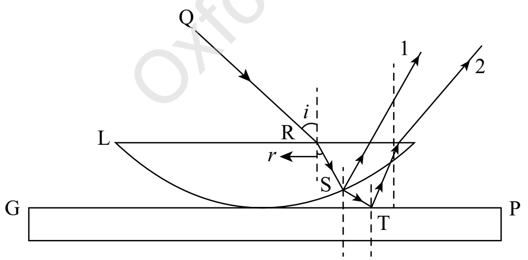

Fig. 1.7 Formation of Newton's rings

A monochromatic ray QR is incident on the plane surface of the plano-convex lens L. RS represents the refracted ray. At the glass–air interface, a portion gets reflected and comes out of the lens in the form of ray 1. The portion transmitted at point S gets reflected at point T on the top surface of the glass plate and finally comes out of the lens as ray 2. Since ray 2 results due to reflection at an air (rarer)–glass (denser) interface, it undergoes a phase change of π. The rays are coherent and produce an interference pattern. The rays are coherent and produce an interference pattern in the form of alternate bright and dark concentric circular rugs known as *Newton rings*.

The effective path difference between rays 1 and 2 is given by expression (1.71), which is valid for the interference pattern obtained with films of variable thickness, namely

$$(\text{PD})_{\text{eff}} = 2\mu t\cos(r+\theta)+\frac{\lambda}{2} \tag{1.82}$$

If the plano-convex lens has a large radius of curvature, the angle θ is extremely small and can be neglected. Equation (1.82) can then be written as follows:

$$(\text{PD})_{\text{eff}} = 2\mu t\cos r+\frac{\lambda}{2} \tag{1.83}$$

For an air film, $\mu = 1$ and, for normal incidence, $r = 0$. Under these conditions, Eq. (1.83) reduces to the following form:

$$(\text{PD})_{\text{eff}} = 2t+\frac{\lambda}{2} \tag{1.84}$$

Newton's Rings by Reflected Light

Figure 1.8 shows the curved surface of the lens as a part of a circle with centre C_1.

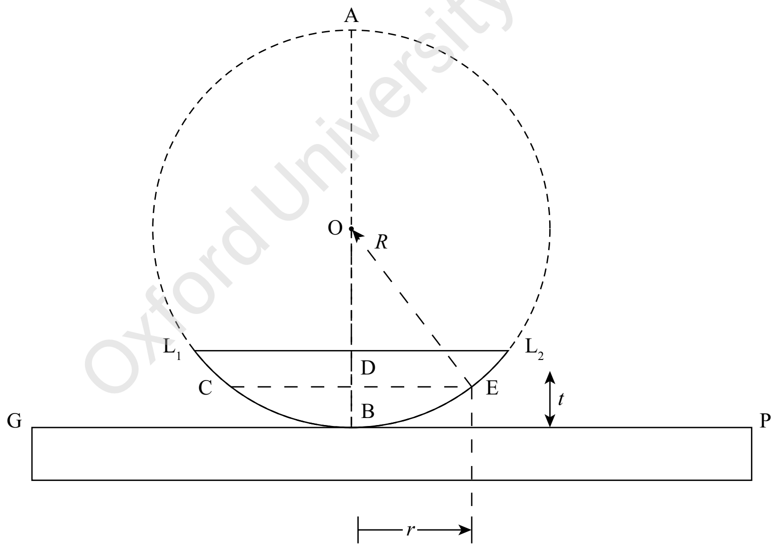

Fig. 1.8 Schematic of curved surface of lens in contact with plane glass plate

L_1BL_2 represents the lens placed on the glass plate GP. The curved surface L_1BL_2 of the lens is part of the spherical surface shown as a dotted circle with centre O in Fig. 1.8. Let R represent the radius of curvature and r the radius of the Newton's ring corresponding to the constant film thickness t.

Using Eq. (1.84), for the nth bright fringe, we have the following relation:

$$2t+\frac{\lambda}{2} = n\lambda$$

which leads to the following form:

$$2t = (2n-1)\frac{\lambda}{2} \tag{1.85}$$

where, $n = 1, 2, 3, \ldots$
For the nth dark ring, we have

$$2t = n\lambda \tag{1.86}$$

where, $n = 0, 1, 2, 3, \ldots$
From the property of the circle, for the circle shown in Fig. 1.7, we can write the following expression: NP × NQ = NO × ND
Substituting values, we get the following equation:

$$r \times r = t(2R - t) = 2Rt - t^2 \approx 2Rt$$

which gives the following relation:

$$r^2 = 2Rt \quad \text{or} \quad t = \frac{r^2}{2R} \tag{1.87}$$

Using Eqs (1.85) and (1.86), we get the following expression for a bright ring:

$$2 \cdot \frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

yielding $r^2 = \dfrac{(2n-1)\lambda R}{2}$

Substituting $r = \dfrac{D}{2}$, we get the following equation: $\dfrac{D^2}{4} = \dfrac{(2n-1)\lambda R}{2}$
which yields the following expression:

$$D = \sqrt{(2\lambda R)(2n-1)} \tag{1.88}$$

or $$D \propto \sqrt{2n-1} \tag{1.89}$$

Thus, diameters of bright rings are proportional to the square roots of the odd numbers $(2n-1)$.
Using Eqs (1.86) and (1.87), we get the following relation for the nth dark ring: $2 \times \dfrac{r^2}{2R} = n\lambda$
which leads to the following expression: $r^2 = n\lambda R$

or $$D^2 = 4n\lambda R \tag{1.90}$$

Thus, $$D = 2\sqrt{n\lambda R} \propto \sqrt{n} \tag{1.91}$$

Diameters of dark rings are proportional to the square roots of natural numbers. Figure 1.9 shows a schematic representation of Newton's rings as seen in reflected light.

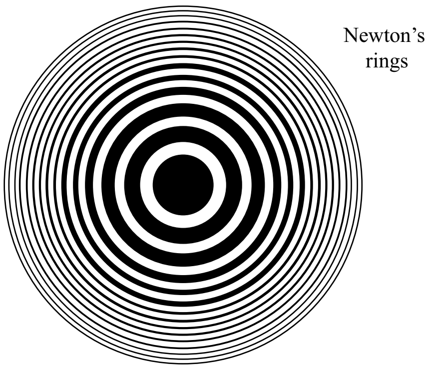

Fig. 1.9 Newton's rings in reflected light

Newton's Rings by Transmitted light

Newton's rings can also be observed in the transmitted light. In this case, for bright rings we have the following condition:

$$2t = n\lambda \tag{1.92}$$

and for dark rings we have the following condition:

$$2t = (2n-1)\frac{\lambda}{2} \tag{1.93}$$

Combining Eqs (1.87) and (1.92), we obtain the following relation for bright rings: $2 \times \frac{r^2}{2R} = n\lambda$

or $\quad r^2 = n\lambda R$

which results in the following equation:

$$D = 2\sqrt{n\lambda R} \propto \sqrt{n} \tag{1.94}$$

Combining Eqs (1.92) and (1.86), we get the following expression for dark rings: $2 \times \frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$

which gives the following relation: $r^2 = \frac{(2n-1)\lambda R}{2}$

Thus, $\quad D = \sqrt{2\lambda R} \times \sqrt{2n-1} \propto \sqrt{(2n-1)} \qquad (1.95)$

The central ring is bright in the transmitted pattern, whereas it is dark in the reflected pattern.

Application: Determination of Wavelength

Let D_n and D_{n+p} represent, respectively, the diameters of the nth and $(n+p)$th dark rings obtained in the reflected pattern. Using Eq. (1.90), we get the following relation:

$$D_n^{\ 2} = 4n\lambda R \tag{1.96}$$

and

$$D_{n+p}^2 = 4(n+p)R\lambda \tag{1.97}$$

From this, we get the following equation:

$$D_{n+p}^2 - D_n^2 = 4pR\lambda \tag{1.98}$$

which results in the following expression:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \tag{1.99}$$

Wavelength λ of monochromatic light (or sodium light) can be determined using Eq. (1.99).

Application: Determination of Refractive Index of a Given Liquid

Suppose a liquid of unknown refractive index μ is used to replace the air film between the lens and the glass plate. If the corresponding diameters of the

nth and $(n+p)$th dark rings are represented by D_n' and $D_{(n+p)}'$, respectively, then we have the following relations:

$$D_n'^2 = \frac{4n\lambda R}{\mu} \tag{1.100}$$

and $$D_{(n+p)}'^2 = \frac{4(n+p)\lambda R}{\mu} \tag{1.101}$$

Using Eqs (1.100) and (1.101), we get the following expression:

$$D_{n+p}'^2 - D_n'^2 = \frac{4p\lambda R}{\mu}$$

or $$\mu = \frac{4p\lambda R}{D_{n+p}'^2 - D_{n'}^2} \tag{1.102}$$

Using Eq. (1.99) in Eq. (1.102), we get the following expression:

$$\mu = \frac{D_{(n+p)}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2} \tag{1.103}$$

Refractive index μ of the liquid can be determined using Eq. (1.103).

Example 1.14 In a Newton's ring set-up, the diameter of the fourth ring was found to be 0.4 cm and that of the 24th ring was 0.8 cm. The radius of curvature of the plano-convex lens is 100 cm. Calculate the wavelength of light used.

Solution Using Eq. (1.99), we get the following relation:

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \tag{1.104}$$

In the given problem, $n+p=24$ and $n=4$. Thus $p=20$.

Substituting these values into Eq. (1.104), we get the following value:

$$\lambda = \frac{(0.8)^2 - (0.4)^2}{(4\times 20\times 100)} = 6\times 10^{-5} \text{ cm}$$

Example 1.15 In a Newton's ring experimental set-up, the diameter of the ninth ring changes from 1.42 to 1.28 cm when a liquid of refractive index μ replaces air in the space between the lens and the plate. Determine the refractive index of the liquid.

Solution From Eq. (1.100), we get the following relation:

$$D_n'^2 = \frac{4n\lambda R}{\mu} \tag{1.105}$$

With the liquid occupying the space, we get the following equation:

$$D_n'^2 = \frac{4\times 9\times \lambda R}{\mu} \tag{1.106}$$

For air as a medium, we have the following expression:

$$D_9^2 = 4\times 9\times \lambda R \tag{1.107}$$

Using Eqs (1.105) and (1.106), we can obtain the following value: $\mu = \frac{D_9^2}{D_9'^2} = \frac{(1.42)^2}{(1.28)^2} = 1.231$.

Example 1.16 In a Newton's ring set-up, the diameter of the third ring has been found to be 0.2 cm and that of the 20th ring 0.7 cm. The radius of curvature of the plano-convex lens is 90 cm. Calculate the wavelength of the light used.

Solution Let us consider the following expression: $\lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}$

Where $n + p = 20$; $n = 3$ giving $p = 17$

Substituting these values, we get the following result:

$$\lambda = \frac{(0.7)^2 - (0.2)^2}{4 \times 17 \times 90} = \frac{(0.49 \times 0.04)}{4 \times 17 \times 90} = 7.35 \times 10^{-5}\ \text{cm}$$

Example 1.17 In a Newton's ring experimental set-up, the diameter of the eighth ring changes from 1.25 to 1.14 cm when a liquid of refractive index μ replaces air in the space between the lens and the plate. Determine the refractive index of the liquid.

Solution The refractive index is $\mu = \dfrac{D_8^2}{D_8'^2} = \dfrac{(1.25)^2}{(1.14)^2} = 1.20$

1.9 INTERFEROMETER

Interferometers are instruments that can be used to study the phenomenon of interference. These instruments can also be used to apply the principle of interference to evaluate some real-life situations. One such evaluation involved the validity of presence of ether using the Michelson–Morley experiment. We will be discussing the construction and working principle of Michelson and Fabry–Perot interferometers in detail in this section. A brief outline of Twyman–Green interferometer will also be presented in this section.

1.9.1 Michelson Interferometer

Michelson interferometer is an optical instrument that was introduced by Albert Michelson in 1881. The instrument is used in investigations where small changes in optical path length is involved. It can be used for accurate comparison of wavelength, measurement of refractive index of gases and transparent solids, and determination of small changes in length. Some useful applications of Michelson interferometer are as follows:

- Experimental evidence of special relativity
- Discovery of hyperfine structure in energy levels of atoms
- Measurement of tidal effects due to moon
- Use of wavelength of light as international standard of meter

Working Principle

The Michelson interferometer uses the principle of division of amplitude to provide interference fringes. Hence, the interference in Michelson interferometer is similar to that in the thin films. An incident beam of light falls on a beam splitter. The beam splitter reflects half the intensity in one direction and transmits other half in another direction. The two beams then travel different optical paths and finally interfere in a common region. The difference in the optical path length decides the characteristics of the formed interference fringes.

Construction

Figure 1.10 gives a basic schematic of Michelson interferometer.

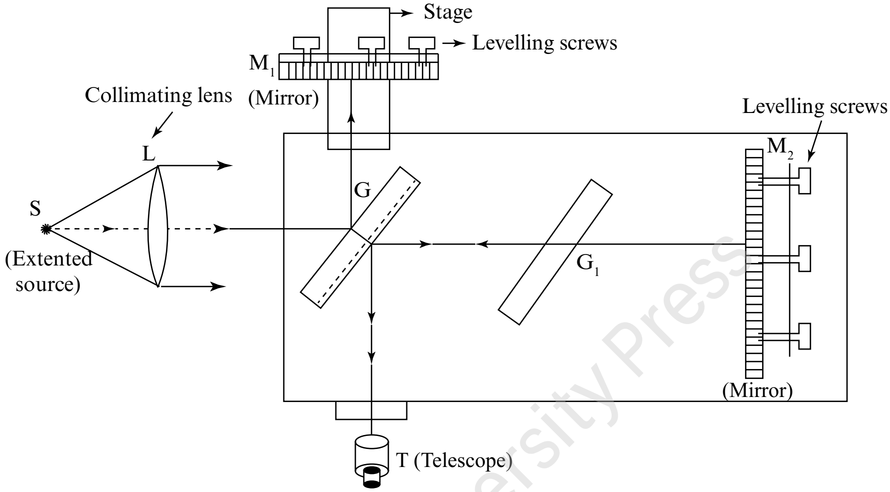

Fig. 1.10 Schematic of Michelson interferometer

The instrument consists of an extended source S and a collimating lens L. Two glass plates G and G_1 are placed at an angle of 45° with respect to the horizontal. The glass plate G is semi-silvered. The apparatus also contains two mirrors M_1 and M_2, each provided with three levelling screws. The mirror M_1 is mounted on a carriage with a provision of very accurate upward and downward movement. The system also has a telescope T to observe the formed interference fringes.

The extended source S emits a monochromatic beam. This monochromatic beam is rendered parallel due to the collimating lens L. This parallel beam is then incident on the semi-silvered glass plate G, which is placed at an angle of 45° with respect to the incident beam. This splits the beam into two parts. One part is reflected from the semi-silvered surface of the glass plate G and moves towards mirror M_1. The other part gets transmitted through the glass plate G and moves towards mirror M_2. Mirrors M_1 and M_2 are held perpendicular to the respective incident beams, which then retrace their original paths, as shown in Fig. 1.10. The reflected rays then meet at the semi-silvered surface of the plate G and finally enter the telescope, where the interference pattern is formed.

A compensating plate G_1 is used to ensure that the optical paths of rays from glass plate G to mirrors M_1 and M_2 are made equal. The mirror M_1 is mounted on a carriage and can therefore be moved through a precise distance, to introduce a desired path difference between the two interfering rays.

To visualize fringe formation, let us imagine that one of the arms of the interferometer is rotated so that the instrument has a single optical axis. The reflections from mirrors M_1 and M_2 is then analogous to reflection from two surfaces with an air gap of thickness 'd'. This is shown schematically in Fig. 1.11.

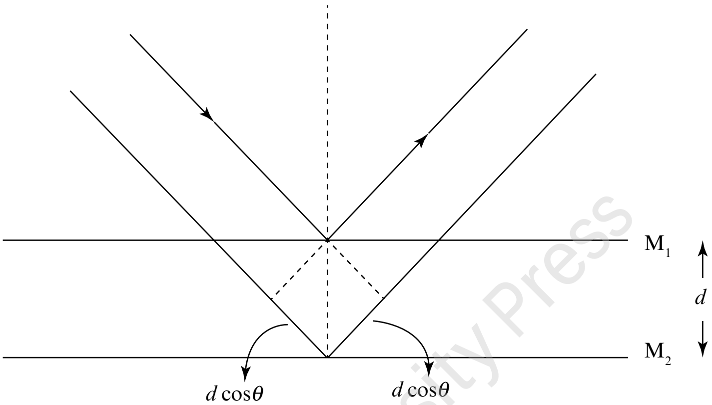

Fig. 1.11 Formation of Fringes in Michelson interferometer

The phase shift introduced due to reflection is same for both the mirrors. The condition for constructive interference is therefore given by,

$$2d\cos\theta = m\lambda \tag{1.108}$$

where m represents the order of interference.

If the two mirrors are aligned perfectly perpendicular to each other, then constant path difference exists over all the regions of the mirrors. The resultant fringe pattern consists of a series of concentric rings. Each ring corresponds to a particular angle of view measured with respect to normal to the mirror M_1. These fringes are therefore called fringes of equal inclination. These fringes are analogous to the interference fringes formed when light from extended source falls on a thin film. As the mirror M_1 is moved in a direction that reduces the path difference to zero, fringe pattern collapses and the fringes disappear.

1.9.2 Fabry–Perot Interferometer

The Fabry–Perot interferometer was invented in 1897 by Charles Fabry and Alfred Perot. It is also known as an etalon. In Fabry–Perot interferometer, light transmitted through two partially reflecting mirrors interfere to produce an interference pattern. On the other hand, the interference pattern in Michelson interferometer is formed by the reflected light. Fabry–Perot interferometer is widely used in the fields of telecommunication, lasers, and spectroscopy to control and determine the wavelengths of light.

Working Principle

An incident ray undergoes multiple reflections and therefore results in a series of parallel transmitted rays. This results in the formation of circular interference fringes. These fringes are sharp and easy to observe and analyze.

Construction

The Fabry–Perot interferometer consists of two plane parallel glass plates C and D separated by a fixed distance as shown schematically in Fig. 1.12.

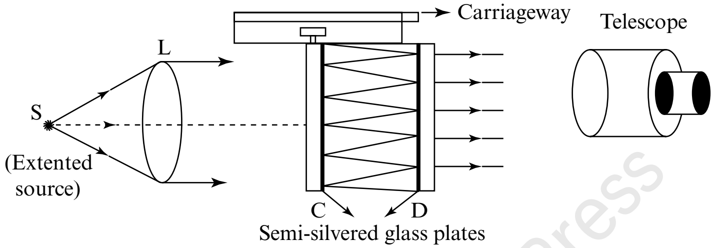

Fig. 1.12 Schematic of Fabry–Perot interferometer

The inner surfaces of the two glass plates are silvered resulting in 80% to 90% reflection. One of the glass plates is attached to a carriageway that enables it to be moved in a precise manner perpendicular to its plane. The separation t between the two plates can thus be adjusted. Light from the extended source, S, is rendered parallel by the collimating lens. Multiple reflections then take place within the two plates, C and D. This results in the formation of circular fringes of equal inclination in the optical plane of the objective of the observing telescope.

1.9.3 Twyman–Green Interferometer

It is a variant of Michelson interferometer was introduced by Frank Twyman and Arthur Green in 1961. The Twyman–Green interferometer uses a monochromatic point source instead of the extended light source, which is used in the Michelson interferometer. This interferometer also uses the principle of division of amplitude for production of interference fringes. This interferometer is used in determining defects in lenses, prisms, plane mirrors, etc.

Construction

The monochromatic source in Twyman–Green Interferometer is kept at the principal focus of a well-corrected lens. The complete schematic of the instrument is shown in Fig. 1.13.

The two mirrors M_1 and M_2 are held perpendicular to each other whereas the beam splitter makes an angle of 45° with respect to the normal to each mirror. By adjusting the portion of mirror M_1, a path difference can be introduced between the two interfering beams. If the path difference is given by $\frac{m\lambda}{2}$ then constructive interference takes place whereas if the path difference

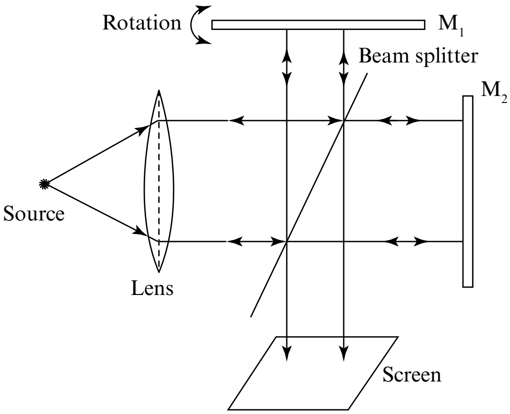

Fig. 1.13 Schematic representation of Twyman–Green interferometer

satisfies the condition $\left(m+\frac{1}{2}\right)\frac{\lambda}{2}$, then complete destructive interference takes place.

The mirror M_1 can also be rotated. This rotation leads to the formation of fringes of equal thickness on the screen. To test optical components, fringes of equal thickness are first obtained by tilting the mirror M_1. The component to be tested is then placed in the path of one of the arms of interferometer. The change in the fringe pattern in used to determine the optical quality of the component.

Note: **Other Interferometers**

Mach-Zehnder Interferometer It can be considered as a variant of the Michelson/Twyman–Green interferometers. It produces an interference pattern with the light only making a single-pass through the sample. It has relatively large and freely accessible working space and flexibility in the location of the fringes. This interferometer is used an important diagnostic tool. It is frequently used in the fields of plasma physics, aerodynamics, and heat transfer to measure density, pressure, and temperature changes in gases.

Fizeau interferometer It was developed by Hippolyte Fizeau. It is a variation of Fabry–Perot, but it is generally easier to use. It is widely used for doing optical and engineering measurements.

IMPORTANT CONCEPTS

1. When two or more waves travel simultaneously through a medium, the resultant displacement at any point of the medium is given by the vector sum of the displacements of the individual waves. This is called the principle of superposition of waves.
2. Two waves are coherent if they have a constant phase between them and also have the same frequency.

3. If the phase difference between any two points along the direction of propagation is independent of time, then the wave is said to be temporarily coherent.
4. If the phase difference between any points located transverse to the direction of propagation is independent of time, then the wave is said to be spatially coherent.
5. Constructive interference occurs due to the superposition of two waves at a point such that the crest of one wave falls on the crest of the other, that is, the path difference between two waves is an integral multiple of the wavelength ($n\lambda$). The intensity is maximum at these points (n is an integer or zero).
6. Destructive interference occurs due to the superposition of two waves at a point such that the crest of one wave falls on the trough of the other, that is, the path difference between the two waves is $\left(n+\frac{1}{2}\right)\lambda$, where λ is the wavelength and n is an integer or zero. The intensity is minimum at these points.
7. The phenomenon of interference requires two wavefronts to interact. These wavefronts can be obtained in two different ways, resulting in two different types of interference: (a) division of wavefront and (b) division of amplitude.
8. Newton's rings are observed when monochromatic light is incident on a film formed between a plano-convex lens and a plane surface.

APPLICATIONS

1. Many optical coatings use optical interference to deliver specific properties. One important example is the use of antireflection coatings. Destructive interference of reflected rays ensures the absence of chosen wavelengths in the reflected light. A similar principle is also used to fabricate narrow-bandpass or band-reject filters. These filters are extensively used in optical systems.
2. Another interesting application of destructive interference can be observed in noise cancelling headphones. These headphones have an inbuilt mechanism and circuitry, which produce their own sound waves that imitate the incoming noise in every respect, except that the sound waves produced by the headphone circuitry is 180° out of phase with the intruding waves.

IMPORTANT FORMULAE

1. A wave is represented as follows:
$$\psi = A\sin\omega t$$
2. $I = A_1^2 = nA^2$
3. $\Delta L = c\,\Delta t$
4. $\delta = \dfrac{2\pi x}{L}$
5. $A^2 = \psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}\cos\delta$
$$\tan\theta = \frac{\psi_{20}\sin\delta}{\psi_{10} + \psi_{20}\cos\delta}$$
6. $I_{\max} = k\left(\psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}\right)$
7. $I_{\min} = k\left(\psi_{10}^2 + \psi_{20}^2 - 2\psi_{10}\psi_{20}\right)$
8. For constructive interference:
$$\text{PD} = 0, \ldots, n\lambda$$
9. For destructive interference:
$$\text{PD} = \left(n+\frac{1}{2}\right)\lambda$$
10. $\dfrac{I_{\max}}{I_{\min}} = \dfrac{(\psi_{10}+\psi_{20})^2}{(\psi_{10}-\psi_{20})^2}$
11. $\beta = \text{fringe width} = \dfrac{\lambda D}{2d}$
12. For interference in thin films:
$$2\mu t\cos r = n\lambda$$
13. For Newton's rings, for the nth dark ring in reflected light:
$$D^2 = 4n\lambda R$$

For nth bright ring in reflected light:

$$D = \sqrt{(2\lambda R)(2n-1)}$$

14. For Newton's rings:

$$\mu = \frac{D^2_{(n+p)} - D^2_n}{D'^2_{(n+p)} - D'^2_n}$$

SELF-ASSESSMENT

Multiple-choice Questions

1.1 A travelling disturbance is represented by

(a) $\psi = A\sin(\omega t + \delta)$ (b) $\psi = A\sin^2(\omega t + \delta)$

(c) $\psi = A\cos(\omega t + \delta)$ (d) $\psi = A\cos^2(\omega t + \delta)$

1.2 For superposition of n waves of equal amplitude, we have

(a) $I = n^2A^2$ (b) $I = n^3A^2$ (c) $I = nA^2$ (d) $I = nA$

1.3 Coherence length and coherence time are related through the expression

(a) $\Delta L = \frac{c}{\Delta t}$ (b) $\Delta L = c^2\Delta t$ (c) $\Delta L = (c\Delta t)^2$ (d) $\Delta L = c\Delta t$

1.4 For constructive interference, we have

(a) $I_{\max} \propto (\psi^2_{10} + \psi^2_{20} + 2\psi_{10}\psi_{20})$ (b) $I_{\max} \propto (\psi^2_{10} + \psi^2_{20} - 2\psi_{10}\psi_{20})$

(c) $I_{\max} \propto (\psi^2_{10} + \psi^2_{20})$ (d) $I_{\max} \propto (\psi^2_{10} - \psi^2_{20})$

1.5 Linear distance between two consecutive fringes is given by

(a) $\frac{\lambda(2d)}{D}$ (b) $\frac{\lambda D}{d}$ (c) $\frac{\lambda D}{2d}$ (d) λDd

1.6 For interferences in thin films, the condition for bright fringe is

(a) $2\mu t\cos r = n\lambda$ (b) $2\mu t\cos r \pm \frac{\lambda}{2} = n\lambda$

(c) $2\mu t\cos r \pm \lambda = n\lambda$ (d) $2\mu t\cos r - \lambda = n\lambda$

1.7 Dark rings in a Newton's ring set-up obey the relation

(a) $D \propto \sqrt{n}$ (b) $D \propto \sqrt{2n}$ (c) $D \propto \sqrt{(2n-2)}$ (d) $D \propto \sqrt{(2n-1)}$

1.8 Intensity of a travelling disturbance with amplitude B is proportional to

(a) B (b) $\frac{1}{B}$ (c) $\frac{1}{B^2}$ (d) B^2

1.9 Two overlapping waves produce a stable interference pattern; their amplitudes must be

(a) vastly different (b) equal (c) comparable (d) unrated

1.10 Condition for temporal coherence is

(a) $\delta_2 - \delta_1 = \delta_2' + \delta_1'$ (b) $\delta_2 - \delta_1 = \delta_2' - \delta_1'$

(c) $\delta_2 - \delta_1 = 2(\delta_2' - \delta_1')$ (d) $\delta_2 - \delta_2 = 2(\delta_1' - \delta_2')$

1.11 Coherence time for a perfectly monochromatic sinusoidal wave is

(a) infinity (b) 0 (c) 1 (d) 2

1.12 A path difference of one half-wavelength introduces a phase difference of

(a) 2π (b) π (c) $\frac{3\pi}{2}$ (d) $\pi/2$

1.13 Constructive interference will not take place for a phase value equal to

(a) 0 (b) 2π (c) π (d) 4π

1.14 In a Young's double-slit experiment, the position of bright fringes is given by

(a) $x = \frac{ndD}{2\lambda}$ (b) $x = \frac{ndD}{\lambda}$ (c) $x = \frac{n\lambda d}{D}$ (d) $x = \frac{n\lambda D}{2d}$

1.15 When light gets reflected from a denser medium, it suffers a phase change of

(a) 2π (b) $\pi/2$ (c) π (d) 3π

1.16 In an interference pattern produced by identical coherent sources of monochromatic light, the intensity at the site of central maximum is I. If intensity at the same spot when either of the two slits is closed is I_0, we must have the condition that

(a) $I = I_0$ (b) $I = 2I_0$
(c) $I = 4I_0$ (d) I and I_0 are not related

1.17 What happens when monochromatic light used in Young's slit experiment is replaced by white light?

(a) Bright fringes become white.
(b) The central fringe is white and all other are coloured.
(c) All fringes are coloured.
(d) No fringes are observed.

1.18 A path difference of $3\pi/2$ between two waves corresponds to a phase difference of

(a) $3\pi/2$ (b) $\pi/3$ (c) 3π (d) $2\pi/3$

1.19 Newton's ring experiment is based on

(a) division of amplitude (b) division of wavefront
(c) none of these (d) combination of (a) and (b)

Review Questions

1.1 What is the difference between temporal coherence and spatial coherence?

1.2 If the amplitudes of two coherent light waves are in the ratio 1:4, find the ratio of maximum to minimum intensity in the interference pattern.

1.3 Find an expression for the intensity distribution when two sinusoidal coherent waves with amplitudes A_1 and A_2 and a phase difference of ϕ superpose to produce interference.

1.4 Find an expression for the fringe width in the interference pattern of Young's double-slit experiment.

1.5 Two independent sources of light of the same wavelength cannot produce interference. Justify.

1.6 Explain why an extended source of light is required for fringes in a Newton's ring experiment. When white light is used in place of a monochromatic light, what change is expected?

1.7 Can you measure the refractive index of a liquid by Newton's ring experiment? Explain.

1.8 Explain interference of light due to thin films.

1.9 Explain the principle of superposition of waves.

1.10 Derive an expression for interference in thin films due to reflection.

1.11 Explain why a convex lens is placed between a monochromatic light source and a microscope while performing experiments on Newton's rings.

1.12 Describe in detail, with the necessary theory, an experiment to determine the refractive index of a transparent liquid using Newton's rings.

1.13 Why are very narrow slits used in Young's double-slit interference experiment?
1.14 Describe, with the necessary equation, how you will determine the refractive index of water using Newton's ring apparatus.
1.15 With the help of a suitable ray diagram, describe the production of Newton's rings.
1.16 What is a coherent source? Explain the different methods used to obtain coherent sources.
1.17 Prove that the diameter of the *n*th dark ring in a Newton's ring set-up is directly proportional to the square root of the ring number.
1.18 Describe the origin of colour on a thin film, along with the derivation of constructive and destructive conditions.
1.19 What is interference?
1.20 Show that for *n* interfering waves, $I = A_1^2 = nA^2$.
1.21 What is coherence?
1.22 How many types of coherence are generally observed?
1.23 Define coherence time and coherence length.
1.24 Show that $I_{\max} = k\left(\psi_{10}^2 + \psi_{20}^2 + 2\psi_{10}\psi_{20}\right)$
1.25 Derive the following expression: $\dfrac{I_{\max}}{I_{\min}} = \dfrac{\left(\psi_{10} + \psi_{20}\right)^2}{\left(\psi_{10} - \psi_{20}\right)^2}$.
1.26 Derive an expression for fringe width for a Young's double-slit experiment.
1.27 Derive the expression $2\mu t \cos r = n\lambda$ for interference patterns observed in thin films.
1.28 Show that for interference from wedge-shaped thin films, the following relation holds: $\beta = \dfrac{\lambda}{2\sin\theta} \approx \dfrac{\lambda}{2\theta}$
1.29 Describe a set-up that can be used to observe Newton's rings.
1.30 Show that for the *n*th bright ring in a Newton's ring set-up in reflected light the diameter is given by the following expression: $D = \sqrt{(2\lambda R)(2n-1)}$
1.31 Show that for Newton's rings, $\mu = \dfrac{D_{(n+p)}^2 - D_n^2}{D_{(n+p)}^{'2} - D_n^{'2}}$
1.32 What are Interferometers?
1.33 Draw a schematic of a Michelson interferometer.
1.34 Explain the working principle of a Michelson interferometer.
1.35 Explain the working principle of a Fabry–Perot interferometer

Numerical Problems

1.1 Two coherent sources whose intensity ratio is 49:1 produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity.

$$\left[\textit{Hint}\text{: } \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\right]$$

1.2 Determine the ratio of intensity of the centre of a bright fringe to the intensity found at a point $\dfrac{1}{6}$ of the distance between two fringes from the centre.

$$\left[\textit{Hint}\text{: } I = 2\psi_{10}^2(1 + \cos\delta)\right]$$

1.3 Two straight and narrow parallel slits 0.8 mm apart are illuminated using a monochromatic light source. A screen placed at a distance of 100 cm is used to obtain fringes. It is found that the distance between consecutive fringes is 0.5 mm. Determine the wavelength of light.

$$\left[\textit{Hint}\text{: } \beta = \frac{\lambda D}{2d}\right]$$

1.4 Two coherent sources are placed 1.2 mm apart, which generate interference fringes on a screen 0.9 m away. The second dark fringe is formed at a distance of 1 mm from the central fringe. Calculate the wavelength of the monochromatic light used. $\left[Hint: x_n = \dfrac{(2n+1)\lambda D}{4d}\right]$

1.5 In Young's experiment, let light of wavelengths 5×10^{-7} m and 8×10^{-8} m be used in turn, keeping the geometry same. Compare the fringe width in the two cases. $\left[Hint: \beta = \dfrac{\lambda D}{2d}\right]$

1.6 In a thin film, between points A and B, six fringes are seen with a light of wavelength 5400 Å. If the light used is of wavelength 4100 Å, what are the number of fringes obtained between A and B? $[Hint: 2\mu t \cos r = n\lambda]$

1.7 A soap film has a refractive index of 4/3 and is 2×10^{-4} cm thick. It is illuminated by white light incident at an angle of 45°. On examining the reflected light using a spectroscope, a dark band is found corresponding to a wavelength of 6×10^{-5} cm. Calculate the order of the interference band. $[Hint: 2\mu t \cos r = n\lambda]$

1.8 White light is incident normally on a soapy water film of thickness 5.5×10^{-5} cm and $\mu = 1.35$. Determine any wavelength that is reflected strongly in the visible region. $[Hint: 2\mu t \cos r = (2n + 1) \lambda/2]$

1.9 In a Young's double-slit experiment, the slits are separated by a distance of 0.3 mm and the screen is placed 1.42 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.1 cm. Calculate the wavelength of light. $\left[Hint: \beta = \dfrac{\lambda D}{2d}\right]$

1.10 In a Newton's ring set-up, the diameter of the eighth ring has been found to be 0.42 cm and that of the 25th ring 0.84 cm. The radius of curvature of the plano-convex lens is 95 cm. Determine the wavelength of the light used. $\left[Hint: \lambda = \dfrac{D_{n+p}^2 - D_n^2}{4pR}\right]$

1.11 In a Newton's ring experimental set-up, the diameter of the eighth ring changes from 1.35 to 1.17 cm when a liquid of refractive index μ replaces air in the space between the lens and the plate. Calculate the refractive index of the liquid. $\left[Hint: D_8'^2 = \dfrac{4 \times 8 \times \lambda R}{\mu}\right]$

1.12 Two coherent sources have their intensities in the ratio 81:9. An interference pattern is obtained using these two sources. Calculate the ratio of maximum intensity to minimum intensity. $\left[Hint: \dfrac{I_{max}}{I_{min}} = \dfrac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\right]$

1.13 The wavelength of the monochromatic light source in Problem 1.4 is changed to 6000 Å. Calculate the new distance of the second dark fringe from the central fringe. $\left[Hint: x_n = \dfrac{(2n+1)\lambda D}{4d}\right]$