

PRINCIPLES OF Basic Electrical Engineering

As per the latest AICTE syllabus

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Preface

The textbook *Principles of Basic Electrical Engineering* has been developed based upon the Model Curriculum of UG Courses in Engineering and Technology (Jan. 2018) of AICTE. The book fulfils the needs of the core course in Basic Electrical Engineering (ESC 101) common to all courses at the first year level. The centre of the text is our hallmark and internationally popular text *Basic Electrical Engineering* (Third edition).

About the Book

The book has been structured into six modules, each comprising various chapters. In writing the text, lucidity in explaining the theory and applications of various laws in electrical engineering has been our prime focus. Along with the numerous typical solved examples and recapitulation, the readers (both faculty and students) will discover understanding the principles of electrical engineering a joyful experience.

Key Features

- The contents in each chapter have been selected and developed according to the revised AICTE syllabus.
- Learning objectives and overview at the beginning of each chapter provide synopsis of the contents.
- Applications of the various principles and laws have been amplified through innumerable solved examples.
- Chapter-end assessment questions, multiple choice objective questions and unsolved problems with answers will equip the students to meet the challenges of the examinations.
- Finally, the smooth and easy flow of subject writing will be conducive for easy understanding of concepts.

Structure of the Book

Module 1 (8 hours): DC Circuits - consists of two chapters. Chapter 1 provides an introduction to electrical engineering and discusses the basics of electricity, electrical quantities, elements, and various associated laws governing the principles of electrical engineering. Chapter 2 on network analysis and network theorems after introducing circuit elements (R, L, and C) defines current laws and explains the analysis of DC circuits with the help of theorems along with time domain analysis of first order RL and RC circuits.

Module 2 (8 hours): AC Circuits- Chapter 3 after defining ac quantities, such as peak, rms, phasor representation, power factor, etc. explains the techniques for analyzing single phase (series and parallel) ac circuits. Current and voltage relations in star-delta connections in balance three phase circuits are also explained.

Module 3 (6 hours): Transformers - Chapter 4 on magnetic circuits commences with the definition of Biot–Savart law and based on dipole movement of electrons, the characteristics and properties of magnetic materials are explained. Mesh analysis technique is explained for solving magnetic circuits and finally application of dot convention to determine the precise direction of statically induced emf in coupled circuits are explained. Chapter 5 is on transformer principles and begins with a description of the constructional features and goes on to explain the principles of operation and develops an equivalent circuit of a transformer. OC and SC tests have been described along with their application in calculating efficiency, regulation, etc.

Module 4 (8 hours): Electrical Machines- Chapter 6 on synchronous machines discusses the constructional details of the stator and rotor of a synchronous machine and goes on to explain, and conceptualizes the setting up of a synchronously rotating magnetic field using graphic and phasor representation. Development of a phasor diagram and equivalent circuit of a synchronous generator has also been included. Chapter 7 - induction motors - describes in detail the construction and principle of operation of three-phase induction motors. It also

discusses the various methods of starting of motors. Chapter 8 on DC machines discusses the development of electromagnetic torque and explains commutator action. It also elucidates armature reaction leading to cross and demagnetization mmf. Additionally, field applications of dc machines have also been included.

Module 5 (6 hours): Power Converters - Chapter 9 explains the need of power converters in present-day electronic devices and classifies the same, based on transformation of DC sources. After providing a bird's eye view of various power semiconductor devices, the working of rectifier and inverter circuits is explained. The three basic types of DC–DC converter circuits have also been described. Chapter 10 on inverters provides a treatise on modern-day electronic products. After providing an introduction to pulse-width modulation schemes for inverters, sinusoidal pulse width modulation has been explained. Working of single-phase half wave, single-phase full bridge, and three-phase inverters have been discussed with the help of circuits.

Module 6 (6 hours): Electrical Installations- This chapter describes the different types of domestic wiring including staircase lighting. Various types of switchgear for LV/LT systems used in energy distribution have been described along with a note on wires and cables. Electricity tariffs charged to cost energy for domestic users are explained. Finally, a detailed description of the different types of earthing methods is also included.

Acknowledgement

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T. K. Nagsarkar
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INTRODUCTION TO ELECTRICAL ENGINEERING

1

Learning Objectives

This chapter will enable the reader to

- Understand the nature of structure of an atom and significance of free electrons
- Differentiate between conductors, semiconductors, and insulators based upon the energy levels of electrons
- Compute the resistance of a conductor from its physical dimensions at different temperatures
- Familiarize with electrostatic phenomena associated with electric charges and define electric field intensity, electric potential and potential difference, electric flux, and electric flux density
- Get familiar with basic electrical quantities: current, voltage, emf, and electric power
- Define Ohm's law for a resistor and compute the resistance of a conductor from its physical dimensions at different temperatures
- Compute the induced voltage due to varying current, power, and energy stored in an inductor
- Based upon an understanding of the charge storing nature of a capacitor, compute its capacitance, current, power, and energy stored
- Define Ampere's law and use it to estimate the force on a current carrying conductor when placed in a magnetic field, and use Fleming's left hand rule to determine direction of the force
- Use Faraday's laws of electromagnetic induction to compute the magnitude of dynamic or static induced voltage and apply Fleming's right hand rule or Lenz's law to determine the direction of the induced voltage
- Define Kirchhoff's voltage and current laws and apply these to compute currents and voltages in a circuit made up of resistors, inductors, and capacitors

1.1 ESSENCE OF ELECTRICITY

It is believed that electricity is present in nature. It is amazing how humankind has been able to put electricity to myriad uses for its own progress and comfort without having an exact knowledge of the nature of electricity. In fact, based on experimentation and observations, theories have been developed to explain the behaviour of electricity.

Electrical energy has been accepted as a form of energy that is most suited for transformation into other forms of energy, such as heat, light, mechanical energy, etc. Electricity can be converted into many different forms to bring about new and enabling technologies of high value. Conversion of electrical energy into pulses and electromagnetic waves has given rise to computers and communication systems. Its conversion into microwaves finds use in microwave ovens, industrial processes, and radars. Electricity in the arc form serves in arc furnaces and welding. Efficient lighting, lasers, visuals, sound, robots, medical tools are among many other examples of the use of electricity.

Electrical engineering deals with the generation, transmission, utilization, and control of electric energy. Electric energy is generated at electric power generating stations such as hydroelectric, thermal, and nuclear power stations. In a hydroelectric power station, the potential energy of the head of water stored in dams is converted

into kinetic energy by regulating the flow of the stored water through turbines. This kinetic energy, in turn, gets transformed into electric energy by the process of electro-mechanical energy conversion. In a thermal station, the chemical energy of coal, oil, natural gas, and synthetic derivatives is converted by combustion into heat energy. Heat energy is also produced by nuclear fission of nuclear fuels in a nuclear reactor. It is then converted into mechanical energy, which in turn is transformed by electro-mechanical energy conversion to electric energy, through thermodynamic processes. Conversion of limitless energy from the sun into usable electric energy through photovoltaic energy conversion is achieved by using solar cells. Commercially, electricity is also being generated from renewable energy sources such as wind, biomass, and geothermal sources. Wind energy is converted into electrical form through a wind turbine coupled to an electrical generator. Geothermal power generation converts energy contained in hot rocks into electricity by using water to absorb heat from rocks and transport it to the earth's surface, where it is converted into electric energy through turbine generators. The majority of biomass electricity is generated using a steam cycle where biomass material is first converted into steam in a boiler; the resultant steam is then used to turn a turbine connected to a generator.

Electricity permits the source of generation to be remote from the point of application. Electric energy transmission systems are varied, such as power transmission systems and electronic communication systems. Electric energy for conversion into light energy, heat energy, and mechanical energy for use in industries, commercial establishments, and households would require bulk transmission of electric power from the source, which produces energy, to the load centre, where the electric energy is utilized. Electrical power transmission systems consist of chains of transmission towers on the earth's surface, from which the line conductors carrying current are suspended by porcelain insulators.

An electric system may be viewed as consisting of generating devices, transformers, and transmission systems which interconnect terminal equipment for converting electrical energy into light, heat, or mechanical energy and vice versa. All devices and equipment can be represented by idealized elements called circuit elements. These elements can be interconnected to form networks, which can be used for modelling and analysing the system behaviour. Conversely, networks may be designed to achieve the required performance from a system.

Electrical engineering is concerned with the study of all aspects of electric power, i.e., its generation, transmission, and utilization. Therefore, it is necessary to become familiar with the basic concepts and terms associated with electricity.

1.2 ATOMIC STRUCTURE AND ELECTRIC CHARGE

Atom is the smallest particle of an element. As per Bohr–Rutherford's planetary model of atom, the mass of an atom and all its positive charge is concentrated in a tiny nucleus, while negatively charged electrons revolve around the nucleus in elliptical orbits like planets around the sun (see Fig. 1.1). The nucleus contains protons and neutrons. A neutron carries no charge and its mass is 1.675×10^{-27} kg, while a proton carries a positive charge $+e$ and its mass is 1.672×10^{-27} kg. The electron carries a negative charge $-e = 1.602 \times 10^{-19}$ C and its mass is 9.109×10^{-31} kg. Thus an electron is lighter than a proton by a factor of about 1840. There are exactly as many protons in the nucleus of an atom as planetary electrons. Thus, the nucleus of an atom can be viewed as a core carrying a positive charge, and the negative charge of the encircling electrons is equal to the positive charge of protons.

An atom as a whole is electrically neutral. The orbits for the planetary electrons are called shells or energy levels. The electrons in successive shells named *K, L, M, N, O, P,*

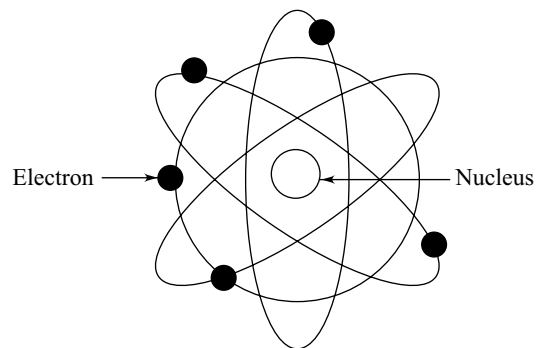


Fig. 1.1 Structure of an atom

and Q are at increasing distance outwards from the nucleus. Each shell has a maximum number of electrons for stability. For most elements, the maximum number of electrons in a filled inner shell equals $2n^2$, where n is the shell number in sequential order outward from the nucleus. Thus the maximum number of electrons in the first shell is 2, for the second shell is 8, for the third shell is 18, and so on. These values apply only to an inner shell that is filled with its maximum number of electrons. To illustrate this rule, a copper atom with 29 electrons is chosen. In this case, the number of electrons in the K , L , M , and N is 2, 8, 18, and 1, respectively.

1.3 CONDUCTORS, SEMICONDUCTORS, AND INSULATORS

As stated in the preceding section, electrons revolve in orbits around the nucleus. The electrons closer to the nucleus possess lower energies than those further from it, which is very much similar to a mass m possessing increasing potential energy as its distance above the earth's surface increases. Thus the position occupied by an electron in an orbit signifies a certain potential energy. Due to the opposite charge, there is a force of attraction between the electron and the nucleus. The closer an electron is to the nucleus, more strongly it is bound to the nucleus. Conversely, further away an electron is from the nucleus, lesser is the force of attraction between the electron and the nucleus. Since the bond between the outer electrons and the nucleus is weak, it is easy to detach such an electron from the nucleus.

When many atoms are brought close together, the electrons of an atom are subjected to electric forces of other atoms. This effect is more pronounced in the case of electrons in the outermost orbits. Due to these electric forces, the energy levels of all electrons are changed. Some electrons gain energy while others lose it. The outermost electrons suffer the greatest change in their energy levels. Thus the energy levels, which were sharply defined in an isolated atom, are now broadened into energy bands. Each band consists of a large number of closely packed energy levels. In general, two bands result, namely, the conduction band associated with the higher energy level and the valence band. A region called forbidden energy gap separates these two bands. Each material has its own band structure. Band structure differences may be used to explain the behaviour of conductors, semiconductors, and insulators.

In metals, atoms are tightly packed together such that the electrons in the outer orbits experience small, but significant, force of attraction from the neighbouring nuclei. The valence band and the conduction band are very close together or may even overlap. Consequently, by receiving a small amount of energy from external heat or electric sources the electrons readily ascend to higher levels in the conduction band and are available as electrons that can move freely within the metal. Such electrons are called free electrons and can be made to move in a particular direction by applying an external energy source. This movement of electrons is really one of negative electric charge and constitutes the flow of electric current. In metals the density of electrons in the conduction band is quite high. Such metals are categorized as conductors. In general metals are good conductors, with silver being the best and copper being the next best.

In semiconductors the valence and conduction bands are separated by a forbidden gap of sufficient width. At low temperatures, no electron possesses sufficient energy to occupy the conduction band and thus no movement of charge is possible. At room temperatures it is possible for some electrons to gain sufficient energy and make the transition to the conduction band. The density of electrons is not as high as in metals and thus cannot conduct electric current as readily as in conductors. Carbon, germanium, and silicon are semiconductors conducting less than the conductor but more than the insulators.

A material with atoms that are electrically stable, that is, with the outermost shell complete, is an insulator. In such materials the forbidden gap is very large, and as a result the energy required by the electron to cross over to the conduction band is impractically large. Insulators do not conduct electricity easily, but are able to hold or store electricity better than conductors. Insulating materials such as glass, rubber, plastic, paper, air, and mica are also called dielectric materials.

1.4 ELECTROSTATICS

Electrostatics is associated with materials in which electrical charge moves only slowly (insulating materials) and with electrically isolated conductors. Charges are static as insulation and isolation prevent easy migration of charge. Electrostatic phenomena arise from the forces that electric charges exert on each other. There are many examples as simple as the attraction of the plastic wrap to one's hand after it is removed from a package, to the operation of photocopiers.

Electrostatics involves the buildup of charge on the surface of objects due to contact with other surfaces. Although exchange of charge happens whenever any two surfaces contact and separate, the effects of charge exchange are usually noticed only when at least one of the surfaces has a high resistance to electrical flow. This is because the charges that transfer to or from the highly resistive surface are more or less trapped there for a long enough time for their effects to be observed. These charges then remain on the object until they either bleed off the ground or are quickly neutralized by a discharge.

The space surrounding a charged object is affected by the presence of the charge and an electric field is established in that space. A charged object creates an electric field—an alteration of the space or field in the region that surrounds it. Electric field is a vector quantity whose direction is defined as the direction in which a positive test charge would be pushed when placed in the field. Thus, the electric field direction about a positive source charge is always directed away from the positive source. And the electric field direction about a negative source charge is always directed toward the negative source.

1.4.1 Coulomb's Law

Coulomb's law states that the force of attraction or repulsion F , between two charges q_1 and q_2 coulombs, concentrated at two different points in a medium, is directly proportional to the product of their magnitudes and inversely proportional to the square of the distance r between them. Mathematically, it may be expressed as

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2} \text{ newton (or N)} \quad (1.1)$$

where ϵ is the absolute permittivity of the surrounding medium and is given by

$$\epsilon = \epsilon_0 \epsilon_r \quad (1.2)$$

where ϵ_0 is the permittivity of free space and is equal to 8.84×10^{-12} F/m; ϵ_r is the relative permittivity of the medium.

If the charges are of like polarity, the force between them is repulsive, and if the charges are of opposite polarity, the force is attractive.

1.4.2 Electric Field Intensity

When a stationary electric charge is placed within an electrostatic field, it experiences a force of attraction or repulsion depending on the nature of the charge and its position in the field. The ratio of the force exerted on the charge to the magnitude of the charge is defined as the electric field intensity. Thus, if a charge of magnitude q coulomb, when placed within an electric field, experiences a force of F newton, then the electric field intensity E will be given by

$$E = \frac{F}{q} \text{ N/C or V/m} \quad (1.3)$$

The force F experienced by charge q_2 due to the presence of charge q_1 is given by Eq. (1.1). Hence, the field strength at the point where charge q_2 is located will be [from Eq. (1.3)]

$$E = \frac{F}{q_2} \text{ N/C or V/m}$$

Substituting for F from Eq. (1.1) in the above equation yields

$$E = \frac{q_1 q_2}{4\pi\epsilon r^2} \times \frac{1}{q_2} = \frac{q_1}{4\pi\epsilon r^2} \text{ N/C or V/m} \quad (1.4)$$

Example 1.1 Find the force in free space between two like point charges of 0.1 C each and placed 1 m apart.

Solution Using Eq. (1.1), the force may be obtained as

$$F = \frac{0.1 \times 0.1}{4\pi \times 8.84 \times 10^{-12} \times 1} = 9 \times 10^7 \text{ N}$$

It may be noted that the magnitude of the force is gigantic. This calculation shows that 0.1 C of electric charge is a very high value and is normally not encountered in engineering computations.

1.4.3 Electric Potential and Potential Difference

Moving a positive test charge against the direction of an electric field would require work by an external force. This work would in turn increase the potential energy of the charge. On the other hand, the movement of a positive test charge in the direction of an electric field would occur without the need for work by an external force. This motion would result in the loss of potential energy of the charge. Potential energy is the stored energy of position of a charge and it is related to the location of the charge within a field.

The above situation finds an analogy in mechanics where work has to be done against the gravitational force in raising a mass to some height above sea level. The greater the mass, the greater is the potential energy possessed by the mass.

While electric potential energy has a dependency upon the charge experiencing the electric field, electric potential is purely location dependent. It is the potential energy per charge.

The electric potential at any point within an electric field is defined as the amount of work done against the electric field (or the energy required) to bring a unit positive charge from infinity to that point, or alternatively, from a place of zero potential to the point. The unit of potential is volt, and 1 volt is equal to 1 joule/coulomb. An alternate name of this quantity, voltage, is named after the Italian physicist Alessandro Volta.

The potential difference between two points within an electric field is the work done by the field in shifting a unit positive charge from one point to the other. It is to be noted that positive charge always flows from higher potential point to lower potential point, whereas a negative charge flows from a lower potential point to higher potential point.

Both potential and potential difference are scalar quantities as these are position dependent in a field but are not dependent on the path by which the position is reached.

The total work per unit charge associated with the motion of charge between two points is called voltage. If v is the voltage in volts, w is the energy in joules, and q is the charge in coulombs, then

$$v = \frac{dw}{dq} \text{ J/C} \quad (1.5)$$

Example 1.2 Two charges $Q_1 = 2 \times 10^{-9} \text{ C}$ and $Q_2 = 3 \times 10^{-9} \text{ C}$ are spaced 6 m apart in air as shown in Fig. 1.2(a). Derive an expression for the net force on a unit positive charge Q at point A , located at x m from Q_1 . If A and B are respectively located 1 m and 4 m away from the charge Q_1 as shown in Fig. 1.2(b), compute the voltage V_{AB} between the points A and B .

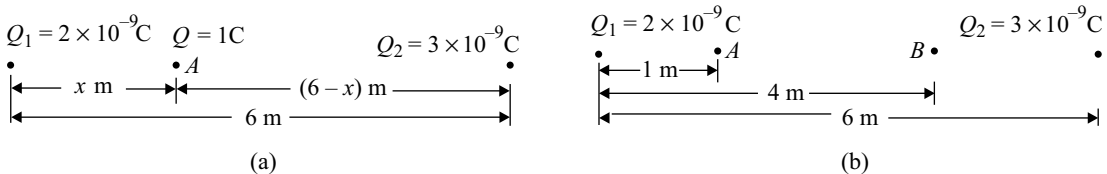


Fig. 1.2

Solution The force of repulsion between Q and Q_1 at point A , directed away from Q_1 , is

$$\frac{1}{4\pi \times 8.84 \times 10^{-12}} \times \frac{2 \times 10^{-9}}{x^2} = \frac{18}{x^2} \text{ N}$$

Similarly, the force of repulsion between Q and Q_2 at point A , directed away from Q_2 , is

$$\frac{1}{4\pi \times 8.84 \times 10^{-12}} \times \frac{3 \times 10^{-9}}{(6-x)^2} = \frac{27}{(6-x)^2} \text{ N}$$

The net force on Q , directed away from Q_2 , is given by

$$F = \frac{27}{(6-x)^2} - \frac{18}{x^2} = 9 \left[\frac{3}{(6-x)^2} - \frac{2}{x^2} \right] \text{ N}$$

The work done in moving Q from point A to point B is given by

$$W_{BA} = \int_a^b F dx = 9 \int_1^4 \left[\frac{3}{(6-x)^2} - \frac{2}{x^2} \right] dx = 9 \left[\frac{3}{(6-x)} + \frac{2}{x} \right]_1^4 = -5.4 \text{ J}$$

Since voltage is defined as work done per unit charge, the voltage between points A and B is given by

$$V_{BA} = \frac{W_{BA}}{Q} = -5.4 \text{ J/C or V}$$

Then, $V_{AB} = -V_{BA} = 5.4 \text{ V}$

1.4.4 Electric Flux

An electric field exists in space between a positively and a negatively charged body. The presence of an electric field is shown by certain imaginary lines through space. They are called *flux lines*. Conventionally, they radiate from a positive charge and converge on equal quantity of negative charge. The electric flux lines are not closed on themselves as a positive and negative charge cannot exist simultaneously. Electric flux lines of an isolated charged conductor are shown in Fig. 1.3.

Both electric charge q and flux ψ are measured in coulomb, and one coulomb of positive charge radiates one coulomb of flux.

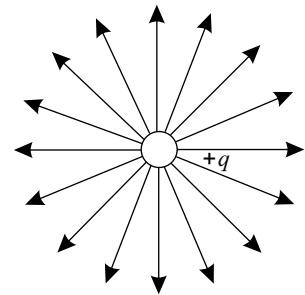


Fig. 1.3 Electric flux lines of an isolated charged conductor

1.4.5 Electric Flux Density

Electric flux density D at any point in a medium is defined as the flux ψ (in coulomb) per unit area a (in m^2), at right angles to the direction of the flux. Thus,

$$D = \frac{\psi}{a} = \frac{q}{a} \text{ C/m}^2 \quad (1.6)$$

From Eq. (1.4), electric field intensity E at a distance r from the centre of a charged body of charge q is

$$E = \frac{q}{4\pi\epsilon r^2}$$

$$\text{or} \quad \epsilon E = \frac{q}{4\pi r^2} \quad (1.7)$$

Now, the electric flux radiating from the charged body is also q coulombs, and $4\pi r^2$ is the total surface area of the sphere, with the centre at the centre of the charged body and a radius of r . The electric flux density is given by

$$D = \frac{q}{4\pi r^2} \quad (1.8)$$

From Eqs. (1.7) and (1.8), we get the following relation:

$$D = \epsilon E \quad (1.9)$$

1.4.6 Gauss' Law

Gauss' law states that the surface integral of the electric flux density over a closed surface enclosing a specific volume is equal to the algebraic sum of all the charges enclosed within the surface, i.e.,

$$\oint D \, ds = \sum q \quad (1.10)$$

1.4.7 Electric Field Due to a Long Straight Charged Conductor

A long conductor, having uniform charge q coulombs per metre, is shown in Fig. 1.4. The electric flux will be radial in all directions perpendicular to the conductor.

Let a point be chosen at a perpendicular distance r from the conductor. The total charge enclosed by an elementary cylindrical surface of length dl will be $q \times dl$, and the total flux ψ coming out of the cylindrical surface will be

$$\psi = q \times dl$$

and the flux density D on the cylindrical surface is

$$D = \frac{\psi}{2\pi r \, dl} = \frac{q \, dl}{2\pi r \, dl} = \frac{q}{2\pi r}$$

Then, the field intensity E is given by

$$E = \frac{D}{\epsilon} = \frac{q}{2\pi \epsilon r} \quad \text{V/m}$$

(1.11)

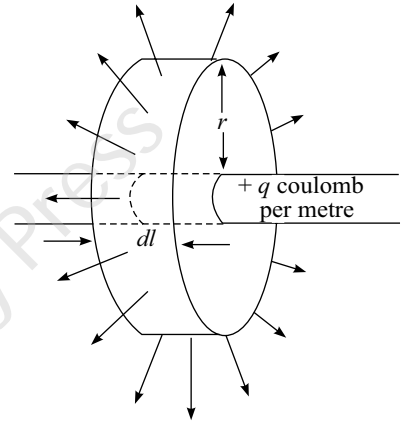


Fig. 1.4 Electric field around a long charged conductor

1.4.8 Electric Field Between Two Charged Parallel Plates

Two parallel plates, with charge $+q$ on one plate and charge $-q$ on the other plate, are shown in Fig. 1.5. The cross-sectional area of each plate is a metre². The flux lines in this case will be perpendicular to the charged plates.

The total flux $\psi = qC$ and the flux density inside the medium is

$$D = \frac{\psi}{a} = \frac{q}{a}$$

and the field intensity is

$$E = \frac{D}{\epsilon} = \frac{q}{\epsilon a} \quad \text{V/m}$$

(1.12)

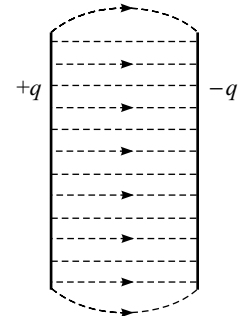


Fig. 1.5 Electric field of parallel charged conductors

1.4.9 Electric Field of a Uniformly Charged Sphere

A hollow metallic sphere with total charge q coulombs is shown in Fig. 1.6. Electric field intensity inside the hollow sphere is zero because of the fact that the electrical charge resides at the surface of the sphere only. Therefore, the electric field intensity outside the charged sphere is to be determined.

The total electric flux ψ going out of the charged sphere is $\psi = q$. The flux density D at a distance r from the centre of the sphere can be determined by considering a spherical shell of radius r with the same centre as the centre of the sphere. The surface area of this spherical shell is $4\pi r^2$ and the flux density will be

$$D = \frac{\psi}{4\pi r^2} = \frac{q}{4\pi r^2}$$

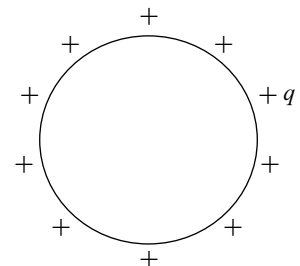


Fig. 1.6 A hollow metallic charged sphere

Also, the electric field intensity is

$$E = \frac{D}{\epsilon} = \frac{q}{4\pi\epsilon r^2} \text{ V/m} \quad (1.13)$$

1.5 ELECTRIC CURRENT

In an isolated metallic conductor, such as a length of copper wire, numerous free electrons exist in the conduction band and yet no current flows. Due to interactive forces between the free electrons themselves and with the positive ions the electrons are in motion which is essentially random in nature, and at any cross section of the copper wire the net movement of electrons is zero. In conductors, an orderly movement of electrons in a given direction can be achieved by applying an external energy source across the ends of the conductor. This makes current to flow across the wire/conductor.

Figure 1.7 shows an arrangement in which an electrochemical cell, commonly called a battery, is connected externally by a conducting wire. Initially, when the energy source is not connected externally, due to the chemical reaction in the battery a large number of electrons gather around one electrode, called cathode, giving it an excess of negative charge.

The other electrode, called anode, has an excess of positively charged nuclei, thereby, charging it positively. The anode is at a higher potential than the cathode. When a conducting wire is connected externally to the battery terminals, electrons in the conduction band are set in motion by the electric force due to accumulated charges at the battery terminals. The motion of these electrons is periodically interrupted by collisions with static atoms and ions. However, at any instant of time the flow of charge at the conductor cross section is constant. There is no accumulation of charge in the conductor; as many charges enter the cross section as leave it. The constant flow of charges constitutes electric current. As long as the chemical reactions in the battery maintain the anode terminal at a higher potential with respect to the cathode terminal, the flow of current continues. Further, the greater the potential difference across the battery terminals, the greater is the accumulated charge, the rate of flow of charge, and the current. If the metallic wire is disconnected from the battery terminals, its electrical neutrality is preserved.

Thus it may be said that the flow of electric current is associated with the movement of electric charge. Flow of electric current in a conductor is possible only when it is connected to the terminals of an electric energy source, such as a battery, and there exists a potential difference across its terminals.

Electric current is defined as the time rate change of charge passing through a cross-sectional area of a conductor. If Δq coulomb is the amount of charge flow in Δt seconds, then the average current i_{av} over a period of time, the instantaneous current i , and the charge q transferred from time t_0 to t_1 are given by

$$i_{av} = \frac{\Delta q}{\Delta t} \text{ C/sec (or amperes)} \quad (1.14)$$

$$i = \frac{dq}{dt} \text{ C/sec} \quad (1.15)$$

$$\text{or } q = \int_0^t i \, dt \text{ C} \quad (1.16)$$

The unit of current is called ampere, named after the French scientist Andre Marie Ampere. A current of 1 A means that the electric charge is flowing at the rate of 1 C/sec.

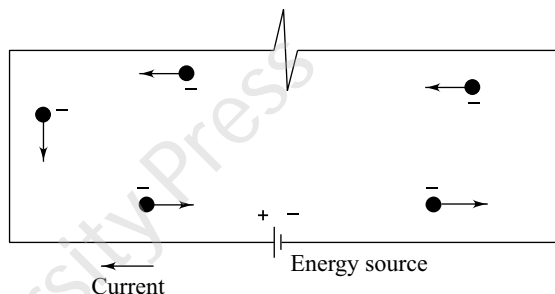


Fig. 1.7 Flow of electrons and current

Currents have direction. In conductors the current consists of the movement of electrons. Conventionally, a positive current is taken to be a flow of positive charge in the direction of a reference arrow used to mark the direction of the current flow, as shown in Fig. 1.7. Thus, the positive direction of the flow of current is taken as opposite to that of the direction of the movement of electrons.

Example 1.3 In a metallic wire, 10^{19} electrons drift across a cross section per second. What is the average current flow in the wire?

Solution Charge on one electron = 1.6×10^{-19} C

From Eq. (1.14),

$$\begin{aligned} I_{av} &= \text{Total charge movement per second} = 1.6 \times 10^{-19} \times 10^{19} \\ &= 1.6 \text{ C/sec} = 1.6 \text{ A} \end{aligned}$$

1.6 ELECTROMOTIVE FORCE

In an isolated metallic conductor, free electrons, which are loosely bonded with their nuclei, can be made to flow in a given direction by applying an electric pressure across the ends of the conductor. Such a pressure is provided by an external energy source, for example, a battery.

Due to the chemical reactions inside an electrochemical cell, commonly called a battery, separation of electric charges takes place. Negative charges accumulate at one terminal, the cathode, and positive charges accumulate at the other terminal, the anode. As the charges of unlike polarity attract each other, work has to be done by an external agency against these attractive forces to separate them. In the case of a battery, the work is done chemically. The greater the number of charges that are separated, the greater is the work that has to be done to achieve this separation and the greater is the potential energy of the separated charges.

The work done per unit charge is a measure of the amount of accumulated charge or a measure of the potential energy that has been established. The work done per unit charge in a battery is the potential difference (pd) between the terminals of the battery.

The pd between the battery terminals is known as the electromotive force, or emf. The emf represents the driving influence that causes a current to flow, and may be interpreted to represent the energy that is used during passing of a unit charge through the source. The term emf is always associated with energy conversion. The emf is usually represented by the symbol E and has the unit volt. When the battery is connected externally through a conductor to a load, energy transfer to the load commences through the conductor. The energy transfer due to the flow of unit charge between the two points in the circuit is termed as potential difference. When all the energy is transferred to the load unit, the pd across the load unit becomes equal to that of the battery emf.

In view of this discussion, it may be stated that both emf and pd are similar entities and have the same units. Thus emf is associated with energy while pd causes the passage of charge, or current. Both potential and potential difference are scalar quantities.

The emf and pd are represented in a diagram following certain conventions. Each is indicated by an arrow, as shown in Fig. 1.8. The arrowhead in each case points to a higher potential. It may be noted that the current leaves the source of emf at the positive terminal and therefore the direction of current flow is the same as that of the emf arrow. The current enters the load at the positive terminal, and thus the direction of current is opposite with respect to the pd arrow of the load.

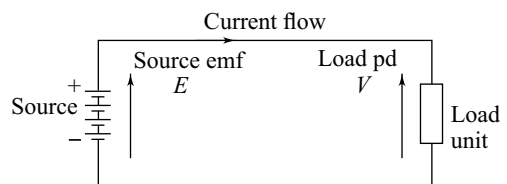


Fig. 1.8 Conventions of representing emf and pd

The unit of pd is volt and the symbol V is used to represent the pd. A volt is defined as the potential difference between two points of a conductor carrying a current of 1 A, when the power dissipated between the points is 1 W. As the pd is measured in volts, it is also termed as voltage drop.

1.7 ELECTRIC POWER

Power is defined as work done or energy per unit time. If force F newton acts for t seconds through a distance d metre along a straight line, then the work done is $F \times d$ joules. Then the power p either generated or dissipated by a circuit element can be represented by the following relationship:

$$p = \frac{F \times d}{t} = F \times u \quad (1.17)$$

where u is the velocity in m/sec.

In the case of rotating machines,

$$p = \frac{2\pi N_r T}{60} \quad (1.18)$$

where N_r is the speed of rotation of the machine in rpm (revolutions per minute) and T is the torque in N m.

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{work}}{\text{charge}} \times \frac{\text{charge}}{\text{time}} = \text{voltage} \times \text{current} \quad (1.19)$$

The unit of power is J/sec or watt (after the Scottish engineer, James Watt). The unit of energy is joule or watt-second. Commercially, the unit of energy is kilowatt-hour (kWh). It represents the work done at the rate of 1 kW for a period of 1 h. The electric supply authorities refer 1 kWh as one 'unit' for billing purposes.

Alternatively, if the current flowing between two points in a conductor is i and the voltage is v , then from the definitions of current and voltage given in Eqs (1.15) and (1.16), it is apparent that the product of current and voltage is power p dissipated between two points in the conductor carrying the current. Thus,

$$p = v \times i \quad (1.20)$$

$$\begin{aligned} &= \frac{dw}{dq} \times \frac{dq}{dt}, \quad \frac{\text{joules}}{\text{coulomb}} \times \frac{\text{coulombs}}{\text{seconds}} \\ &= \frac{dw}{dt}, \quad \frac{\text{joules}}{\text{second}} \quad \text{or} \quad \text{watts} \end{aligned} \quad (1.20a)$$

Just like voltage, power is a signed quantity. Usually the electrical engineering community adopts the passive sign convention. As per this convention, if positive current flows into the positive terminal of an element, the power dissipated is positive, that is, the element absorbs power; while if the current leaves the positive terminal of an element, the power dissipated is negative, that is, the element delivers power.

Example 1.4 A circuit delivers energy at the rate of 30 W and the current is 10 A. Determine the energy of each coulomb of charge in the circuit.

Solution From Eq. (1.20)

$$v = \frac{p}{i} = \frac{30}{10} = 3 \text{ V}$$

$$\text{Also, } v = \frac{p}{i} = \frac{dw}{dt} \times \frac{dt}{dq} = \frac{dw}{dq}$$

$$\therefore dw = v \times dq$$

If $i = 10 \text{ A}$, $dq = i \times dt = 10 \times 1 = 10 \text{ C}$, then

$$dw = 3 \times 10 = 30 \text{ J}$$

Therefore, the energy of each coulomb of charge is $30/10 = 3 \text{ J}$.

Example 1.5 An electric motor is developing 15 kW at a speed of 1500 rpm. Calculate the torque available at the shaft.

Solution Substituting into Eq. (1.18), $15,000 \text{ W} = T \times 2\pi \times \frac{1500}{60}$

$$\therefore T = 95.45 \text{ Nm}$$

1.8 OHM'S LAW

This law is named after the German mathematician Georg Simon Ohm who first enunciated it in 1827. It states that *at constant temperature, potential difference V across the ends of a conductor is proportional to the current I flowing through the conductor*. Mathematically, Ohm's law can be stated as

$$V \propto I \quad \text{or} \quad V = R \times I$$

$$\text{or} \quad R = \frac{V}{I} \quad (1.21)$$

In Eq. (1.21), R is the proportionality constant and is the resistance of the conductor. Its unit is ohm (Ω):

$$1 \Omega = 1 \text{ V/A} \quad (1.22)$$

It may be noted that subsequently it was established that Ohm's law could not be applied to networks containing unilateral elements (such as diodes), or non-linear elements (such as thyrite, electric arc, etc.). A unilateral element is the one that does not exhibit the same V - I characteristic when the direction of the flow of current through it is reversed. Similarly, in non-linear elements the V - I characteristic is not linear.

Using a dc source, voltaic cell, Ohm achieved the experimental verification of his law. Later experiments with time-varying sources showed that this law is also valid when the potential difference applied across a linear resistance is time-varying. In this case, Eq. (1.21) is written as

$$v = R \times i \quad (1.23)$$

where v and i are instantaneous values of the potential difference and current, respectively.

1.9 BASIC CIRCUIT COMPONENTS

Resistor, inductor and capacitor are the three basic components of a network. A resistor is an element that dissipates energy as heat when current passes through it. An inductor stores energy by virtue of a current through it. A capacitor stores energy by virtue of a voltage existing across it. The behavior of an electrical device may be approximated to any desired degree of accuracy by a circuit formed by interconnection of these basic and idealized circuit elements.

1.9.1 Resistors

A resistor is a device that provides resistance in an electric circuit. As already stated in Section 1.5, ordinarily the free electrons in a conductor undergo random movement but the net movement of electrons is zero and hence this does not result in a net current flow. The free electrons in a conductor can be made to flow in a particular direction by applying an external voltage source. The application of the voltage source produces an electric field within the conductor, which produces a directed motion of free electrons. The motion of these free electrons is directed opposite to the electric field. During their motion these electrons collide with the fixed atoms in the lattice structure of the material of the conductor. Such collisions result in the production of irreversible heat loss. Thus resistance is the property of a circuit element which offers hindrance or opposition to the flow of current and in the process electric energy is converted into heat energy. Electric resistance is analogous to pipe friction in a hydraulic system and friction in a mechanical system. The resistance of a conductor opposes the current, pipe friction opposes the water flow through the pipe, and friction opposes the motion of a mechanical system, and the energy dissipated in overcoming this opposition appears as heat.

A physical device whose principal electrical characteristic is resistance is called *resistor*. A resistor is said to be linear if it satisfies Ohm's law, that is, the current through the resistor is proportional to the pd across it.

If the magnitude of resistance varies with the voltage or current, the resistor is said to be non-linear. Resistors made of semiconductor materials are non-linear resistors.

The resistance of a resistor depends on the material of which the conductor is made and the geometrical shape of the conductor. The resistance of a conductor is proportional to its length l and inversely proportional to its cross-sectional area a . Therefore, the resistance of a conductor can be written as

$$R \propto \frac{l}{a} \quad \text{or} \quad R = \frac{\rho \times l}{a} \quad (1.24)$$

The proportionality constant ρ is called the specific resistance or resistivity of the conductor and its value depends on the material of which the conductor is made. Equation (1.24) is valid only if the current is uniformly distributed throughout the cross section of the conductor. In Eq. (1.24), if $l = 1 \text{ m}$, $a = 1 \text{ m}^2$, then $\rho = R$. Thus specific resistance is defined as the resistance of a conductor having a length of 1 m and a cross section of 1 m^2 . The unit of resistivity can be obtained as under:

$$\rho = \frac{R \times a}{l}, \quad \frac{\text{ohm} \times \text{metre}^2}{\text{metre}} = \text{ohm-metre } (\Omega \text{ m})$$

The inverse of resistance is called *conductance* and the inverse of resistivity is called *specific conductance* or *conductivity*. The symbol used to represent conductance is G and conductivity is σ . Thus, from Eq. (1.24), conductivity $\sigma = 1/\rho$, and its units are siemens per metre or mho.

$$G = \frac{1}{R} = \frac{a}{\rho l} = \frac{1}{\rho} \times \frac{a}{l} = \sigma \times \frac{a}{l} \quad \text{mho} \quad (1.25)$$

Example 1.6 Find the resistance of stranded annealed copper wire 200 m long and 25 mm^2 in cross section. Resistivity of copper is $1.72 \times 10^{-8} \Omega \text{ m}$.

Solution

$$R = \frac{\rho l}{a} = \frac{1.72 \times 10^{-8} \times 200}{25 \times 10^{-6}} = 0.1376 \Omega$$

Example 1.7 Find the resistance of the semicircular copper section, shown in Fig. 1.9, between the equipotential faces A and B . The inner radius is 6 cm, radial thickness 4 cm, and axial thickness 4 cm.

Solution The mean radius of the semicircular section is

$$6 + 2 = 8 \text{ cm} = 0.08 \text{ m}$$

Then, the mean length is $l = \pi \times r = \pi \times 0.08$

Area of the cross section $a = 0.04 \times 0.04 = 0.0016 \text{ m}^2$

Resistivity of copper $\rho = 1.72 \times 10^{-8} \Omega \text{ m}$

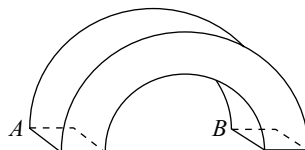


Fig. 1.9

Therefore, the resistance $R = \frac{\rho \times l}{a} = \frac{1.72 \times 10^{-8} \times \pi \times 0.08}{0.0016} = 270.286 \times 10^{-8} \Omega = 2.703 \mu\Omega$

Example 1.8 A coil consists of 4000 turns of copper wire having a cross-sectional area of 0.8 mm^2 . The mean length per turn is 80 cm. The resistivity of copper at normal working temperature is 0.02 m Wm . Calculate the resistance of the coil and the power dissipated when it is connected across a 230-V dc supply.

Solution

$$R = \frac{\rho \times l}{a} = \frac{0.02 \times 10^{-6} \times (4000 \times 80 \times 10^{-2})}{0.8 \times 10^{-6}} = 80 \Omega$$

Now, power dissipated $= V \times I = 230 \times 230/80 = 661.25 \text{ W}$

Example 1.9 An aluminium wire 7.5 m long is connected in parallel with a copper wire 6 m long. When a current of 5 A is passed through the combination, it is found that the current in the aluminium wire is 3 A. The diameter of the aluminium wire is 1 mm. Determine the diameter of the copper wire. Resistivity of copper is $0.017 \mu\Omega \text{ m}$ and that of aluminium is $0.028 \mu\Omega \text{ m}$.

Solution The resistance of aluminium wire is

$$R_{\text{Al}} = \frac{\rho_{\text{Al}} \times l_{\text{Al}}}{a_{\text{Al}}} = \frac{0.028 \times 10^{-6} \times 7.5}{\frac{\pi}{4}(10^{-3})^2} = 0.2675 \Omega$$

The potential drop across aluminium wire is $0.2675 \times 3 = 0.8025 \text{ V}$. Then the potential drop across the copper wire is also 0.8025 V . Therefore,

$$\text{Resistance of copper wire} = \frac{0.8025}{2} = 0.40125 \Omega$$

The cross section of copper wire is

$$\frac{\rho_{\text{Cu}} l_{\text{Cu}}}{R_{\text{Cu}}} = \frac{0.017 \times 10^{-6} \times 6}{0.40125} = 0.2542 \times 10^{-6} \text{ m}^2$$

$$\text{Then } \frac{\pi}{4}(d_{\text{Cu}})^2 = 0.2542 \times 10^{-6}$$

$$\therefore d_{\text{Cu}} = 0.569 \times 10^{-3} \text{ m} = 0.569 \text{ mm}$$

Example 1.10 A porcelain cylinder 5 cm in diameter is wound with a bare high resistance wire having a resistance of $1 \Omega \text{ m}$ length and 1 mm^2 cross section. The distance between consecutive turns equals the diameter of the wire. If the external surface of the cylinder (excluding the ends) can dissipate 0.32 W/cm^2 at the permitted temperature rise, find the length of the cylinder and the diameter and length of wire for a loading of 100 W and a current of 1 A .

Solution The area required to dissipate $100 \text{ W} = 100/0.32 = 312.5 \text{ cm}^2$

Let the length of the cylinder be $L \text{ cm}$, length of the wire be $l \text{ cm}$, and the diameter of the wire be $d \text{ cm}$. Then

$$L = \frac{312.5}{\pi \times (\text{diameter of cylinder})} = \frac{312.5}{\pi \times 5} = 19.846 \text{ cm} \approx 20 \text{ cm}$$

$$\text{Resistance of the wire, } R = \frac{\text{Load, watts}}{(\text{current})^2} = \frac{100}{1^2} = 100 \Omega$$

Spacing between two consecutive turns = $d \text{ cm}$

Distance along the axis of the cylinder between consecutive turns = $2d \text{ cm}$

$$\text{Therefore, Number of turns} = \frac{L}{2d} = \frac{20}{2d} = \frac{10}{d}$$

$$\text{Length of 1 turn of wire} = \pi \times 5 \text{ cm}$$

$$\text{Length of wire } l = \frac{10 \times 5 \times \pi}{d} = \frac{50\pi}{d} \text{ cm}$$

Now, the resistance of wire of length 1 m and area of cross section 1 mm^2 is 1Ω . Then,

$$\rho = \frac{1 \Omega \times 1 \text{ mm}^2}{1 \text{ m}} = \frac{1 \Omega \times 10^{-2} \text{ cm}^2}{100 \text{ cm}} = 10^{-4} \Omega \text{ cm}$$

$$\therefore R = 100 = \frac{10^{-4} \times 50\pi \times 4}{d \times \pi d^2} = \frac{2 \times 10^{-2}}{d^3}$$

Then, $d^3 = 2 \times 10^{-4}$ or $d = 0.058$ cm

and $l = \frac{50\pi}{d} = \frac{50\pi}{0.058} = 2700$ cm = 27 m

1.9.1.1 Temperature Coefficient of Resistance

All current-carrying conductors and resistors dissipate heat when carrying current. When V volts applied across a resistor of R ohm causes a current of I ampere to flow, the electrical energy absorbed by the resistor is at the rate of $V \times I$ or $I^2 R$ which is converted into heat, thereby causing a temperature rise in the resistor. When the resistor becomes warmer than its surrounding medium, it dissipates heat into the surrounding medium. Finally, when the release of heat energy is at the same rate as it receives electric energy, the temperature of the resistor no longer rises. All resistors have a power rating, which is the maximum power that can be dissipated without the temperature rise being damaging to the resistor. Thus a 4 W resistor of 100Ω can pass a current of 20 mA, whereas a $1/4$ W resistor of 100Ω can allow only 50 mA. If the current level exceeds, the resistances are overheated and might burn.

The resistance of most conductors and all metals increases with increase in temperature. However, the resistance of carbon and insulating materials decreases with increase in temperature. Certain alloys such as constantan (60% copper and 40% nickel) and manganin (84% copper, 12% manganese, and 4% nickel) show no change in resistance for a considerable variation in temperature. This makes these alloys ideal for the construction of accurate resistances used in resistance boxes. Investigations reveal that a linear variation of resistance with temperature for copper prevails over a temperature range -50°C to 200°C . The change in resistance is usually proportional to the change in temperature. The temperature coefficient of resistance is the ratio of the change in resistance per degree change in temperature to the resistance at some definite (reference) temperature and is denoted by the Greek letter α .

Figure 1.10 shows the linear variation of the resistance of copper with the change in temperature. It may be seen from the graph that at -234.5°C its resistance becomes theoretically zero. If $R_0 = 1 \Omega$ is the resistance of copper at 0°C , then $R_{-234.5} = 0 \Omega$ at -234.5°C , and by definition the temperature coefficient of copper at 0°C , α_0 is given by

$$\alpha_0 = \frac{\frac{R_0 - R_{-234.5}}{0 - (-234.5)} \frac{\Omega}{^\circ\text{C}}}{R_0 \frac{\Omega}{^\circ\text{C}}} = \frac{234.5}{1} = 0.004264/^\circ\text{C} \quad (1.26)$$

In general, resistance R_2 at any temperature t_2 can be expressed in terms of resistance R_1 at temperature t_1 as

$$R_2 = R_1 [1 + \alpha_1 (t_2 - t_1)] \quad (1.27)$$

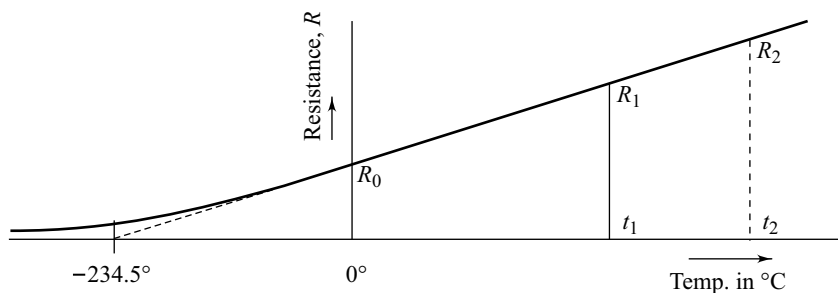


Fig. 1.10 Variation of resistance of copper with temperature

where α_1 is the temperature coefficient at temperature t_1 . Suppose the reference temperature is taken as 0°C . Then

$$\begin{aligned}
 R_1 &= R_0 (1 + \alpha_0 t_1) \\
 R_2 &= R_0 (1 + \alpha_0 t_2) \\
 \therefore R_2 &= R_1 \frac{(1 + \alpha_0 t_2)}{(1 + \alpha_0 t_1)} \quad (1.28)
 \end{aligned}$$

Equating Eqs (1.27) and (1.28) and simplifying, the value of α_1 is given as

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \quad (1.29)$$

Similarly, the specific resistance ρ varies linearly with temperature. The expression for ρ_1 , the resistivity at temperature t_1 , in terms of ρ_0 , the resistivity at 0°C , will be

$$\rho_1 = \rho_0 (1 + \alpha_0 t_1) \quad (1.30)$$

Typical values of resistivity and temperature coefficients of resistances at 20°C are given in Table 1.1.

Table 1.1 Resistivity and temperature coefficient

Material	Resistivity at 20°C , $\Omega \text{ m}$	Temperature coefficient, α_{20}
Copper, annealed	1.69×10^{-8} to 1.74×10^{-8}	0.00393
Aluminium, hard drawn	2.80×10^{-8}	0.0039
Carbon	6500×10^{-8}	-0.000476
Tungsten	5.6×10^{-8}	0.0045
Manganin	48×10^{-8}	0
Constantan (Eureka)	48×10^{-8}	0

Example 1.11 A potential difference of 250 V is applied to a copper field coil at a temperature of 15°C and the current is 5 A. What will be the mean temperature of the coil when the current has fallen to 3.91 A, the applied voltage being the same as before? The temperature coefficient of copper at 0°C is 0.00426.

Solution At 15°C , $R_{15} = \frac{250}{5} = 50 \Omega$

At $t^\circ\text{C}$, $R_t = \frac{250}{3.91} = 63.94 \Omega$

Then $\frac{R_t}{R_{15}} = \frac{1 + \alpha_0 \times t}{1 + \alpha_0 \times 15}$

or $\frac{63.94}{50} = \frac{1 + 0.00426 \times t}{1 + 0.00426 \times 15}$

Hence $t = 84.63^\circ\text{C}$.

Example 1.12 If the resistance temperature coefficient of a conductor is α_1 at $t_1^\circ\text{C}$, derive an expression for the temperature coefficient α_2 at $t_2^\circ\text{C}$ in terms of α_1 and the temperatures.

Solution From Eq. (1.29), it is seen that

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \quad \text{or} \quad \alpha_0 = \frac{\alpha_1}{1 - \alpha_1 t_1}$$

Similarly, $\alpha_2 = \frac{\alpha_0}{1 + \alpha_0 t_2}$

Substitution of α_0 in the expression for α_2 results in

$$\alpha_2 = \frac{\left(\frac{\alpha_1}{1 - \alpha_1 t_1} \right)}{1 + \left(\frac{\alpha_1}{1 - \alpha_1 t_1} \right) t_2} = \frac{\alpha_1}{1 + \alpha_1(t_2 - t_1)} = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)}$$

1.9.2 Inductors

The electrical element that stores energy in association with a flow of current is called *inductor*. The idealized circuit model for the inductor is called an *inductance*. Practical inductors are made of many turns of thin wire wound on a magnetic core or an air core.

A unique feature of the inductance is that its presence in a circuit is felt only when there is a changing current. Figure 1.11 shows a schematic representation of an inductor.

For the ideal circuit model of an inductor, the voltage across it is proportional to the rate of change of current in it. Thus if the rate of change of current is di/dt and v is the induced voltage, then

$$v \propto \frac{di}{dt}$$

$$\text{or } v = L \frac{di}{dt} \text{ V} \quad (1.31)$$

In Eq. (1.31) the proportionality constant L is called inductance. The unit of inductance is henry, named after the American physicist Joseph Henry. Equation (1.31) may be rewritten as

$$L = \frac{v}{\frac{di}{dt}} = \frac{\text{volt-second}}{\text{ampere}} \quad \text{or} \quad \text{henrys (H)} \quad (1.32)$$

Equation (1.32) can be used to define inductance. If an inductor induces a voltage of 1 V when the current is uniformly varying at the rate of 1 A/sec, it is said to have an inductance of 1 H. Integrating Eq. (1.31) with respect to time t ,

$$i = \frac{1}{L} \int_0^t v dt + i(0) \quad (1.33)$$

where $i(0)$ is the current at $t = 0$. From Eq. (1.33) it may be inferred that the current in an inductor cannot change suddenly in zero time.

Instantaneous power p entering the inductor at any instant is given by

$$p = vi = Li \frac{di}{dt} \quad (1.34)$$

When the current is constant, the derivative is zero and no additional energy is stored in the inductor. When the current increases, the derivative is positive and hence the power is positive; and, in turn, an additional energy is stored in the inductor. The energy stored in the inductor, W_L , is given by

$$W_L = \int_0^t v i dt = \int_0^t Li \frac{di}{dt} \times dt = L \int_0^t i di = \frac{1}{2} Li^2 \text{ joule} \quad (1.35)$$

Equation (1.35) assumes that the inductor has no previous history, that is, at $t = 0$, $i = 0$. The energy is stored in the inductor in a magnetic field. When the current increases, the stored energy in the magnetic field also increases. When the current reduces to zero, the energy stored in the inductor is returned to the source from which it receives the energy.

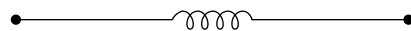


Fig. 1.11 Schematic representation of an inductor

Example 1.13 A current having a variation shown in Fig. 1.12 is applied to a pure inductor having a value of 2 H. Calculate the voltage across the inductor at time $t = 1$ and $t = 3$ sec.

Solution For the period $0 \leq t \leq 1$ sec

Current, $i = 10t$ A

Rate of change of current $\frac{di}{dt} = 10$ A/sec

Therefore, at $t = 1$ sec, voltage across the inductor is

$$L \frac{di}{dt} = 2 \times 10 = 20 \text{ V}$$

For the period $1 \leq t \leq 3$ sec

Rate of change of current $\frac{di}{dt} = -5$ A/sec

Therefore, at $t = 3$ sec, voltage across the inductor is

$$L \frac{di}{dt} = 2 \times -5 = -10 \text{ V}$$

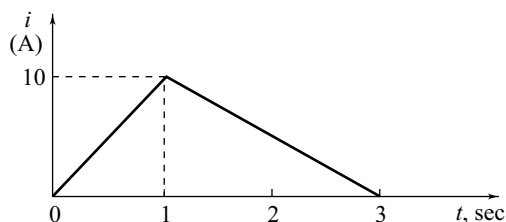


Fig. 1.12

Example 1.14 A voltage wave having the time variation shown in Fig. 1.13 is applied to a pure inductor having a value of 0.5 H. Calculate the current through the inductor at times $t = 1, 2, 3, 4, 5$ sec. Sketch the variation of current through the inductor over 5 sec.

Solution For the period $0 \leq t \leq 1$ sec, $v = 10$ V; $i(0) = 0$. The current i may be expressed using Eq. (1.33) as

$$i = \frac{1}{L} \int_0^t v dt + i(0) = \frac{1}{0.5} \int_0^t 10 dt = 20 \int_0^t dt = 20t$$

Then at $t = 1$ sec, $i = 20 \times 1 = 20$ A

For the period $1 \leq t \leq 3$ sec, $v = -10$ V; $i(1) = 20$ A, then current

$$i = \frac{1}{L} \int_1^t v dt + i(1) = \frac{1}{0.5} \int_1^t -10 dt + 20 = -20 \int_1^t dt + 20 = -20(t-1) + 20$$

Then at $t = 2$ sec, $i = -20 \times (2-1) + 20 = -20 + 20 = 0$ A

And at $t = 3$ sec, $i = -20 \times (3-1) + 20 = -40 + 20 = -20$ A

For the period $3 \leq t \leq 5$ sec, $v = 10$ V; $i(3) = -20$ A,

$$\begin{aligned} i &= \frac{1}{L} \int_3^t v dt + i(3) = \frac{1}{0.5} \int_3^t 10 dt - 20 \\ &= 20 \int_3^t dt - 20 = 20(t-3) - 20 \end{aligned}$$

Then at $t = 4$ sec, $i = 20 \times (4-3) - 20 = 20 - 20 = 0$ A

And at $t = 5$ sec, $i = 20 \times (5-3) - 20 = 40 - 20 = 20$ A

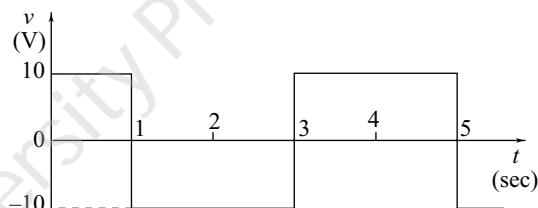


Fig. 1.13

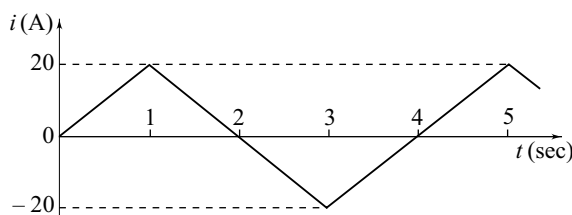


Fig. 1.14 Variation of current through the inductor

Example 1.15 The voltage waveform shown in Fig. 1.15 is applied across an inductor of 5 H. Derive an expression for current in the circuit and sketch the current and energy waveforms against time. Assume zero initial condition in the circuit.

Solution Using Eq. (1.31), the generalized relation for current through the inductor is written as $i = \frac{1}{5} \int_0^t v dt$

For the period $0 \leq t \leq 1$ sec, $v = 12$ V. Therefore, the current through the inductor is given by

$$i = \frac{1}{5} \int_0^t 12 dt = 2.4t + i(0)$$

At $t = 0$, $i(0) = 0$.

Thus, $i = 2.4t$

For the period $1 \leq t \leq 3$ sec, $v = 18$ V. Therefore, the current through the inductor is given by

$$i = \frac{1}{5} \int_0^t 18 dt = 3.6t + i(1.0)$$

At $t = 1.0$ sec, $i = 2.4$ A. Hence,

$$2.4 = 3.6 \times 1.0 + i(1.0) \text{ or } i(1.0) = -1.2 \text{ A}$$

The expression for the inductor current during the period $1 \leq t \leq 3$ sec is

$$i = 3.6t - 1.2$$

For the period $3 \leq t \leq 4$ sec, $v = 12$ V. Hence, the current through the inductor is expressed as

$$i = \frac{1}{5} \int_0^t 12 dt = 2.4t + i(3.0)$$

At $t = 3.0$ sec, $i = 9.6$ A. Hence,

$$9.6 = 2.4 \times 3.0 + i(3.0) \text{ or } i(3.0) = 2.4 \text{ A}$$

$\therefore i = 2.4t + 2.4$

Energy stored in the various periods is as follows.

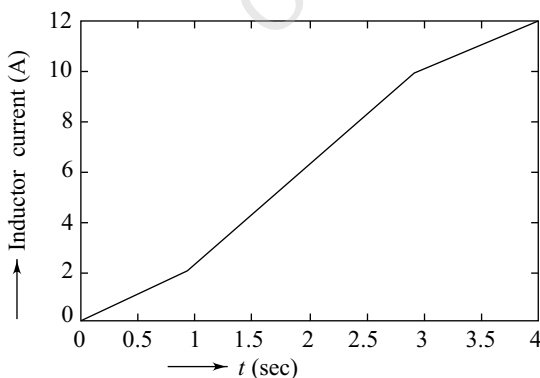


Fig. 1.16

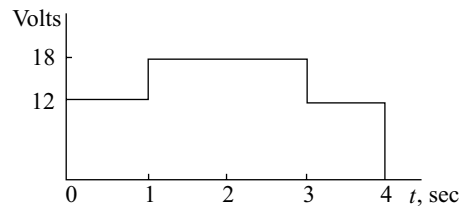


Fig. 1.15

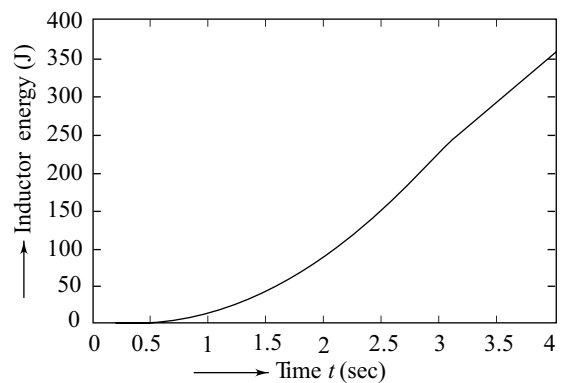


Fig. 1.17

For the period $0 \leq t \leq 1$ sec, $W_L = \frac{1}{2} \times 5 \times (2.4t)^2 = 14.4t^2$ J

For the period $1 \leq t \leq 3$ sec, $W_L = \frac{1}{2} \times 5 \times (3.6t - 1.2)^2 = 32.4t^2 - 21.6t + 3.6$ J

For the period $3 \leq t \leq 4$ sec, $W_L = \frac{1}{2} \times 5 \times (2.4t + 2.4)^2 = 14.4t^2 + 28.8t + 14.4$ J

The variation of inductor current and energy with time is sketched in Figs 1.16 and 1.17.

1.9.3 Capacitors

A *capacitor* is a device that can store energy in the form of a charge separation when it is suitably polarized by an electric field by applying a voltage across it. In the simplest form, a capacitor consists of two parallel conducting plates separated by air or any insulating material, such as mica. It has the characteristic of storing electric energy (charge), which can be fully retrieved, in an electric field. A significant feature of the capacitor is that its presence is felt in an electric circuit when a changing potential difference exists across the capacitor. The presence of an insulating material between the conducting plates does not allow the flow of dc current; thus a capacitor acts as an open circuit in the presence of dc current. Figure 1.18 shows the schematic representation of a capacitor.

The ability of the capacitor to store charge is measured in terms of capacitance C . *Capacitance* of a capacitor is defined as charge stored per volt applied and its unit is farad (F). However, for practical purposes the unit of farad is too large. Hence, microfarad (μF) is used to specify the capacitance of the components and circuits.

In Fig. 1.18(b), it is assumed that the charge on the capacitor at any time t after the switch S is closed is q coulombs and the voltage across it is v volts. Then by definition

$$C = \frac{q}{v} \text{ coulomb} \quad (1.36)$$

Current i flowing through the capacitor can be obtained as

$$i = \frac{dq}{dt} = C \frac{dv}{dt} \text{ ampere} \quad (1.37)$$

Equation (1.37) is integrated with respect to time to get the voltage across the capacitor as

$$v = \frac{1}{C} \int_0^t i dt + v(0) \quad (1.38)$$

where $v(0)$ is an integration constant which defines the initial voltage across the capacitor at $t = 0$. It may be noted from Eq. (1.38) that the voltage across a capacitor cannot change instantaneously, that is, in zero time.

Power p in the capacitor is given as

$$p = vi = Cv \frac{dv}{dt} \text{ watt} \quad (1.39)$$

Energy stored in the capacitor, W_C , is given by

$$W_C = \int p dt = C \int v dv = \frac{1}{2} Cv^2 \text{ joule} \quad (1.40)$$

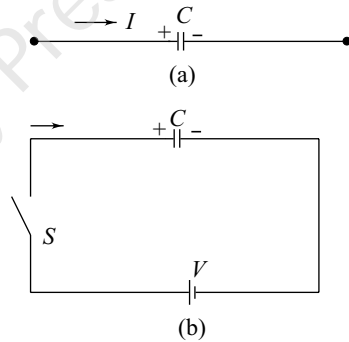


Fig. 1.18 (a) Schematic representation of a capacitor and (b) capacitor across a dc source

From Eq. (1.40) it is evident that the energy stored in the capacitor is dependent on the instantaneous voltage and is returned to the network when the voltage is reduced to zero.

As stated earlier, a capacitor consists of two electrodes (plates) separated by an insulating material (dielectric). If the area of the plates is $A \text{ m}^2$ and the distance between them is $d \text{ m}$, it is observed that

$$C \propto A \quad \text{and} \quad C \propto \frac{1}{d}$$

$$\therefore C = \frac{\epsilon A}{d} \quad (1.41)$$

where ϵ is the absolute permittivity constant. The absolute permittivity constant depends on the type of dielectric employed in the capacitor. The ratio of the absolute permittivity constant of the dielectric ϵ to the permittivity constant of vacuum ϵ_0 is called relative permittivity ϵ_r , that is,

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Hence, $\epsilon = \epsilon_0 \epsilon_r$.

The units for absolute permittivity ϵ can be established from Eq. (1.41) as under:

$$\epsilon = \frac{C \text{ (farads)} \times d \text{ (metres)}}{A \text{ (metres)}^2} = \frac{C \times d}{A} \text{ farads/metre (F/m)}$$

Based on experimental results, the value of the permittivity constant of vacuum has been found to be equal to $8.84 \times 10^{-12} \text{ F/m}$. Therefore, the value of ϵ_r for vacuum is 1.0 and for air is 1.0006. For practical purposes, the value of ϵ_r for air is also taken as 1.

Example 1.16 A voltage wave having a time variation of 20 V/sec is applied to a pure capacitor having a value of $25 \mu\text{F}$. Find (a) the current during the period $0 \leq t \leq 1 \text{ sec}$, (b) charge accumulated across the capacitor at $t = 1 \text{ sec}$, (c) power in the capacitor at $t = 1 \text{ sec}$, and (d) energy stored in the capacitor at $t = 1 \text{ sec}$.

Solution (a) Current through the capacitor i may be obtained using Eq. (1.37) as

$$i = C \frac{dv}{dt} = 25 \times 10^{-6} \times 20 = 500 \mu\text{A}$$

(b) At $t = 1 \text{ sec}$, $v = 20 \text{ V}$. Charge q at $t = 1 \text{ sec}$ may be obtained using Eq. (1.36) as

$$q = C v = 25 \times 10^{-6} \times 20 = 500 \mu\text{C}$$

(c) At $t = 1 \text{ sec}$, power $p = v \times i = 20 \times 500 \times 10^{-6} = 1 \times 10^{-2} \text{ W}$

(d) At $t = 1 \text{ sec}$, energy stored in the capacitor, W_C , can be obtained using Eq. (1.40) as

$$W_C = \frac{1}{2} C v^2 = \frac{1}{2} \times 25 \times 10^{-6} \times (20)^2 = 5 \times 10^{-3} \text{ J}$$

Example 1.17 A current having variation shown in Fig. 1.19 is applied to a pure capacitor having a value of $5 \mu\text{F}$. Calculate the charge, voltage, power, and energy at time $t = 2 \text{ sec}$.

Solution For the period $0 \leq t \leq 1 \text{ sec}$, $i = 100 \times 10^{-3} t = 0.1 t \text{ A}$

$$\text{At } t = 1 \text{ sec, } q = \int_0^t i dt = \int_0^1 0.1 t dt = 0.1 \times \left[\frac{t^2}{2} \right]_{t=0}^{t=1} = 0.05 [t^2]_{t=0}^{t=1} = 0.05 [1 - 0] = 0.05 \text{ C}$$

$$v = \frac{q}{C} = \frac{1}{C} \int_0^t 0.1 t dt = \frac{0.05 t^2}{500 \times 10^{-6}}$$

$$= 100 t^2 = 100 \text{ V}$$

where $t = 1 \text{ sec}$,

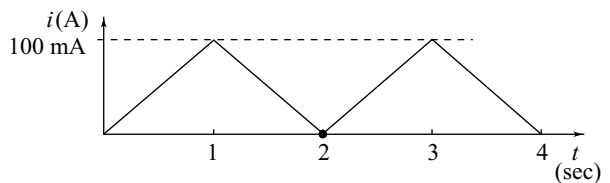


Fig. 1.19

$$p = v \times i = 100 \times 0.1 = 10 \text{ W}$$

$$W_C = \int_0^t vi \, dt = \int_0^t 100t^2 \times 0.01t \, dt = \int_0^1 t^3 \, dt = \left[\frac{t^4}{4} \right]_{t=0}^{t=1} = \frac{1}{4}[1 - 0] = 0.25 \text{ J}$$

For the period $0 \leq t \leq 1$ sec, $i = 0.2 - 0.1t$ A

$$\begin{aligned} \text{At } t = 2 \text{ sec, Charge } q &= q_{t=1} + \int_1^2 i \, dt = 0.05 + \int_1^2 (0.2 - 0.1t) \, dt = 0.05 + \left[0.2t - 0.1 \times \frac{t^2}{2} \right]_{t=1}^{t=2} \\ &= 0.05 + [0.2(2 - 1) - 0.05(2^2 - 1^2)] = 0.05 + 0.05 = 0.1 \text{ C} \end{aligned}$$

$$\begin{aligned} \text{Voltage } v &= \frac{q}{C} = \frac{1}{C} \left[0.05 + \int_1^2 (0.2 - 0.1t) \, dt \right] \\ &= \frac{1}{500 \times 10^{-6}} [0.05 + 0.2t - 0.05t^2]_{t=1}^{t=2} \\ &= \frac{10^6}{500} [0.05 + 0.2(2 - 1) - 0.05(2^2 - 1^2)] = \frac{10^6}{500} \times 0.1 = 200 \text{ V} \end{aligned}$$

$$\text{Power } p = v \times i = 200 \times 0 = 0 \text{ W}$$

$$\begin{aligned} \text{Energy } W_C &= W_{C,t=1} + \int_1^2 vi \, dt \\ &= 0.25 + \int_1^2 \left[\frac{1}{500 \times 10^{-6}} [0.05 + 0.2t - 0.05t^2] \times (0.2 - 0.1t) \right] dt \\ &= 0.25 + \frac{10^6}{500} \int_1^2 [0.01 + 0.035t - 0.03t^2 + 0.005t^3] dt \end{aligned}$$

$$\begin{aligned} \text{Energy } W_C &= 0.25 + \frac{10^6}{500} \left[0.01t + 0.035 \times \frac{t^2}{2} - 0.03 \times \frac{t^3}{3} + 0.005 \times \frac{t^4}{4} \right]_{t=1}^{t=2} \\ &= 0.25 + \frac{10^6}{500} \times 0.01125 = 0.25 + 22.50 = 22.75 \text{ J} \end{aligned}$$

1.10 ELECTROMAGNETISM RELATED LAWS

Some laws related to electromagnetism are discussed in this section.

1.10.1 Magnetic Field Due to Electric Current Flow

Hans Christian Oersted, a Danish physicist, in 1831 discovered that current flowing in a conductor generates a magnetic field all around it. He proved that the magnetic lines of force due to the current flow in a conductor were concentric circles closed on themselves as shown in Fig. 1.20. The direction of the magnetic lines of force depends upon the direction of current. The convention adopted to show the direction of the current flow is that current flowing into the plane of the paper is indicated by a cross sign and current flowing out of the plane of the paper is shown by a dot.

Maxwell's corkscrew rule is a convenient method of determining the direction of the magnetic field set up by a current-carrying conductor. It states that if a right-handed corkscrew is placed along the direction of the current flow,

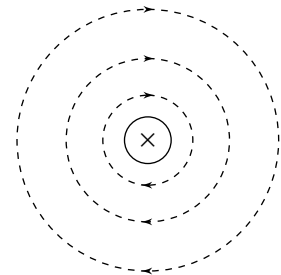


Fig. 1.20 Magnetic field due to current in a straight conductor

the direction of motion of the hand, which would advance the screw in the direction of the current, gives the direction of magnetic field as shown in Fig. 1.21(a).

Another method of determining the direction of the field is to employ the right hand rule. Imagine the current-carrying conductor to be held in the right hand with the thumb pointing in the direction of the current with the fingers wrapped around it. Then the direction of the fingers points in the direction of the magnetic field.

Magnetic field of a solenoid When a current-carrying conductor is given the shape of circular coils of the conductor placed side by side and insulated from one another, it is called a *solenoid* [see Fig. 1.21(b)]. The magnetic field is represented by the dotted lines. If the fingers of the right hand are wrapped around the current-carrying conductor with the fingers pointing in the direction of the current, then the thumb outstretched parallel to the axis of the solenoid points in the direction of the magnetic field inside the solenoid.

If an iron rod is placed inside the solenoid coil, as shown in Fig. 1.21(b), and the coil is connected to a voltage source, the iron rod is magnetized and behaves like a magnet. The magnetic field becomes hundreds of times stronger.

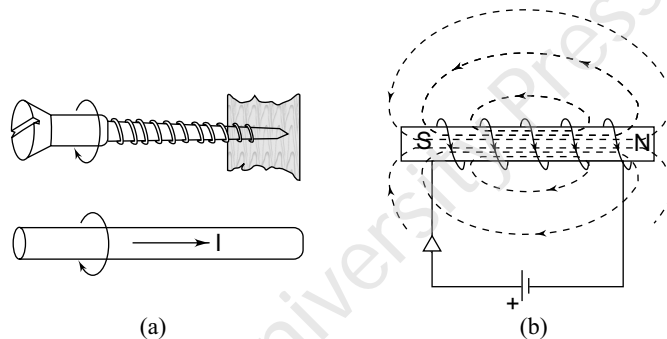


Fig. 1.21 (a) Maxwell's corkscrew rule and (b) solenoid with a magnetic core

1.10.2 Force on a Current-carrying Conductor Placed in a Magnetic Field

If a current-carrying conductor is placed at right angles to the lines of force of a magnetic field, a mechanical force will be exerted on the conductor. The magnitude of the mechanical force can be calculated by using *Ampere's law*.

1.10.2.1 Ampere's Law

When a straight elemental conductor of length l metres and carrying current I_2 amperes is placed in the same horizontal plane at a distance of r metres from a straight, long conductor carrying current I_1 amperes in the opposite direction to I_2 , the small conductor experiences a force of repulsion, F , given by

$$F = \frac{\mu I_1}{2\pi r} I_2 l \text{ newton} \quad (1.42)$$

$$= B I_2 l \text{ newton} \quad (1.43)$$

$$\text{where } B = \frac{\mu I_1}{2\pi r} \text{ tesla (T)} \quad (1.44)$$

In Eq. (1.44) μ is a scalar constant of the medium (called permeability of the medium) and B is the magnetic flux density. From Eq. (1.43) it may be noted that the unit of flux density is taken as the density of the magnetic field such that a conductor carrying 1 A at right angles to that field experiences a force of 1 Nm. The unit is

named tesla (T) after the famous electrical inventor Nikola Tesla. The units of B may also be found as follows:

$$B = \frac{\text{newton}}{\text{ampere} \times \text{metre}} = \frac{\text{joule}}{\text{metre} \times \text{ampere} \times \text{metre}}$$

$$= \frac{\text{watt} \times \text{second}}{\text{ampere} \times \text{metre}^2} = \frac{\text{volt} \times \text{second}}{\text{metre}^2} = \frac{\text{weber}}{\text{metre}^2} \quad \text{or} \quad \text{Wb/m}^2$$

From Faraday's law, it is shown later in Section 1.10.3 that weber = volt \times second.

For a magnetic field having a cross-sectional area $A \text{ m}^2$ and a uniform flux density B tesla, the total flux ϕ is given by

$$\phi = B \times A \quad (1.45)$$

$$\text{and} \quad B = \frac{\phi}{A} \text{ Wb/m}^2 \quad (1.46)$$

Then, for $\phi = 1 \text{ Wb}$,

$$A = 1 \text{ m}^2, \quad B = 1, \quad T = 1 \text{ Wb/m}^2$$

Figure 1.22(a) shows a part of a magnetic field with lines of force in the plane of the paper. Figure 1.22(b) shows a conductor arranged at right angles to the paper and carrying a current whose direction is inwards and produces a magnetic field around the conductor in the plane of the paper. Figure 1.22(c) shows the combined effect where the conductor is situated in the magnetic field of Fig. 1.22(a). On the right hand side of the conductor, the two fields, both due to the permanent magnet and the current-carrying conductor, are in the same direction, whereas on the left hand side they are in opposition. Thus the effect of the current is to transfer some of the lines of force from the left hand side to the right hand side of the conductor, resulting in bending of some of them, as shown. Since the flux lines behave like lastic cords and tend to return to the shortest path, the conductor experiences a mechanical force towards the left.

The magnitude of the force on the conductor, F , in the case of a conductor of length l metres arranged at right angles to the magnetic field B tesla or, Wb/m^2 , and carrying current I is given by Eq. (1.43) as

$$F = BIl \text{ newton} \quad (1.47)$$

The direction of the force can be determined by Fleming's left hand rule, which is illustrated in Fig. 1.23.

The rule states that if the thumb, forefinger, and the middle finger of the left hand are stretched and held at right angles to each other, with the middle finger pointing in the direction of current flow and the forefinger in the direction of the magnetic field, then the thumb will point in the direction of force on the conductor. As an aid to apply Fleming's left hand rule it may be remembered that the forefinger and field associate with each other, the middle finger has an **i** in middle, *i* also represents current, and finally thumb contains an **m** that is associated with motion.

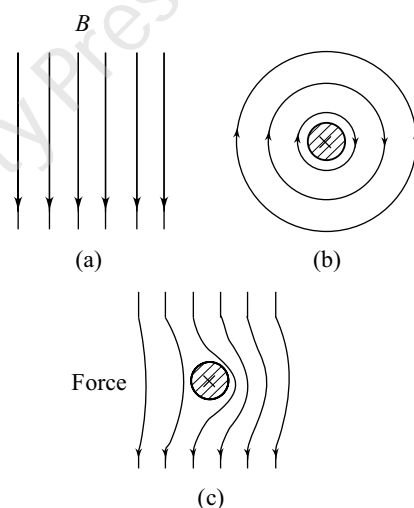


Fig. 1.22 Force on a current-carrying conductor placed in a magnetic field

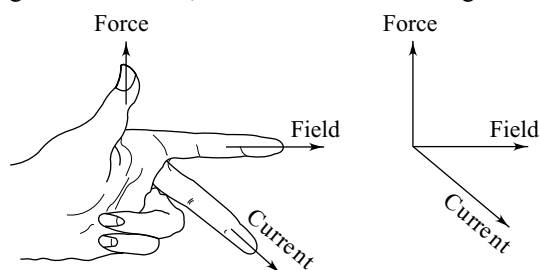


Fig. 1.23 Left-hand rule

1.10.3 Faraday's Laws of Electromagnetic Induction

Michael Faraday in 1831 experimentally demonstrated that a variable magnetic field could produce an electric field. He showed that when a conductor is moved in a stationary magnetic field, an emf is produced between the ends of the conductor; and if the ends of the conductor are joined by a wire, an induced current flows through the conductor. Alternatively, a stationary conductor when placed in a magnetic field that changes with time develops an emf across it. This phenomenon is called *electromagnetic induction*. Based on this, Faraday developed the following laws.

Faraday's first law This law states that whenever the magnetic flux changes with respect to an electrical conductor or a coil, an emf is induced in the conductor.

Faraday's second law This law states that the magnitude of the emf induced in the conductor or the coil by electromagnetic induction is directly proportional to the time rate of change of the flux linkages.

The term flux linkages merely means the product of flux in webers and the number of turns with which the flux is linked. Let the magnetic flux through a coil of N turns be increased by $\Delta\phi$ webers in Δt seconds, then according to Faraday's second law the magnitude of the induced emf, e , in the coil will be given by

$$e \propto \frac{N\Delta\phi}{\Delta t} \propto \frac{d\psi}{dt} \quad (1.48)$$

In the SI system of units, e is given in volts and the constant of proportionality is unity. Hence,

$$e = \frac{d\psi}{dt} = N \frac{d\phi}{dt} \text{ volt} \quad (1.49)$$

It may be noted from Eq. (1.49)

$$\text{volt} = \frac{\text{weber}}{\text{second}} \text{ or } \text{volt} \times \text{second} = \text{weber}$$

The direction of the induced emf can be determined in two ways, namely, (i) Fleming's right hand rule and (ii) Lenz's law.

Fleming's right hand rule To apply the right hand rule, hold the thumb, forefinger, and second (or middle) finger at right angles to each other as shown in Fig. 1.24. If the thumb points in the direction of motion of the conductor and forefinger in the direction of the magnetic field, then the second finger gives the direction of the induced emf (voltage).

Lenz's law German physicist Heinrich Lenz in 1834 enunciated a simple rule, presently known as Lenz's law. The law states that the direction of the induced emf is always such that it tends to establish a current which opposes the change of flux responsible for inducing that emf. In accordance with Lenz's law, a negative sign is assigned to the expression for emf and Eq. (1.49) get modified as

$$e = -N \frac{d\phi}{dt} \quad (1.50)$$

1.10.3.1 Types of Induced emf

There are two different ways of producing emf in accordance with Faraday's laws. First, emf may be produced by a relative movement of a conductor or a coil with respect to the magnetic field. Such an emf is called *dynamically induced emf* (*motional emf*). Second, if the strength of the magnetic field is varied without changing its orientation and a

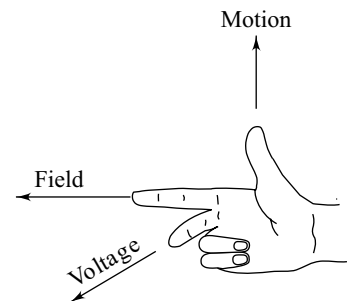


Fig. 1.24 Right-hand rule

conductor or a coil is placed in this field, an emf is induced in the conductor or the coil. This emf is called *statically induced emf* (transformer emf).

Dynamically induced emf in a conductor Figure 1.25 shows an isometric view of a pair of NS poles and the plan of a conductor AA placed in the air gap between the pair of poles. If the conductor length be l metres and current I amperes flows axially into the plane of the paper and B is the flux density in tesla, the conductor experiences a force Bil newtons tending to move the conductor to the left (Lenz's law). Thus a force of this magnitude is to be applied in the opposite direction to move the conductor from position A to position B through distance d metres. Work done, W , in moving the conductor is given by

$$W = Bil \times d \quad \text{Nm} \quad \text{or} \quad \text{J}$$

If the movement of conductor A from position A to B takes place at a uniform velocity in t seconds, so as to cut at right angles the lines of force of the uniform magnetic field of the air gap, then a constant emf, say E volts, is induced in it. Electrical power generated due to the movement of the conductor is $E \times I$ watts and the corresponding energy produced is $E \times I \times t$ W/sec or joules. As the mechanical energy required for moving the conductor horizontally in the air gap is converted into electrical energy, the following equation results:

$$E \times I \times t = Bil \times d$$

$$\text{or} \quad E = Bl \times \frac{d}{t} = Blu \quad (1.51)$$

where u is the velocity in m/sec.

If the conductor velocity u makes an angle θ with the direction of the magnetic field then the emf induced is given by

$$E = Blu \sin \theta \quad (1.52)$$

In Eq. (1.51), $Bl d$ is the total magnetic flux, ϕ webers, in the area shown shaded in Fig. 1.25. The conductor cuts this flux ϕ when it moves from AA to BB in t seconds. Thus

$$E \text{ volts} = \frac{\phi}{t} \quad \text{Wb/sec} \quad (1.53)$$

In general if a conductor cuts a flux of $d\phi$ webers in dt seconds, then the generated emf e volts is given as

$$e = \frac{d\phi}{dt} \text{ V} \quad (1.54)$$

The sign of the emf induced can be determined from the physical considerations. Therefore, the negative sign in Eq. (1.54) is left out.

Magnitude of induced emf in a coil Assume a coil of N turns. The flux in it is increased by $d\phi$ webers in dt seconds by moving a permanent magnet towards the coil. Since there are N turns in the coil, all the turns link this flux. From Eq. (1.49) the emf developed, e , may be written

$$e = N \frac{d\phi}{dt} = \frac{d\psi}{dt} \quad (1.55)$$

ψ is called the flux linkage and is equal to $N\phi$.

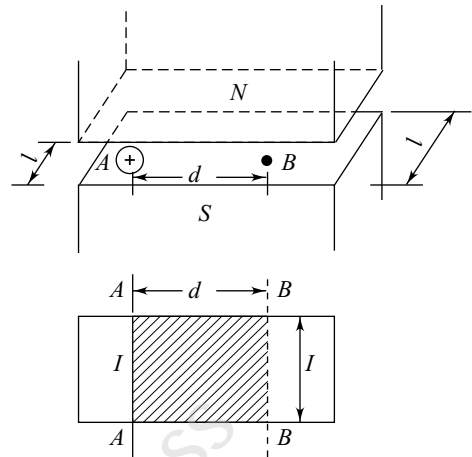


Fig. 1.25 Current-carrying conductor moving across a magnetic field

Example 1.18 A straight conductor 100 cm long and carrying a direct current of 50 A lies perpendicular to a uniform magnetic field of 1.5 Wb/m^2 (tesla). Find (a) the mechanical force on the conductor, (b) the mechanical power in watts to move the conductor against the force at a uniform speed of 5 m/sec, and (c) the electromotive force generated in the conductor.

Solution From Eq. (1.47), $F = BIl$

(a) Force on the conductor is $1.5 \text{ [T]} \times 50 \text{ [A]} \times 1 \text{ [m]} = 75 \text{ N}$

(b) The mechanical power to move the conductor against the force is $F \times u = 75 \text{ [N]} \times 5 \text{ [m/sec]} = 375 \text{ W}$

(c) From Eq. (1.51) $E = B \times l \times u$

The emf generated is $1.5 \text{ [T]} \times 1 \text{ [m]} \times 5 \text{ [m/sec]} = 7.5 \text{ V}$

Example 1.19 A wire of length 50 cm moves in a direction at right angles to its length at 40 m/s in a uniform magnetic field of density 1.5 Wb/m^2 . Calculate the electromotive force induced in the conductor when the direction of motion is (a) perpendicular to the field, (b) inclined at 30° to the direction of the field.

Solution From Eq. (1.52), $E = B \times l \times u \times \sin \theta$

When $\theta = 90^\circ$, force on the conductor is $1.5 \text{ [T]} \times 0.5 \text{ [m]} \times 40 \text{ [m/sec]} \times \sin 90^\circ = 30 \text{ V}$

When $\theta = 30^\circ$, force on the conductor is $1.5 \text{ [T]} \times 0.5 \text{ [m]} \times 40 \text{ [m/sec]} \times \sin 30^\circ = 15 \text{ V}$

1.11 KIRCHHOFF'S LAWS

Gustav Robert Kirchhoff (1824–1887), a German physicist, published the first systematic description of the laws of circuit analysis. These laws are known as *Kirchhoff's current law (KCL)* and *Kirchhoff's voltage law (KVL)*. His contribution forms the basis of all circuit analyses problems.

Kirchhoff's current law states that the algebraic sum of the currents at a node (junction) in a network at any instant of time is zero. KCL may be expressed mathematically as

$$\sum_{j=1}^n i_j = 0 \quad (1.56)$$

where i_j represents current in the j th element and n is the number of elements connected to the node k . This means that the algebraic sum of the currents meeting at a junction is zero. If the currents entering the node are taken as positive, then the currents leaving the node are negative, or vice versa. The KCL may be thought of to be a consequence of the conservation of electric charge—charge cannot be created nor destroyed but must be conserved.

As an example of KCL, consider the node k in Fig. 1.26 where currents i_1 , i_2 , i_3 , i_4 , and i_5 flowing in the five branches meet. For node k , KCL may be written in the form

$$-i_1 + i_2 - i_3 + i_4 - i_5 = 0 \quad (1.57)$$

$$\text{or} \quad i_2 + i_4 = i_1 + i_3 + i_5 \quad (1.58)$$

In Eq. (1.58) currents i_2 and i_4 are flowing towards node k and hence a positive sign is assigned to these currents while the currents i_1 , i_3 , and i_5 , which leave node k , are negative.

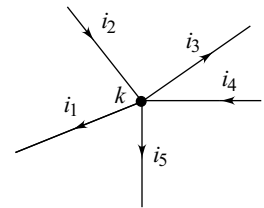


Fig. 1.26 Applications of KCL

Example 1.20 For the circuit shown in Fig. 1.27 determine the value of i_5 for the following values of voltages: $v_1 = 3 \sin t$, $v_2 = 10 \sin t$, $v_3 = 10 \cos t$, $i_4 = \cos t$.

Solution From Eq. (1.37)

$$i_1 = \frac{1}{3} \frac{d}{dt} (3 \sin t) = \cos t$$

Applying Ohm's law,

$$i_2 = \frac{10 \sin t}{5} = 2 \sin t$$

From Eq. (1.31)

$$10 \cos t = 2 \frac{di_3}{dt}$$

$$\therefore i_3 = 5 \int \cos t \, dt = 5 \sin t$$

Applying KCL to the node

$$-i_1 + i_2 - i_3 - i_4 + i_5 = 0$$

$$\begin{aligned} \therefore i_5 &= i_1 - i_2 + i_3 + i_4 \\ &= \cos t - 2 \sin t + 5 \sin t + \cos t \\ &= 3 \sin t + 2 \cos t \end{aligned}$$

If $3 = K \cos \phi$ and $2 = K \sin \phi$, then $K = 5$ and $\phi = \tan^{-1}(2/3)$.

Thus, $i_5 = 5 \sin [t + \tan^{-1}(2/3)]$.

Kirchhoff's voltage law states that at any instant of time the sum of voltages in a closed circuit is zero. KVL may be expressed mathematically as

$$\sum_{j=1}^n v_j = 0 \quad (1.59)$$

where v_j represents the individual voltage in the j th element around the closed circuit having n elements.

If the voltage drop from the positive polarity to the negative polarity is assigned a positive sign, then the voltage rise from the negative polarity to the positive polarity is assumed negative, or vice versa. KVL is a consequence of the fact that no energy is lost or created in an electric circuit. In other words, KVL states that in a closed loop, at any instant of time, the algebraic sum of the emfs acting around the loop is equal to the algebraic sum of the pds around the loop.

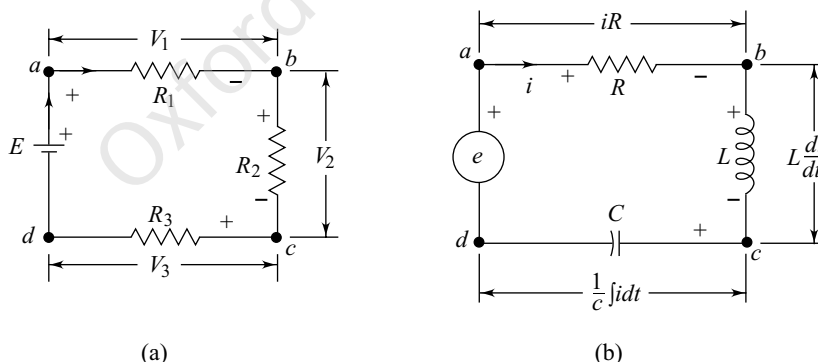


Fig. 1.28 Application of KVL

For the closed loop shown in Fig. 1.28(a) the dc source (battery) causes a constant current flow in the loop. Applying KVL, using the sign convention for the positive direction of voltage drop and emf, the following expression can be written:

$$E = V_1 + V_2 + V_3$$

$$\text{or} \quad V_1 + V_2 + V_3 - E = 0 \quad (1.60)$$

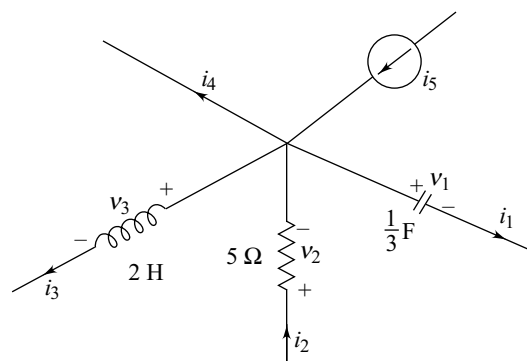


Fig. 1.27

KVL applies equally well when the source voltage is time-varying, causing a time-varying current flow. For the closed loop shown in Fig. 1.28(b) a time-varying voltage source e produces a time-varying current i in the closed loop. Then, by KVL the following expression can be written:

$$e = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt \quad (1.61)$$

Example 1.21 For the closed circuit shown in Fig. 1.29 determine i_3 for the following data: $v_1 = 5 \sin t$, $i_2 = 2 \cos t$, $v_4 = 4 \cos t$, and $i_5 = \sin t$.

Solution The voltage across the capacitor is

$$v_2 = \frac{1}{C} \int i_2 dt = 2 \int 2 \cos t dt = 4 \sin t,$$

and $v_5 = R \times i_5 = 4 \sin t$

Applying KVL around the closed circuit shown in Fig. 1.29,

$$-v_1 + v_2 - v_3 - v_4 + v_5 = 0$$

$$\text{or } v_3 = -v_1 + v_2 - v_4 + v_5 = -5 \sin t + 4 \sin t - 4 \cos t + 4 \sin t = 3 \sin t - 4 \cos t$$

$$\text{Now, } v_3 = L \frac{di_3}{dt} = \frac{1}{3} \frac{di_3}{dt}$$

$$i_3 = 3 \int (3 \sin t - 4 \cos t) dt = -9 \cos t - 12 \sin t = -(9 \cos t + 12 \sin t)$$

Suppose $12 = K \cos \phi$ and $9 = K \sin \phi$. Then $K = 15$ and $\phi = \tan^{-1}(3/4)$. Therefore,

$$i_3 = -15(\sin \phi \cos t + \cos \phi \sin t) = -15 \sin(t + \phi)$$

Example 1.22 For the circuit shown in Fig. 1.30 determine the values of I_2 and V_S .

Solution Let the node C be taken as the reference node in Fig. 1.30. Applying KCL to node B ,

$$I_3 + I_4 = 6 - 4 = 2 \text{ A}$$

Now, $I_3 = I_4 = 1 \text{ A}$, being current through two 2Ω resistances in parallel across nodes A and B . Then the potential of node B with respect to node C is

$$V_B = 2 \times 4 = 8 \text{ V}$$

The voltage drop across nodes B and A ,

$$V_{BA} = I_4 \times 2 = 1 \times 2 = 2 \text{ V}$$

Then the potential of node A with respect to node C is

$$V_A = V_B - V_{BA} = 8 - 2 = 6 \text{ V}$$

$$\text{Therefore, } I_2 = \frac{V_A}{2} = \frac{6}{2} = 3 \text{ A}$$

Applying KCL to node A ,

$$I_S + I_3 + I_4 - I_2 = 0$$

$$\therefore I_S = 3 - 1 - 1 = 1 \text{ A}$$

Then, $V_S = V_A + 2 \times I_S = 6 + 2 \times 1 = 8 \text{ V}$.

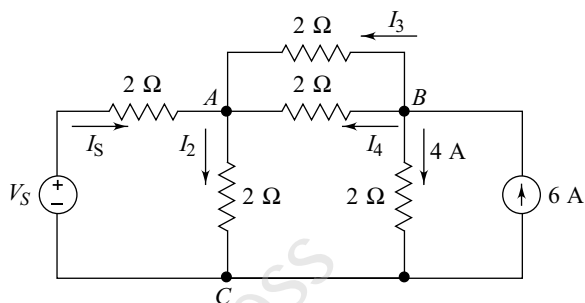


Fig. 1.30

Recapitulation

Charge on an electron, $e = -1.602 \times 10^{-19}$ C

Mass of an electron $= 9.11 \times 10^{-31}$ kg

Charge on a proton $= 1.602 \times 10^{-19}$ C

Mass of a proton $= 1.673 \times 10^{-27}$ kg

Coulomb's law: $F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$ newton (or N)

Electric field intensity, $E = \frac{F}{q}$ N/C or V/m

Voltage, $v = \frac{dw}{dq}$ J/C

Electric flux density, $D = \frac{q}{4\pi r^2} = \epsilon E$

Gauss' law: $\oint D \, ds = \sum q$

Current, $I = \frac{Q}{t}$ C/sec or A

Quantity of electricity, $Q = I \times t$ C

Potential difference $V = \frac{P}{I} = \frac{W}{Q}$

Ohm's law: $I = \frac{V}{R}$

Resistance of a conductor, $R = \frac{\rho l}{a}$ Ω

Resistivity $\rho = \frac{R \times a}{l}$ Ω m

Resistance of a conductor at $(t_1)^\circ\text{C}$, $R_1 = R_0 (1 + \alpha_0 t_1)$

Temperature coefficient of resistance at t_1 , $\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$

Temperature coefficient of resistivity, $\rho_1 = \rho_0 (1 + \alpha_0 t_1)$

Conductance of a conductor, $G = \frac{\sigma \times a}{l}$

Capacitance $C = \frac{q}{V}$ farad

Energy stored in a capacitor, $W_C = \frac{1}{2} CV^2$

Voltage induced in an inductor, $v = L \frac{di}{dt}$ V

Inductance of an inductor, $L = \frac{v}{di/dt}$ H

Energy stored in an inductor, $W_L = \frac{1}{2} Li^2$ J

Flux density, $B = \frac{\phi}{A}$ tesla or Wb/m²

Instantaneous value of induced voltage,

$$e = -\frac{d(N\phi)}{dt} = -N \frac{d\phi}{dt} = -N \frac{d\phi}{dt} \text{ V}$$

Assessment Questions

1. Write a short essay on the fundamental nature of electricity.
2. Describe the atomic structure and therefrom distinguish between (a) conductors, (b) semi-conductors, and (c) insulators.
3. Define Coulomb's law and electric field intensity.
4. Distinguish between (a) electric potential and potential difference and (b) electric flux and electric flux density.
5. Define Gauss' law and describe the electric field set up due to a long straight charged conductor.
6. Derive expressions for electric fields set up between (a) two charged parallel plates and (b) a uniformly charged sphere.
7. Describe the nature of current and express the co-relation between charge and current.
8. Explain (a) emf and (b) electric power. What is passive sign convention?
9. State and explain Ohm's law.
10. Enumerate the basic circuit elements and briefly describe their properties.
11. Specify the parameters that govern the resistance of a conductor. Distinguish between linear and non-linear resistors.
12. Explain what is meant by the temperature coefficient of resistance of a material.
13. State how the physical parameters of a capacitor are related to its capacitance and discuss the significance of the permittivity of the dielectric.
14. State and explain Ampere's law.
15. Define inductance and derive an expression for the energy stored in the inductor. State how the direction of induced emf is determined.

16. Define Kirchhoff's laws. What is the basis of these laws?
17. Apply Kirchhoff's laws and develop voltage and current equations for a hypothetical resistive circuit.

Problems

- 1.1 Plot the variation of charge when the electric current varies as shown in Fig. 1.31.
- 1.2 A pd of 1.5 V causes a current of 270 μA to flow in a conductor. Calculate the resistance of the conductor. [5.56 k Ω]
- 1.3 What is the voltage across an electric heater of resistance 5 Ω through which passes a current of 46 A? [230 V]

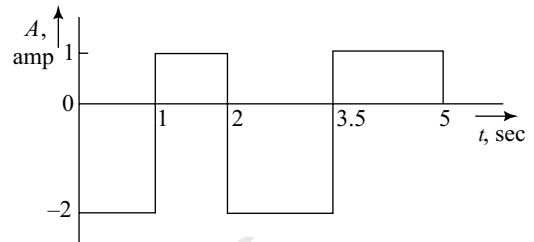


Fig. 1.31

- 1.4 Calculate the current in a circuit due to a pd of 20 V applied to a 20 k Ω resistor. If the supply voltage is doubled while the circuit resistance is trebled, what is the new current in the circuit? [1 mA, 0.67 mA]
- 1.5 A pd of 12 V is applied to a 4.7 k Ω resistor. Calculate the circuit current. [2.55 mA]
- 1.6 A current in a circuit is due to a pd of 20 V applied to a resistor of resistance 200 Ω . What resistance would permit the same current to flow if the supply voltage were 200 V? [2 k Ω]
- 1.7 A pd of 12 V is applied to a 7.5 Ω resistor for a period of 10 sec. Calculate the electric charge transferred during this time. [16 C]
- 1.8 What is the charge transferred in a period of 8 sec by current flowing at the rate of 3.5 A? [28 C]
- 1.9 For the assumed directions of current flows and voltages of the elements shown in Fig. 1.32. State whether the element is absorbing or dissipating power.
- 1.10 A dc motor connected to a 230 V supply, developing 20 kW at a speed of 1000 rpm, has an efficiency of 0.85. Calculate (a) the current and (b) cost of energy absorbed if the load is maintained constant for 12 h. Assume the cost of electrical energy to be Rs 2.50 per kWh. [(a) 102.3 A, (b) Rs 705.90]
- 1.11 An electric motor runs at 600 rpm when driving a load requiring a torque of 400 N m. If the motor input is 30 kW, calculate the efficiency of the motor and the heat lost per minute by the motor. Assume its temperature to remain constant. [83.8%, 292.8 kJ]
- 1.12 A conductor of length l and radius r has a resistance of $R\Omega$. If the volume of the conductor is V , show that

$$(i) \ r = \sqrt[4]{\frac{\rho V}{\pi^2 R}} \quad \text{and} \quad (ii) \ l = \sqrt{\frac{VR}{\rho}}$$

Assume that the resistivity of the conductor is ρ .

- 1.13 A voltage of 440 V is applied across a parallel plate liquid resistor. If the resistor absorbs 50 kW, calculate the distance between the plates. Assume a resistivity of 25 $\Omega\text{-cm}$ for the liquid and a current density of 0.30 A/cm². [58.67 cm]
- 1.14 A conductor has a resistance of R_1 ohms at $t_1^\circ\text{C}$ and is made of copper with a resistance-temperature coefficient α referred to 0°C . Find an expression for the resistance R_2 of the conductor at temperature $t_2^\circ\text{C}$.
- 1.15 The resistance temperature coefficients of two conductors A and B, at a temperature of $t^\circ\text{C}$ are α_A and α_B respectively. The resistors are connected in series such that their resistances are in the ratio of $(R_A/R_B) = a$.

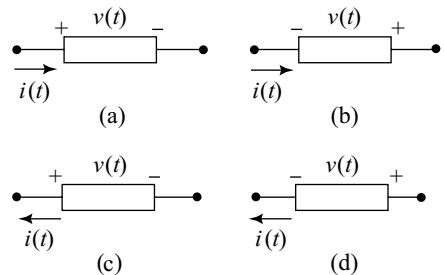


Fig. 1.32

- (a) Derive an expression for the resistance temperature coefficient α and the temperature t for the circuit. (b) If $a = 4$, $\alpha_A = 0.003/^\circ\text{C}$, and $\alpha_B = 0.0003/^\circ\text{C}$, determine the value of α and the temperature t . (c) What is the ratio of the resistors when $\alpha = 0.005/^\circ\text{C}$, $\alpha_A = 0.0004/^\circ\text{C}$, and $\alpha_B = 0.0025/^\circ\text{C}$?

$$\left[(a) \frac{a\alpha_A - \alpha_B}{(a-1)}, \frac{1}{\alpha - (\alpha_A + \alpha_B)}, (b) 1666.7^\circ\text{C}, (c) 1.84 \right]$$

- 1.16 The field coil of a motor has a resistance of $500\ \Omega$ at 15°C . By how much will the resistance increase if the motor attains an average temperature of 45°C when it is running? Take $\alpha = 0.00428$ per $^\circ\text{C}$ referred to 0°C . [60.33 Ω]
- 1.17 A copper rod, 0.6 m long and 4 mm in diameter, has a resistance of $825\ \mu\Omega$ at 20°C . Calculate the resistivity of copper at that temperature. If the rod is drawn out into a wire having a uniform diameter of 0.8 mm, calculate the resistance of the wire when its temperature is 60°C . Assume the resistivity to be unchanged and the temperature coefficient of resistance of copper to be 0.00426 per $^\circ\text{C}$. [0.01727 $\mu\Omega/\text{m}$, 0.6035 Ω]
- 1.18 A coil of insulated copper wire has a resistance of $160\ \Omega$ at 20°C . When the coil is connected to a 240 V supply, the current after several hours is 1.35 A. Calculate the average temperature throughout the coil, assuming the temperature coefficient of resistance of copper at 20°C to be 0.0039 per $^\circ\text{C}$. [48.5 $^\circ\text{C}$]
- 1.19 The voltage waveform shown in Fig. 1.33 is applied across a parallel combination of a capacitor of 0.4 F and a resistor of $4\ \Omega$. Plot the waveforms of the currents through the capacitor, resistor, and the total current and determine the (a) energy dissipated in the resistor, (b) maximum energy stored in the capacitor, (c) energy supplied by the source, (d) total charge flow through the resistor, and (e) average resistor voltage.

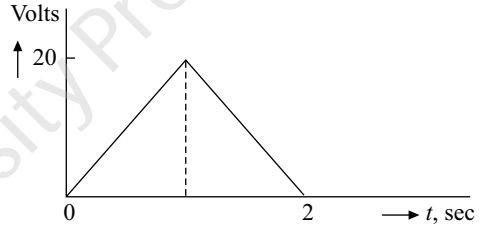


Fig. 1.33

[(a) 66.67 J, (b) 80 J, (c) 66.67 J, (d) 5 C, (e) 13.33 V]

- 1.20 The distance between the plates of a capacitor is 6 mm and its dielectric material has a relative permittivity of 3. Another sheet of dielectric material of relative permittivity ϵ_r and thickness 9 mm is inserted by moving the plates apart. If the capacitance of the composite capacitor is half of the original capacitor, determine the value of ϵ_r . [4.5]
- 1.21 A voltage of 25 kV is applied to a parallel plate capacitor whose capacitance is $2.5 \times 10^{-4}\ \mu\text{F}$. If the area of each plate is $110\ \text{cm}^2$ and the plates are separated by a dielectric material of thickness 3 mm, calculate the (a) total charge in coulombs, (b) per sq m charge density, (c) relative permeability of the dielectric, and (d) potential gradient. [(a) 6.25 μC , (b) 568.18 $\mu\text{C}/\text{m}^2$, (c) 7.7, (d) 83.33 kV/cm]
- 1.22 A parallel plate condenser has an area of $A\ \text{cm}^2$ and the distance between the plates is $d\ \text{mm}$. If the relative permittivity of the dielectric material is ϵ_r , determine how the energy stored in the capacitor will vary with each factor, when a voltage of V volts is applied across it.

$$\left[W_C \propto \frac{AV^2}{d} \right]$$

- 1.23 A conductor of $l\ \text{m}$ is carrying a current of $I\ \text{A}$ (whose direction is perpendicular to and coming out of the plane of the paper) and is placed at right angles to a magnetic field of magnitude $B\ \text{T}$. If the magnetic lines of force are in the plane of the paper and have a direction from top to bottom, determine the magnitude and direction of the force experienced by the conductor. How does the direction of the force change if (i) the direction of the conductor current is reversed, (ii) the direction of the magnetic field only is reversed, and (iii) the direction of the current flow and that of the magnetic field both are reversed? State the law used to determine the direction of the force.
- 1.24 A current-carrying conductor is situated at right angles to a uniform magnetic field having a density of 0.4 T. (a) Calculate the force (in N m length) on the conductor when the current is 100 A. (b) Calculate the current in the conductor when the force per metre length of the conductor is 25 N. [(a) 40 N m, (b) 62.5 A]

- 1.25 The voltage across an inductor of 5 H varies as shown in Fig. 1.34. Determine the inductor current and energy at $t = 1, 2, 3$, and 5 sec. Sketch the plot illustrating variation of current with time.

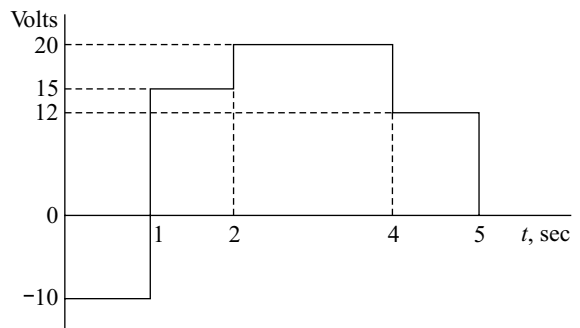


Fig. 1.34

- 1.26 The coil of a moving-coil loudspeaker has a mean diameter of 40 mm and is wound with 1000 turns. It is situated in a radial magnetic field of 0.4 T. Calculate the force on the coil, in newtons, when the current is 10 mA. [0.5024 N]

- 1.27 A square coil of side l cm and T number of turns revolves about its axis at right angles inside a magnetic field of density B Wb/m². If the speed of the coil is N rpm, derive an expression for the instantaneous value of the induced emf. If $l = 15$ cm, $B = 0.5$ Wb/m², and $N = 1200$ rpm, determine (a) maximum and (b) minimum values of the induced emf. (c) What are the respective angles made by the plane of the coil with the magnetic field? (d) Calculate the angle made by the plane of the coil with the magnetic field when the instantaneous value of the induced emf is 185 V. [(a) 212.06 V, 90°, (b) 0 V, 0°, (c) 60.74°]

- 1.28 A conductor, 750 mm long, is moved at a uniform speed at right angles to its length and in a uniform magnetic field having a density of 0.4 T. If the generated emf in the conductor is 3 V and the conductor forms part of a closed circuit having a resistance of 0.5 Ω , calculate: (a) the velocity of the conductor in m/sec, (b) the force acting on the conductor in newtons, (c) the work done in joules when the conductor has moved 500 mm. [(a) 10 m/sec, (b) 1.8 N, (c) 0.9 J]

- 1.29 In a coil of 120 turns, the flux is varying with time as shown in Fig. 1.35. If $\phi_m = 0.025$ Wb and $T = 0.04$ sec, determine the value of the statically induced emf. Sketch to scale the waveform of the induced voltage.

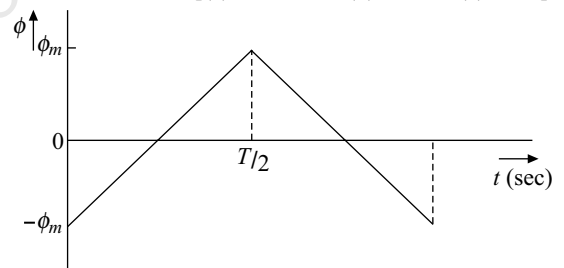


Fig. 1.35

- 1.30 The axle of a certain motorcar is 1.6 m long. Calculate the generated emf in the car when it is travelling at 120 km/h. Assume the vertical component of the earth's magnetic field to be 40 μ T. [2.13 mV]

- 1.31 A coil of 2500 turns gives rise to a magnetic flux of 5 mWb when carrying a certain current. If this current is reversed in 0.2 sec, what is the average value of the emf induced in coil? [125 V]

- 1.32 A short coil of 500 turns surrounds the middle of a bar magnet. If the magnet sets up a flux of 60 μ Wb, calculate the average value of the emf induced in the coil when the latter is removed completely from the influence of the magnet in 0.04 sec. [0.75 V]

- 1.33 For the circuit shown in Fig. 1.36 determine the value of the source current I_S for the following operating conditions:

- (a) the source voltage $V_S = 12$ V, and $I_{AB} = 0$
(b) the source voltage $V_S = 15$ V, and $I_{AB} = 3$ A

[(a) 4 A, (b) -2 A]

- 1.34 In the circuit shown in Fig. 1.37, $v(t) = 3e^{-t}$. Use Kirchhoff's laws and the volt-ampere relations for the elements to determine the source current $i_S(t)$. [4.8 e^{-t}]

- 1.35 Repeat Problem 1.34 for $v(t) = 4 \sin t$. [3.2 $\sin t + 9.6 \cos t$]

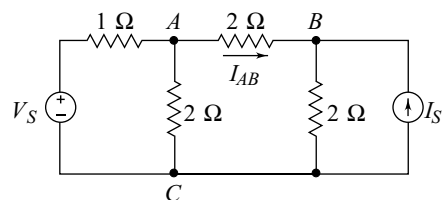


Fig. 1.36

- 1.36 In the circuit shown in Fig. 1.37, $i(t) = -15e^{-2t}$. Use Kirchhoff's laws and the volt-ampere relations for the elements to determine the source voltage $v_S(t)$. [$22e^{-t}$]
- 1.37 Repeat Problem 1.36 for $i(t) = 20 \sin t$. [$40.67 \sin t + 43.33 \cos t$]

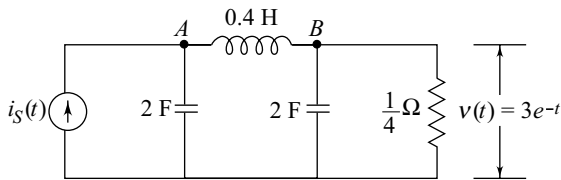


Fig. 1.37

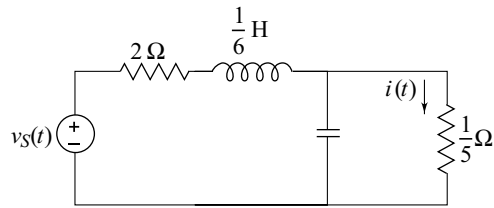


Fig. 1.38

Objective Type Questions

- In Bohr's model of atomic structure, the nucleus consists of
 - electrons
 - electrons and protons
 - protons
 - protons and neutrons
- If 1 A current flows in a circuit, the number of electrons flowing in a circuit is
 - 0.625×10^{19}
 - 1.6×10^{19}
 - 1.6×10^{-19}
 - 0.625×10^{-19}
- The resistivity of a conductor depends on the
 - area of the conductor
 - length of the conductor
 - type of material
 - none of these
- Current flowing in a series circuit having four equal resistances is I amperes. What is the magnitude of the current if the four resistances are connected in parallel?
 - $0.25I$
 - I
 - $4I$
 - $8I$
- How many coulombs of charge flow through a circuit carrying a current of 10 A in 1 min?
 - 10
 - 60
 - 600
 - 1200
- Two parallel plates separated by a distance d are charged to V volts. The field intensity E is given by
 - $V \times d$
 - V/d
 - $V \times d^2$
 - V^2/d
- A capacitor carries a charge of 0.15 C at 10 V. Its capacitance is
 - 0.015 F
 - 1.5 F
 - 1.5 μ F
 - none of these
- Four capacitors each of 20 μ F are connected in parallel, the total capacitance is
 - 80 μ F
 - 5 μ F
 - 16 μ F
 - none of these
- One farad is equal to
 - 1 Ω
 - 1 V/C
 - 1 C/V
 - none of these
- Point A has an absolute potential of 20 V and point B is at an absolute potential of -5 V. V_{BA} has a value of
 - -25 V
 - 15 V
 - 25 V
 - none of these
- The unit of resistivity is
 - Ω
 - Ω/m
 - Ω/m^2
 - $\Omega \text{ m}$
- Two resistors connected in parallel across a battery of 1 V draw a current of 1 A. When one of the resistors is disconnected, the current drawn is 0.2 A. The resistance of the disconnected resistor is
 - 1 Ω
 - 1.25 Ω
 - 5 Ω
 - none of these
- The effect of temperature on metals and insulating materials is that the
 - resistance of both increases
 - resistance of both decreases
 - resistance of metals decreases and that of insulating material increases
 - resistance of metals increases and that of insulating materials decreases
- Two resistors each of 100 Ω are rated at 100 W and 0.25 W. Which has a higher current rating?
 - 100 W
 - 0.25 W
 - both have the same rating
 - none of these
- The unit of inductance is henry. It is represented by
 - V/A
 - V sec/A

- (iii) V s (iv) V/sec
16. Instantaneous power in an inductor is proportional to the
 (i) product of instantaneous current and rate of change of current
 (ii) square of instantaneous current
 (iii) the induced voltage
 (iv) none of these
17. The voltage induced in an inductor of L henry is represented by
 (i) Li (ii) $\frac{L}{i}$
 (iii) $L \frac{di}{dt}$ (iv) none of these
18. Absolute permittivity of a dielectric medium is represented by
 (i) $\frac{\epsilon_0}{\epsilon_r}$ (ii) $\frac{\epsilon_r}{\epsilon_0}$
 (iii) $\epsilon_0 \epsilon_r$ (iv) none of these
19. A parallel plate capacitor has a capacitance of C farads. If one of the sides of the plates is doubled and the distance between them is halved, the capacitance of the capacitor is
 (i) $0.5C$ F (ii) C F
 (iii) $2C$ F (iv) $4C$ F
20. Which of these is not an expression for the energy stored in a capacitor?
 (i) $\frac{1}{2} CV^2$ (ii) $C \int v dv$
- (iii) $\int p dt$ (iv) QV^2
21. Magnetic flux has the unit of
 (i) newton (ii) ampere turns
 (iii) coulomb (iv) weber
22. A 1-m-long conductor carries a current of 50 A at right angles to a magnetic field of 100×10^{-3} T. The force on the conductor is
 (i) 5000 N (ii) 500 N
 (iii) 50 N (iv) 5 N
23. Whenever the magnetic flux changes with respect to an electrical conductor or a coil, an emf is induced in the conductor is Faraday's
 (i) first law (ii) second law
 (iii) third law (iv) none of these
24. A conductor of length l m is moving at right angles to a magnetic field of constant magnitude at a velocity of v m/sec. The magnitude of the induced emf is proportional to
 (i) $l \times v$ (ii) l/v
 (iii) v/l (iv) none of these
25. A coil wound around a magnetic ring is required to produce a flux of 800×10^{-6} Wb. What is the magnitude of the mmf required to set up the flux if the reluctance of the ring is 1.675×10^{-6} AT / Wb?
 (i) 13.4 AT (ii) 134 AT
 (iii) 1340 AT (iv) 1.34×10^6

Answers

- | | | | | | | |
|----------|----------|----------|-----------|----------|----------|----------|
| 1. (iv) | 2. (i) | 3. (iii) | 4. (iii) | 5. (iii) | 6. (ii) | 7. (i) |
| 8. (i) | 9. (iii) | 10. (i) | 11. (iv) | 12. (ii) | 13. (iv) | 14. (i) |
| 15. (ii) | 16. (i) | 17. (iv) | 18. (iii) | 19. (iv) | 20. (iv) | 21. (iv) |
| 22. (iv) | 23. (i) | 24. (i) | 25. (iii) | | | |